PRODUCT-TO-SUM IDENTITIES

THESE WILL BE VERY USEFUL IN CALCULUS

WRITE $\sin\alpha \sin\beta$ AS A SUM WITHOUT PRODUCTS

WRITE $\cos\alpha \cos\beta$ AS A SUM WITHOUT PRODUCTS

Write $\sin\alpha\cos\beta$ as a sum without products

Product-to-sum Identities

$$\sin\alpha\sin\beta \equiv \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta \equiv \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta \equiv \frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$$

Sum-to-product Identities

These are useful when solving equations. Turn a sum into a product, then factor like you did in basic algebra.

Write $\sin\alpha \pm \sin\beta$ as a product

WRITE $\cos\alpha + \cos\beta$ AS A PRODUCT

Write $\cos\alpha - \cos\beta$ as a product

Sum-to-product Identities

$$\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cos\left(\frac{\alpha \mp \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha - \beta}{2}\right)\cos\left(\frac{\alpha + \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)$$

WRITE $\sin(5\theta)\cos(3\theta)$ AS A SUM

WRITE $\sin(5\theta)+\sin(3\theta)$ AS A PRODUCT

PROVE $$\frac{\sin(4\theta) + \sin(2\theta)}{\cos(4\theta) + \cos(2\theta)} \equiv \tan(3\theta)$$

PROVE $\sin\theta[\sin(3\theta)+\sin(5\theta)] \equiv \cos\theta[\cos(3\theta)-\cos(5\theta)]$

PROVE $1 + \cos(2\theta) + \cos(4\theta) + \cos(6\theta) \equiv 4\cos\theta\cos(2\theta)\cos(3\theta)$

PROVE $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$

WHEN $\pi = \alpha + \beta + \gamma$