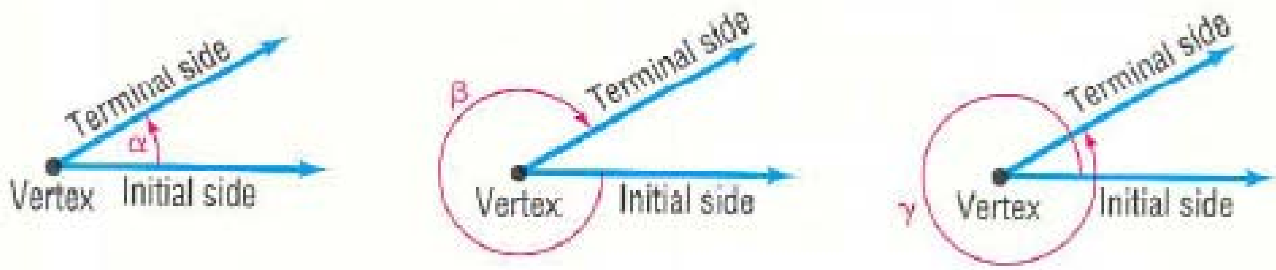
WHAT, EXACTLY, IS AN ANGLE?

Angle Vocabulary



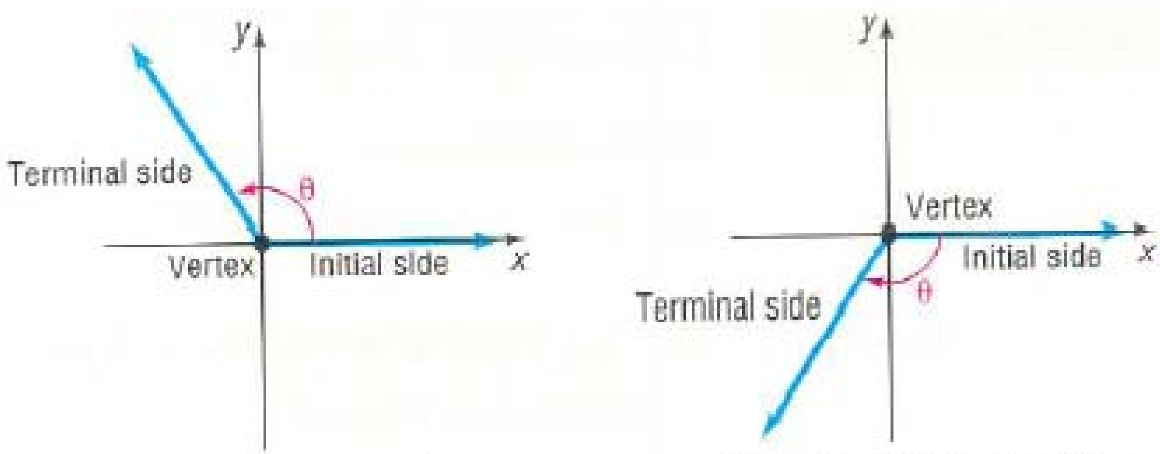
Counterclockwise rotation
Positive angle
(a)

Clockwise rotation
Negative angle
(b)

Counterclockwise rotation
Positive angle
(c)

An angle θ is said to be in **standard position** if its vertex is at the origin of a rectangular coordinate system and its initial side coincides with the positive x-axis. See Figure 3.

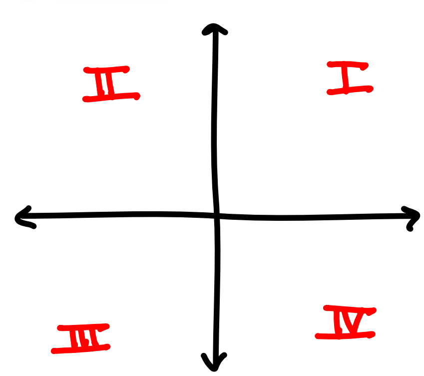
Figure 3



(a) θ is in standard position; θ is positive

(b) θ is in standard position; θ is negative

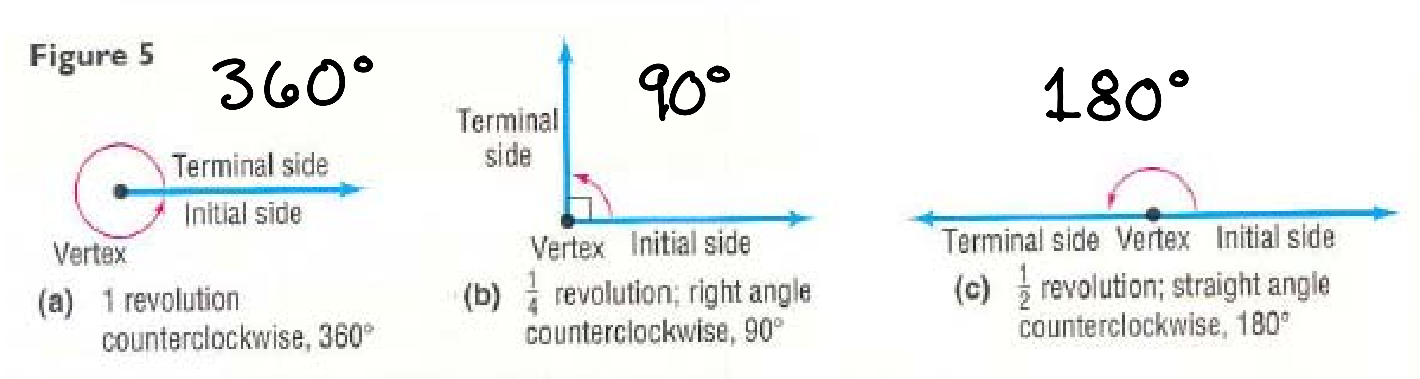
The Four Quadrants:



Degrees

If you rotate the initial side of an angle counter-clockwise until it meets itself, the angle formed is 360 degrees.

- $1^\circ = \frac{1}{360}$ revolutions
- A RIGHT ANGLE HAS 90 degrees.
- A STRAIGHT ANGLE HAS 180 degrees.



UNIT CONVERSION AND THE "SMART ONE"

$1 = \frac{2}{2} = \frac{7}{7}$

$1 = \frac{100¢}{1\$} = \frac{1\$}{100¢}$

$1^\circ = 60$ minutes denoted $60'$

$1' = 60$ seconds denoted $60''$

$1 = \frac{360^\circ}{1 \text{ rev}}$ $\quad 1 = \frac{1^\circ}{60'}$ $\quad 1 = \frac{1'}{60''}$

EXAMPLE 2 **Converting between Degrees, Minutes, Seconds, and Decimal Forms by Hand**

ANSWERS

$\approx 50° + 0.1° + 0.005833°$
$= 50.105833°$

$= 21° + 15.36'$
$= 21° + 15' + 21.6''$
$\approx 21°15'22''$

(a) Convert $50°6'21''$ to a decimal in degrees.

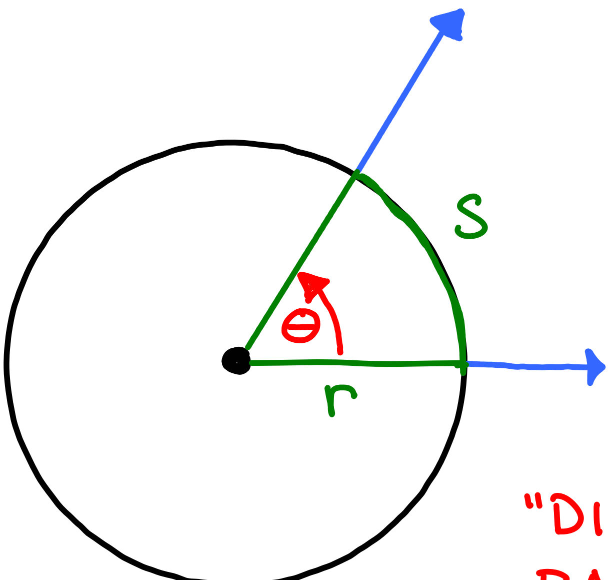
(b) Convert $21.256°$ to the D°M'S'' form.

Radians

A **central angle** is an angle whose vertex is at the center of a circle. The rays of a central angle subtend (intersect) an arc on the circle.

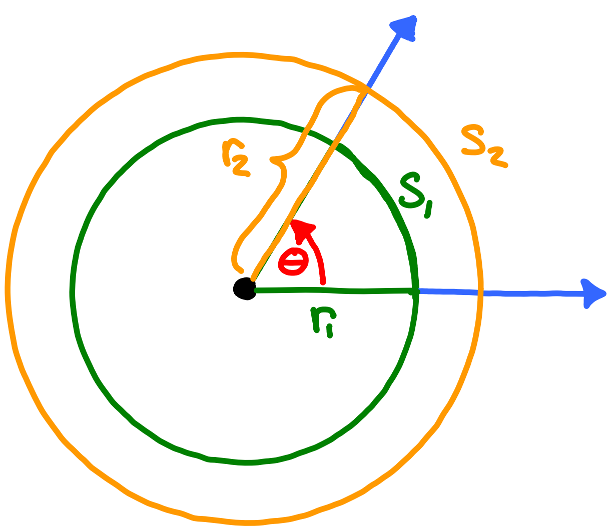
THE MEASURE OF AN ANGLE IS INDEPENDENT OF THE SIZE OF ANY CIRCLE DRAWN AROUND IT.

DRAW A CIRCLE OF ANY RADIUS r AROUND AN ANGLE. WE DEFINE THE MEASURE OF AN ANGLE TO BE THE RATIO OF THE ARCLENGTH (s) SUBTENDED BY THE ANGLE TO THE RADIUS OF THE CIRCLE



$$\boxed{\theta := \frac{s}{r}}$$

S AND r HAVE UNITS OF LENGTH. ALTHOUGH THEIR RATIO IS A PURE NUMBER, WE CAN REFER TO THIS RATIO AS HAVING "DIMENSION-LESS UNITS" CALLED RADIANS.



$$\theta = \frac{s_1}{r_1} = \frac{s_2}{r_2}$$

WHY USE THE "UNIT" CIRCLE?

