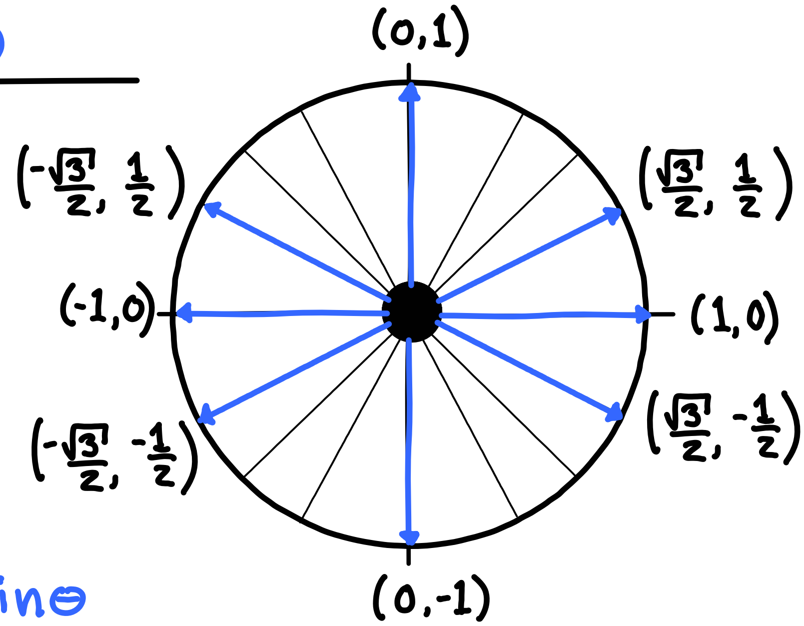
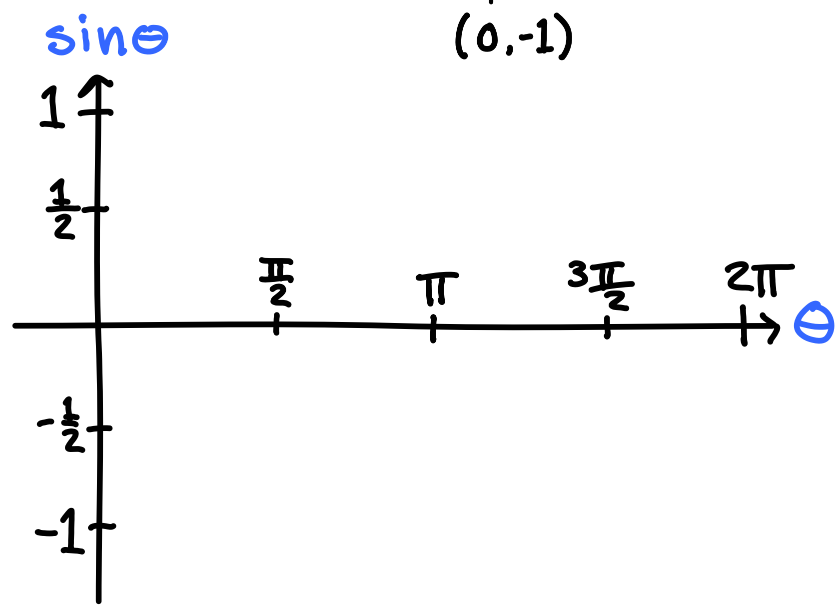
Graphing $\sin(\theta)$

