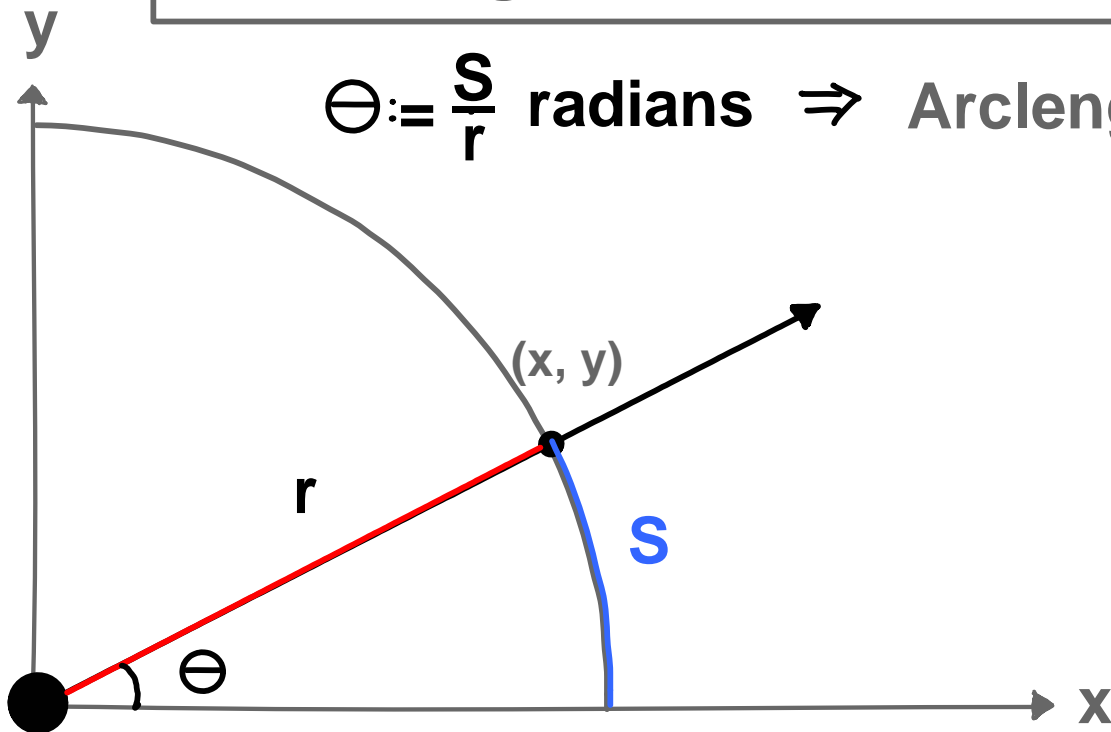


TRIG IN A NUTSHELL

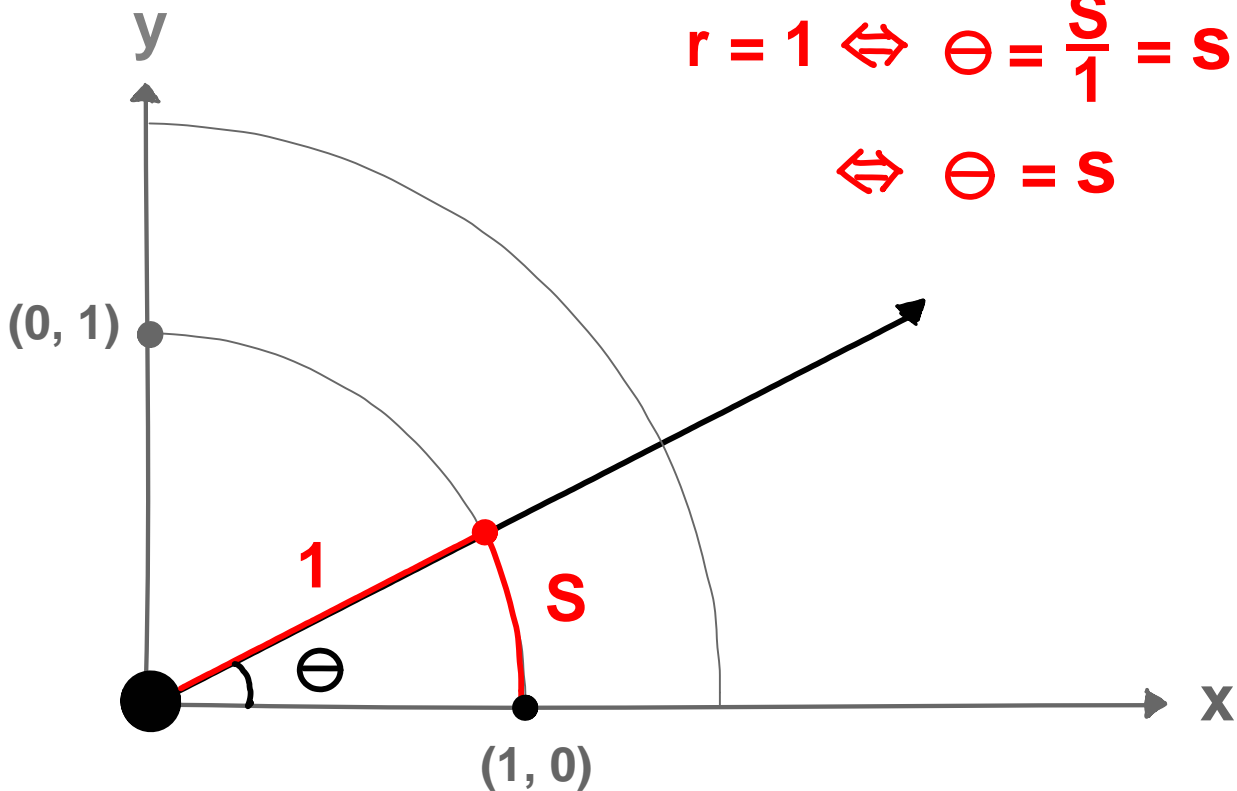
Angles and Circles

$$\Theta := \frac{S}{r} \text{ radians} \Rightarrow \text{Arclength: } s = r\theta$$

