"Dollar" and "Cent" Functions

Consider the following two functions

$$\$_7(x) := x - \frac{x^3}{3\cdot2\cdot1} + \frac{x^5}{5\cdot4\cdot3\cdot2\cdot1} - \frac{x^7}{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

$$¢_6(x) := 1 - \frac{x^2}{2\cdot1} + \frac{x^4}{4\cdot3\cdot2\cdot1} - \frac{x^6}{6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

Factorial Notation

$$N! := N\underbrace{(N-1)(N-2)(N-3)\cdots3\cdot2\cdot1}_{(N-1)!}, \quad N \in \mathbb{N}$$

$$\Rightarrow N! = N(N-1)!$$

Examples:

$$7! = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 7\cdot6!$$
$$3! = 3\cdot2\cdot1 = 3\cdot2!$$
$$2! = 2\cdot1 = 2\cdot1!$$
$$1! = 1$$

For $N=1$, $$N! = N(N-1)!$$

$$\Rightarrow 1 = 1! = 1(1-1)! = 1\cdot0! = 0!$$

$$\Rightarrow \boxed{0! = 1}$$

$0! = 1$ is not any more strange than $7^0 = 1$

$7^3 = 7 \cdot 7 \cdot 7$

$7^2 = 7 \cdot 7$

$7^1 = 7$

$1 = 7^0$

$7^{-1} = \frac{1}{7}$

$7^{-2} = \frac{1}{7^2}$

$7^2 \cdot 7^3 =$

$7^m \cdot 7^n =$

$$\$_\infty(x) := \$(x) := \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$¢_\infty(x) := ¢(x) := \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

NOTE: $\frac{x^1}{1!} = x$ AND $\frac{x^0}{0!} = 1$

CAN YOU EVALUATE EITHER FUNCTION FOR ANY INPUT VALUES?

$$\$(x) := \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$\$(0) =$

$\$(1) =$

$\$(\pi) =$

SIGMA NOTATION FOR SUMS Σ

USE THIS WHEN ADDING MANY ITEMS OF THE SAME FORM.

EXAMPLES:

$$\sum_{n=1}^{5} 3n = 3\cdot 1 + 3\cdot 2 + 3\cdot 3 + 3\cdot 4 + 3\cdot 5$$

$$\sum_{n=3}^{7} \frac{(-1)^n}{2n} = \frac{(-1)^3}{2\cdot 3} + \frac{(-1)^4}{2\cdot 4} + \frac{(-1)^5}{2\cdot 5} + \frac{(-1)^6}{2\cdot 6} + \frac{(-1)^7}{2\cdot 7}$$

$$= -\frac{1}{6} + \frac{1}{8} - \frac{1}{10} + \frac{1}{12} - \frac{1}{14}$$

$$\$(x) = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!} = \$(x)}$$

$$\text{¢}(x) = \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n}}{(2n)!} = \text{¢}(x)}$$

HOW DO WE EVALUATE A FUNCTION THAT HAS AN INFINITE SUM? LEARN CALCULUS.

$$\$\left(\frac{\pi}{2}\right) = \frac{\pi/2}{1!} - \frac{(\pi/2)^3}{3!} + \frac{(\pi/2)^5}{5!} - \frac{(\pi/2)^7}{7!} + \cdots = 1$$

CONSIDER THE CONSEQUENCES OF A FUNCTION, SUCH AS ¢(x), HAVING ONLY EVEN POWERS OF IT'S INPUT VARIABLE.

THESE TWO FUNCTIONS ARE ACTUALLY CALLED

SINE AND COSINE

AND ARE ACTUALLY WRITTEN

SIN(x) AND COS(x)

INSTEAD OF \$(x) AND ¢(x)

$$\boxed{\cos(x) = \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n}}{(2n)!}}$$

$$\boxed{\sin(x) = \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}}$$

Compare the graphs of:

$$¢_0(x) = 1$$

$$¢_2(x) = 1 - \frac{x^2}{2!}$$

$$¢_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$¢_6(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$¢_8(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

NOTICE COSINE IS AN EVEN FUNCTION

SINCE $\cos(-x) = \cos(x) \quad \forall x \in \mathbb{R}$

$$\cos(x) = \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\Rightarrow \cos(-x) =$$

NOTICE SINE IS AN ODD FUNCTION

SINCE $\sin(-x) = -\sin(x) \quad \forall x \in \mathbb{R}$

A GEOMETRIC APPROACH ALLOWS US TO EVALUATE SINE AND COSINE FOR <u>SOME</u> INPUT VALUES WITHOUT USING A CALCULATOR. WE USE A CIRCLE TO DO THIS.

GIVE ME ANY REAL NUMBER TO PLUG INTO SINE OR COSINE. USUALLY WE'D USE THE LETTER x TO REPRESENT THAT NUMBER BUT WE WILL USE x FOR SOMETHING ELSE SO CALL YOUR NUMBER S FOR NOW.

FIRST, WE RELATE YOUR NUMBER TO A CIRCLE WITH RADIUS OF 1 UNIT (THE UNIT CIRCLE).