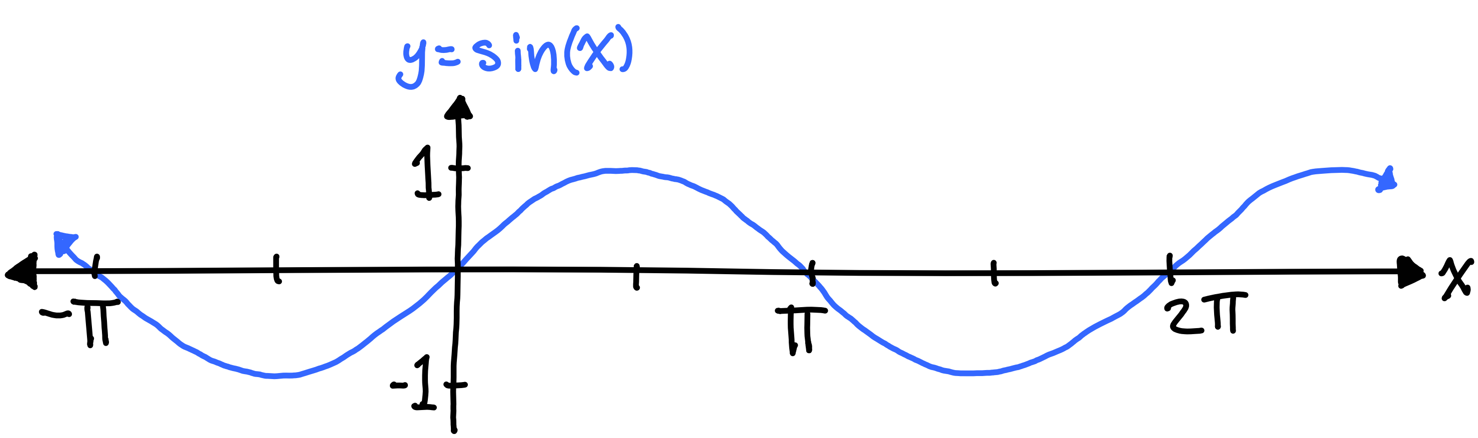
θ	$\sin\theta$	$(\theta, \sin\theta)$
0		$(0, \quad)$
$\frac{\pi}{6}$		$(\frac{\pi}{6}, \quad)$
$\frac{\pi}{2}$		$(\frac{\pi}{2}, \quad)$
$\frac{5\pi}{6}$		$(\frac{5\pi}{6}, \quad)$
π		$(\pi, \quad)$
$\frac{7\pi}{6}$		$(\frac{7\pi}{6}, \quad)$
$\frac{3\pi}{2}$		$(\frac{3\pi}{2}, \quad)$
$\frac{11\pi}{6}$		$(\frac{11\pi}{6}, \quad)$
2π		$(2\pi, \quad)$





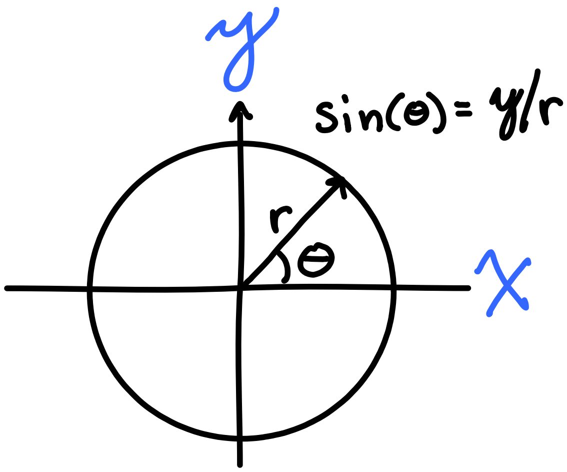
We prefer $f(x) = y$ so,

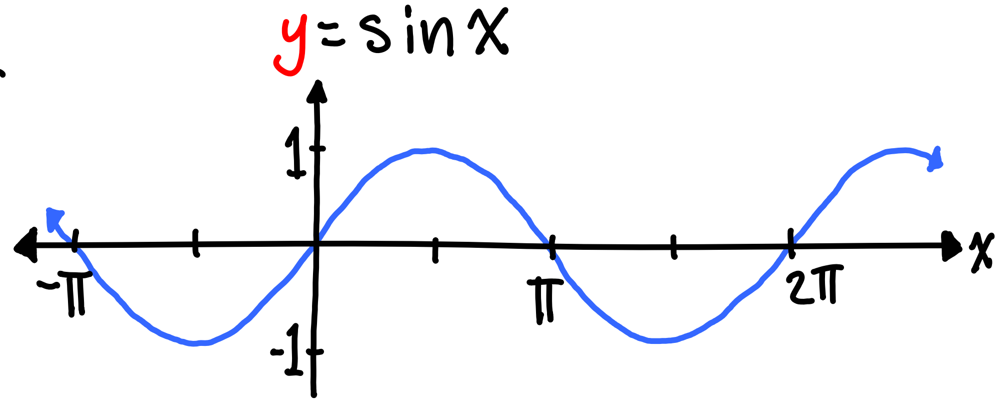
$\theta \longleftrightarrow x$ (Not the same x as on the unit circle)



Two Visual Representations

Both Drawn on X-Y Coordinate Systems





Different X's and Y's. Different Worlds.