SINCE THE MEASURE OF AN ANGLE DOESN'T DEPEND ON THE SIZE OF THE CIRCLE DRAWN AROUND IT, WE LIKE TO CHOOSE THE EASIEST CIRCLE TO DEAL WITH.

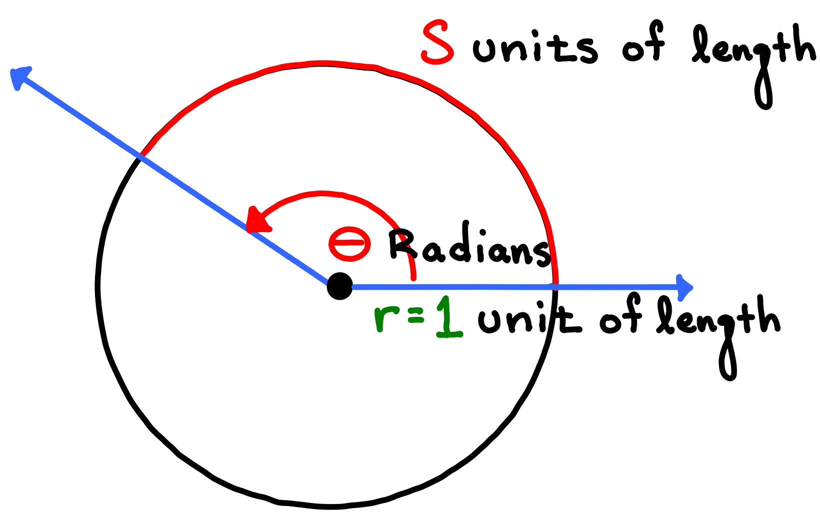
THE UNIT CIRCLE HAS RADIUS $R=1$. THUS

$$R=1 \Rightarrow \theta = \frac{S}{R} = \frac{S \text{ units of length}}{1 \text{ unit of length}} = S = \theta$$

THUS,

$$\boxed{R=1 \iff \theta = S}$$

THIS MEANS WE CAN "SEE" AN ARCLENGTH WHOSE LENGTH (in units of length) HAS THE SAME VALUE AS THE ANGLE (in units of radians)



S is the same number as θ when $r=1$

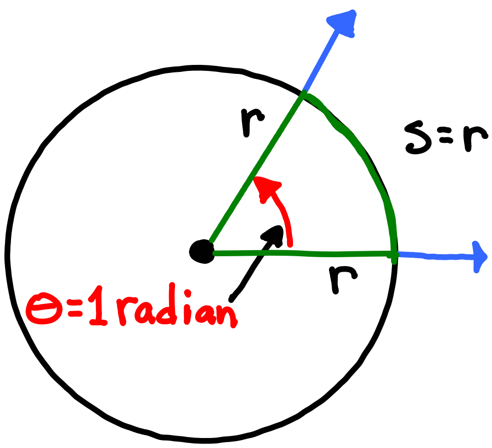
How Big is an Angle of size 1 Radian?

When does $1 = \theta := \frac{s}{r}$?

$$s = r \iff \theta = 1 \text{ radian}$$

If the radius of the circle is r and the length of the arc subtended by the central angle is also r, then the measure of the angle is **1**

$\pi = 3.1415\ldots$ is a number called "Pi".



There happens to be 2π radians in a revolution.

The circumference of a circle, C, is the arclength subtended by an angle equal to 1 rev = 360° = 2π radians. In other words,

$$s = C \iff \theta = 2\pi \text{ radians}$$

Also,

$$\theta \text{ radians} := s/r \iff s = r\theta$$

Therefore, $C = 2\pi r$ or $\pi = C/2r = C/D$

where $D := 2r$, the diagonal of the circle. Thus,

$$\boxed{\pi = 3.1415\ldots = C/D}$$

"π is the ratio of the circumference to the diameter of a circle."

Changing Units

$2\pi \text{ radians} = 360°$

$\Leftrightarrow \quad \pi \text{ radians} = 180°$

$\Leftrightarrow \quad 1° = \frac{\pi}{180} \text{ radians}$

$\Leftrightarrow \quad 1 \text{ radian} = \left(\frac{180}{\pi}\right)°$

Examples: Degrees to Radians

$30° =$

$45° =$

$60° =$

$36° =$

Why might Radians Be More Useful Than Degrees?

Count revolutions around a circle:

2π, 4π, 6π Radians for 1, 2, and 3 revolutions

OR

$360°$, $720°$, $1080°$ for 1, 2, and 3 revolutions

Which is easier?

There are many more reasons, such as

Our Formulas Work With Radians Not Degrees.