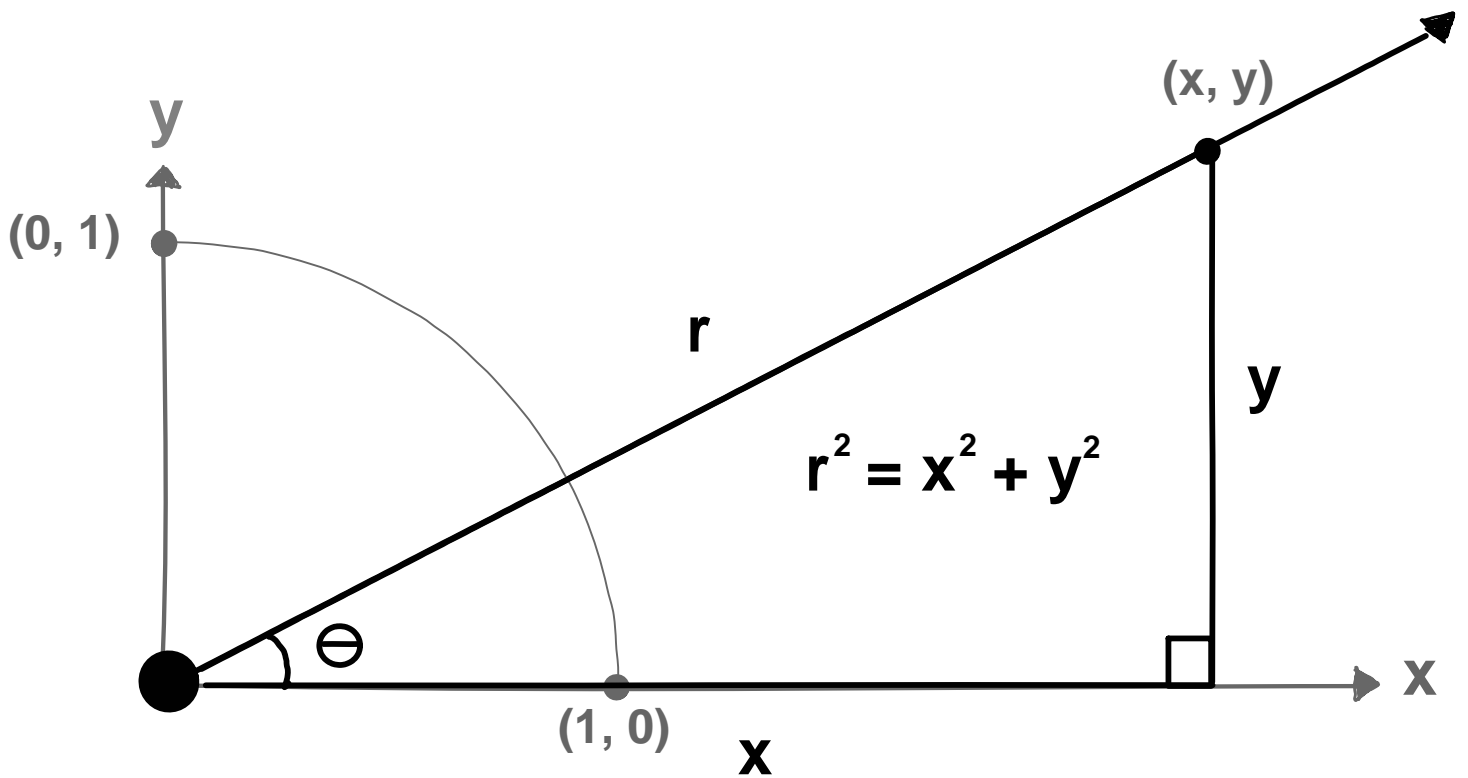


The Unit Circle: $r = 1$

$$r = 1 \Leftrightarrow \Theta = \frac{S}{1} = S$$
$$\Leftrightarrow \Theta = S$$