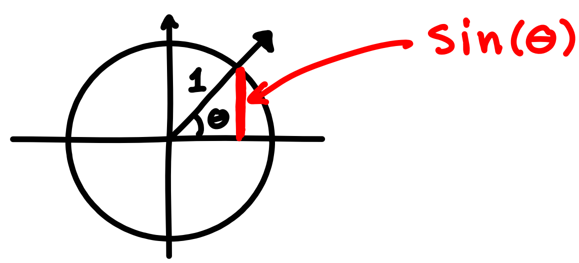
Like Parallel Universes!

Multiple Perspectives of the Same Concepts:

Taylor Series: $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

Trigonometric Ratio: $\sin(\theta) := y/r$

Unit Circle provides a **Geometric** perspective



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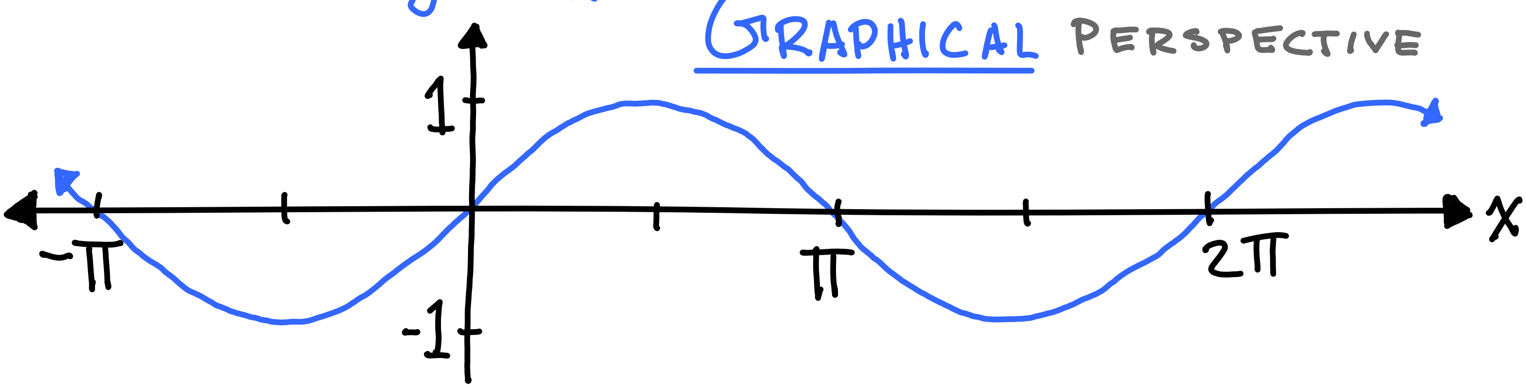
y = sin(x)