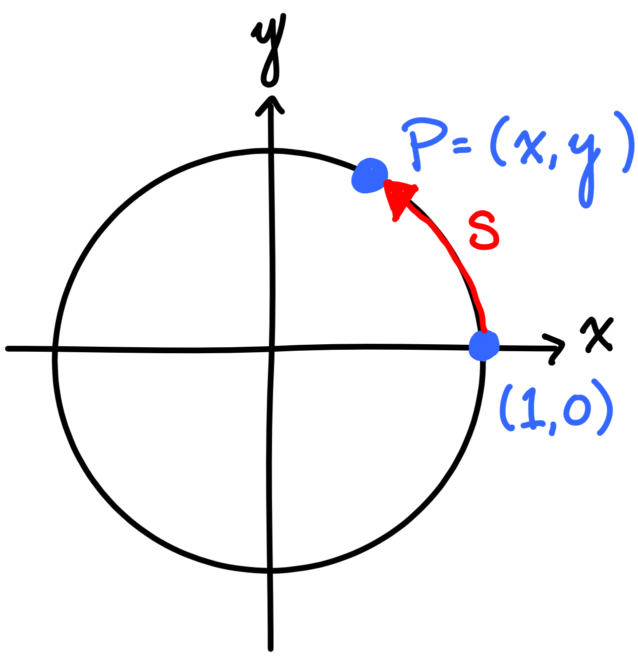
THIS RELATIONSHIP IS GIVEN BY

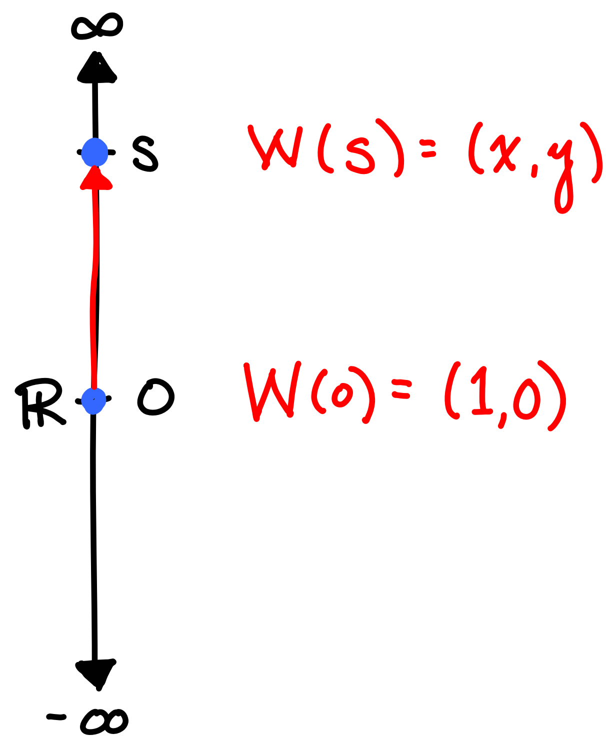
"THE WRAPPING FUNCTION"

SO CALLED BECAUSE IT WRAPS THE REAL NUMBER LINE AROUND THE UNIT CIRCLE.

IT ASSOCIATES YOUR NUMBER WITH A POINT P ON THE UNIT CIRCLE.

$$W(s) = P = (x, y) \qquad s \in \mathbb{R}$$





YOUR NUMBER IS REPRESENTED BY AN ARC THAT IS S UNITS OF LENGTH.

(R = 1 UNITS OF LENGTH)

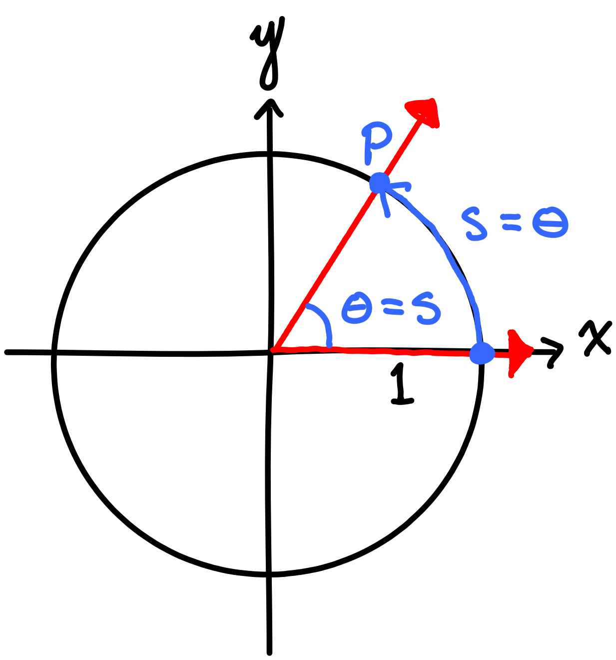
CONSIDER, BY DEFINITION $\theta := \frac{s}{r}$

SO IF $r=1$ THEN $\theta = s$

$(r=1 \Leftrightarrow \theta = s)$

THIS MEANS THAT ON THE UNIT CIRCLE (CIRCLE OF RADIUS $r=1$) s AND θ ARE THE SAME NUMBER.

SO, YOU CAN THINK OF YOUR NUMBER AS AN ANGLE IN RADIANS OR AS AN ARCLENGTH



SINCE $\theta = s$ LET'S THINK OF YOUR NUMBER AS AN ANGLE AND CALL IT θ (THETA).