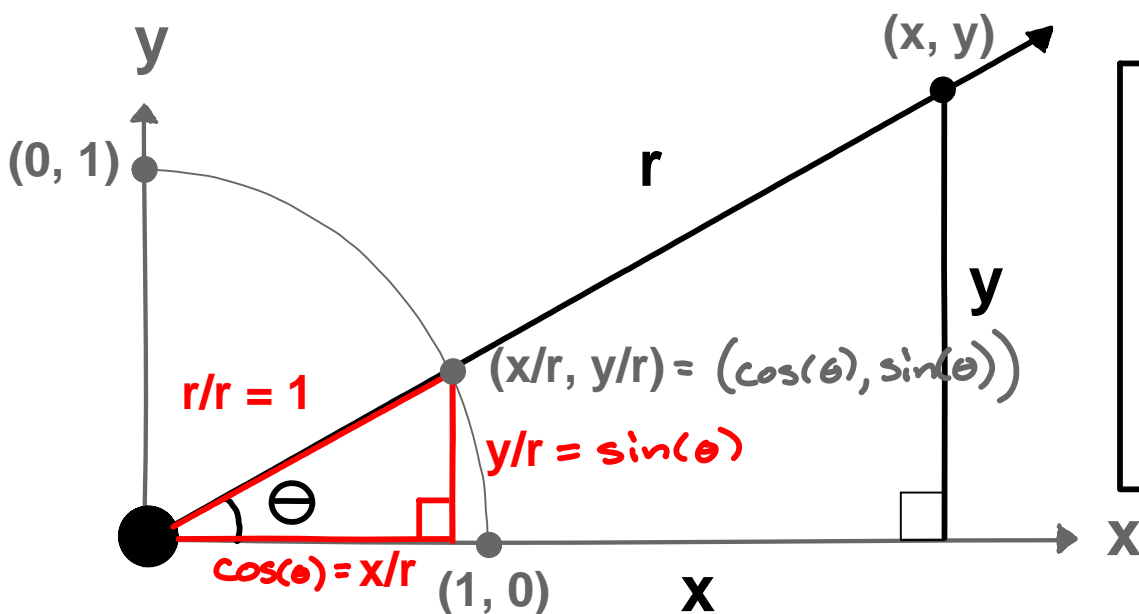


Angles and Triangles



Triangle with Hypotenuse = 1

$$\left(\frac{r}{r}\right)^2 = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 \quad 1 = \cos^2(\theta) + \sin^2(\theta)$$



Sine

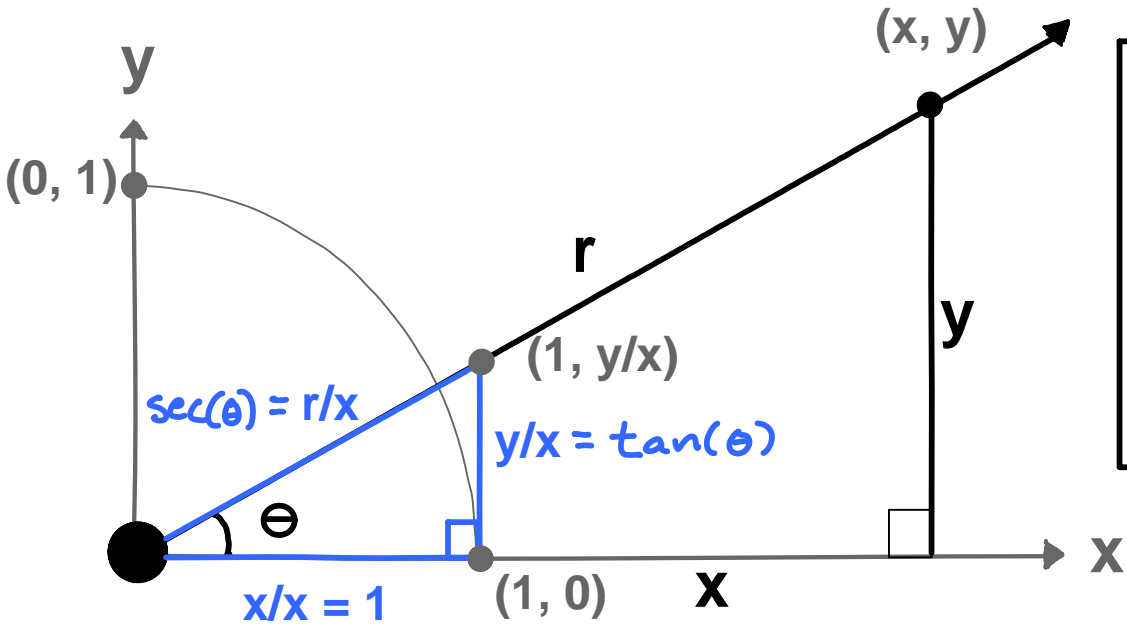
$$\sin(\theta) := \frac{y}{r}$$

$$\cos(\theta) := \frac{x}{r}$$

Cosine

Triangle with Adjacent Side = 1

$$\left(\frac{r}{x}\right)^2 = \left(\frac{x}{x}\right)^2 + \left(\frac{y}{x}\right)^2 \qquad \sec^2(\theta) = 1 + \tan^2(\theta)$$