Graphical Perspective



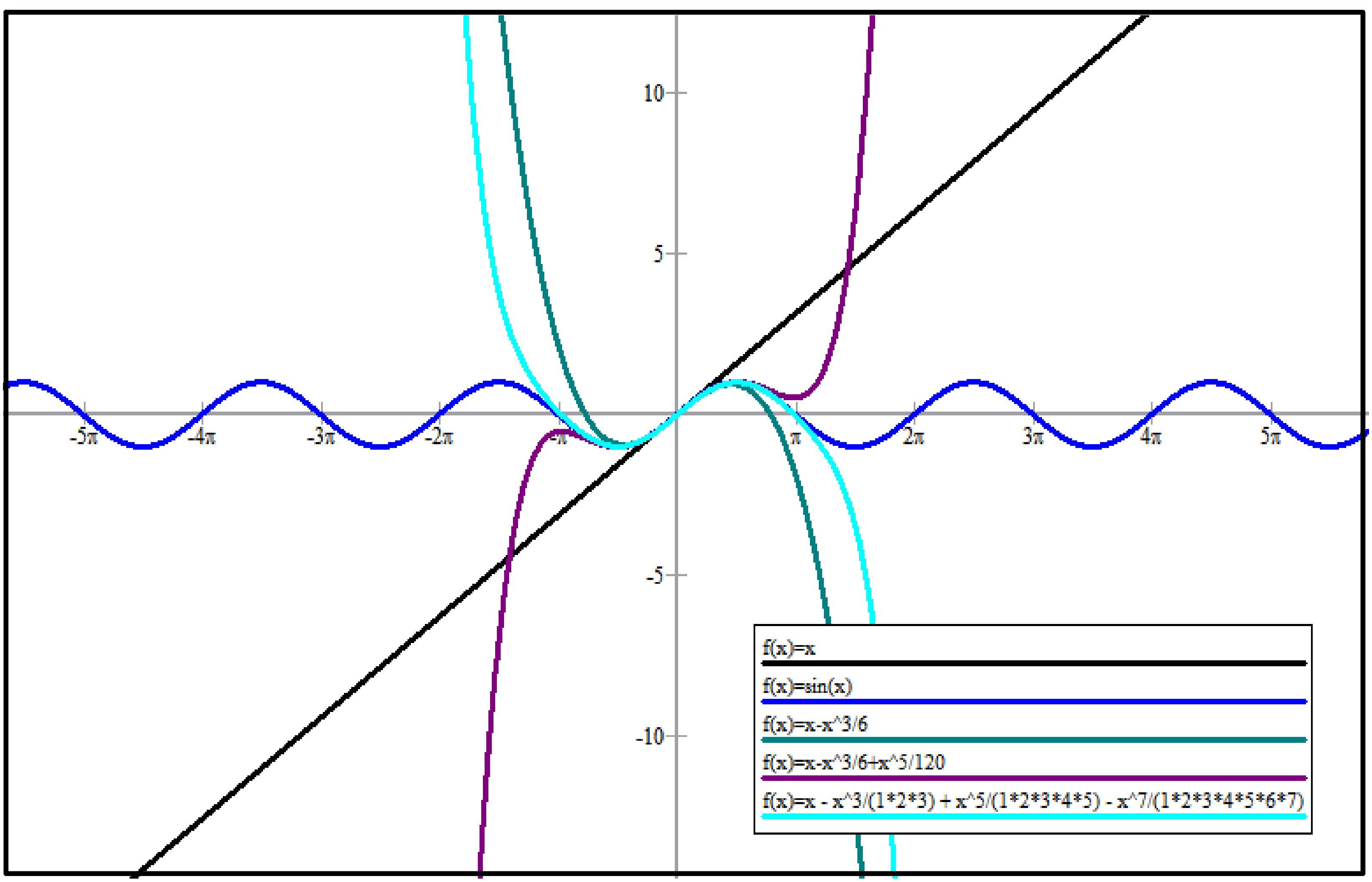
Aproximating sin(x) using it's Taylor series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

This Taylor series for sine provides good aproximations for small values of x.

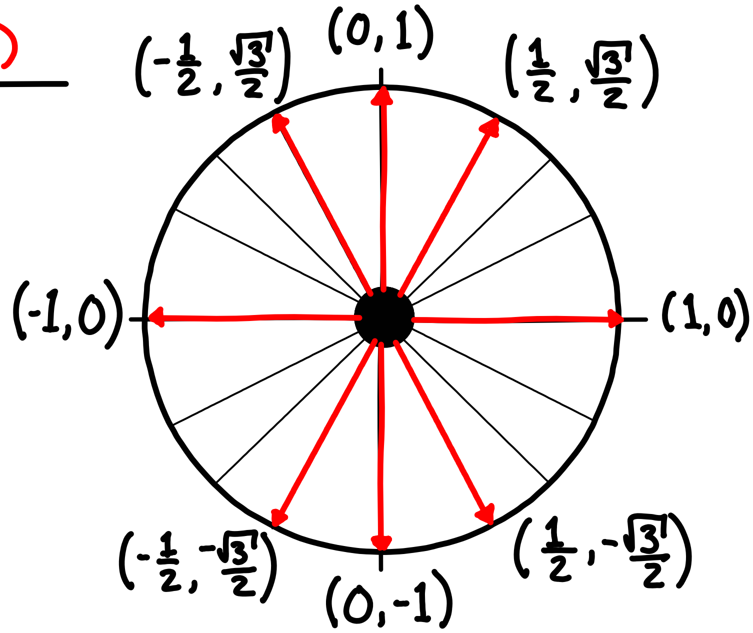
Consider,

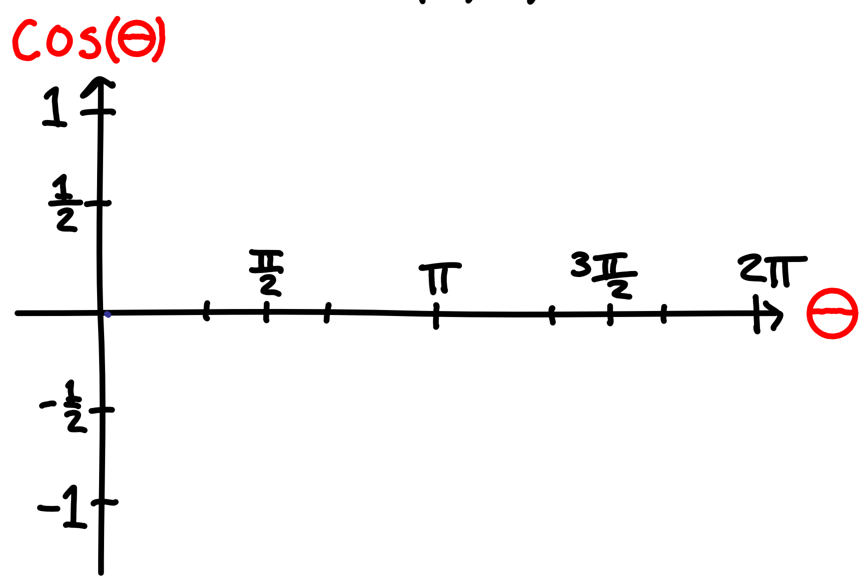
$$\sin(1/10) = ?$$

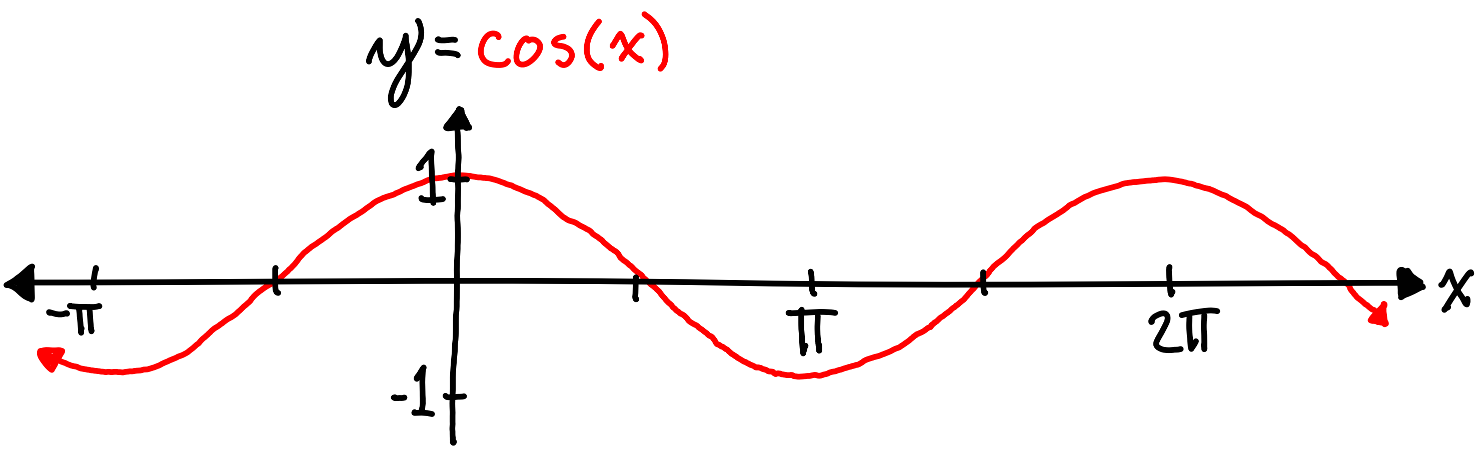


GRAPHING COS(Θ)