Tangent

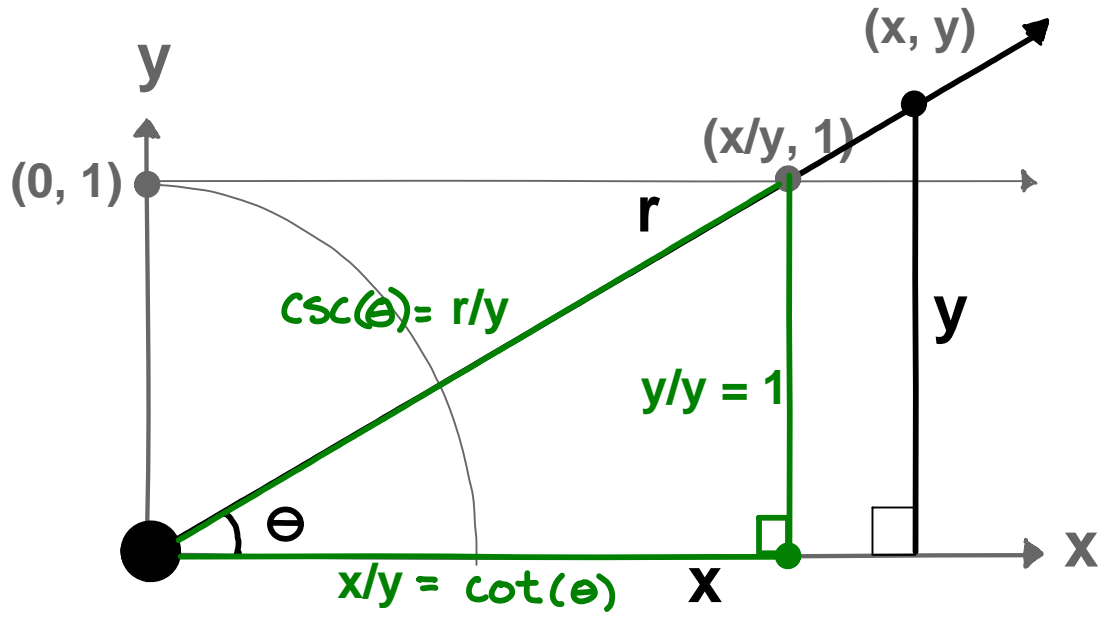
$$\tan(\theta) := \frac{y}{x}$$

$$\sec(\theta) := \frac{r}{x}$$

Secant

Triangle with Opposite Side = 1

$$\left(\frac{r}{y}\right)^2 = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{y}\right)^2 \qquad \csc^2(\theta) = \cot^2(\theta) + 1$$



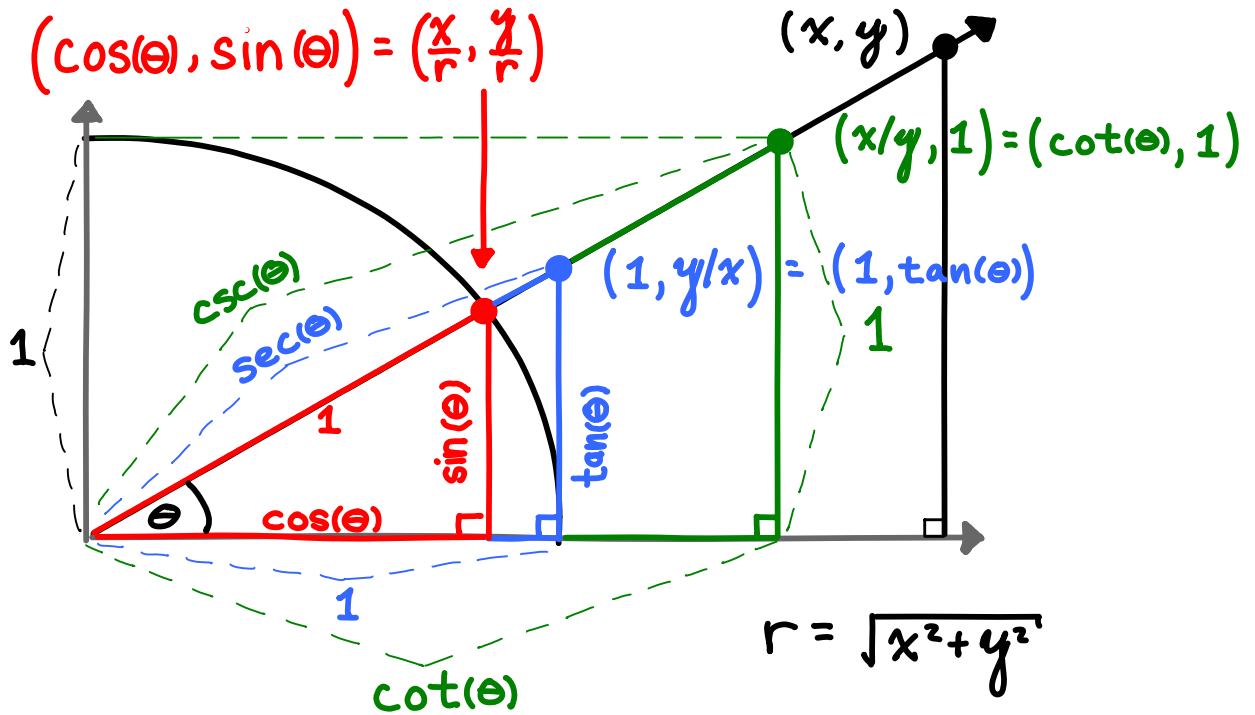
Cotangent

$$\cot(\theta) := \frac{x}{y}$$

$$\csc(\theta) := \frac{r}{y}$$

Cosecant

Trigonometry Summary Diagram



Geometric definitions of the six trig functions and the "Reciprocal Identities"

| Sine | Cosecant |
|--|--|
| $\sin(\theta) := \frac{y}{r} = \frac{1}{\csc(\theta)}$ | $\csc(\theta) := \frac{r}{y} = \frac{1}{\sin(\theta)}$ |
| Cosine | Secant |
| $\cos(\theta) := \frac{x}{r} = \frac{1}{\sec(\theta)}$ | $\sec(\theta) := \frac{r}{x} = \frac{1}{\cos(\theta)}$ |
| Tangent | Cotangent |
| $\tan(\theta) := \frac{y}{x} = \frac{1}{\cot(\theta)}$ | $\cot(\theta) := \frac{x}{y} = \frac{1}{\tan(\theta)}$ |

The "Quotient Identities"

$$\tan(\theta) := \frac{y}{x} = \frac{y/r}{x/r} = \frac{\sin(\theta)}{\cos(\theta)} \Leftrightarrow \tan(\theta) \equiv \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) \equiv \frac{1}{\tan(\theta)} = \frac{1}{\frac{\sin(\theta)}{\cos(\theta)}} = \frac{\cos(\theta)}{\sin(\theta)} \Leftrightarrow \cot(\theta) \equiv \frac{\cos(\theta)}{\sin(\theta)}$$

The "Pythagorean Identities"

$$x^2 + y^2 = r^2 \Leftrightarrow \frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2} \Leftrightarrow \cos^2(\theta) + \sin^2(\theta) = 1$$

$$x^2 + y^2 = r^2 \Leftrightarrow \frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2} \Leftrightarrow 1 + \tan^2(\theta) = \sec^2(\theta)$$

$$x^2 + y^2 = r^2 \Leftrightarrow \frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2} \Leftrightarrow \cot^2(\theta) + 1 = \csc^2(\theta)$$

EXAMPLES

If $(4, 3)$ is on the terminal side of an angle θ ,
what is $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$?

If $\tan(\theta) = \frac{4}{3}$ what is $\sin(\theta)$, $\cos(\theta)$, $\cot(\theta)$,
 $\sec(\theta)$, and $\csc(\theta)$? NOTE: $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

If $\theta = 45^\circ$, what is $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$?

If $\sin(\theta) = \frac{1}{3}$ what is $\cos(\theta)$, $\cot(\theta)$, $\sec(\theta)$,
and $\csc(\theta)$?