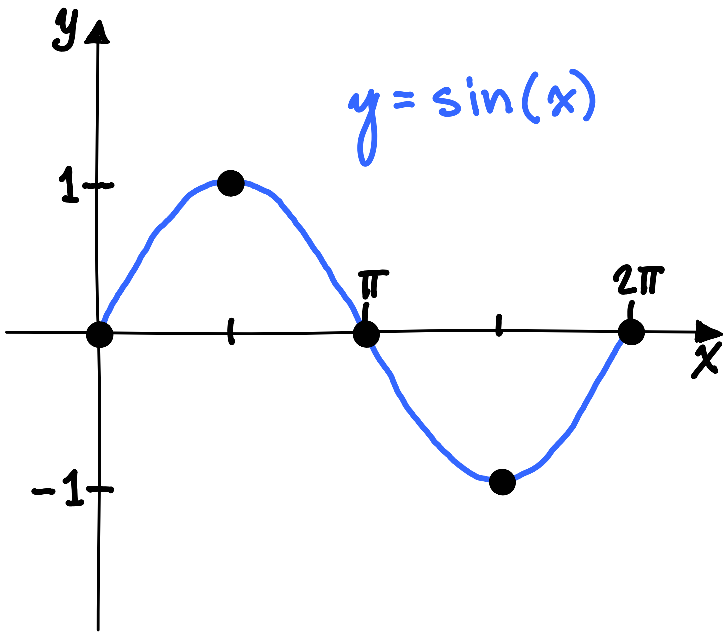
θ	$\cos(\theta)$	$(\theta, \cos(\theta))$
0		$(0,\quad)$
$\frac{\pi}{3}$		$(\frac{\pi}{3},\quad)$
$\frac{\pi}{2}$		$(\frac{\pi}{2},\quad)$
$\frac{2\pi}{3}$		$(\frac{2\pi}{3},\quad)$
π		$(\pi,\quad)$
$\frac{4\pi}{3}$		$(\frac{4\pi}{3},\quad)$
$\frac{3\pi}{2}$		$(\frac{3\pi}{2},\quad)$
$\frac{5\pi}{3}$		$(\frac{5\pi}{3},\quad)$
2π		$(2\pi,\quad)$

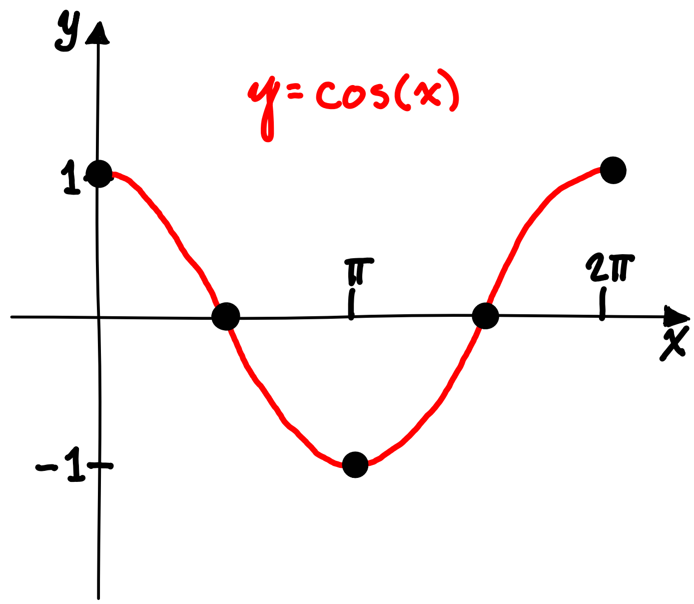


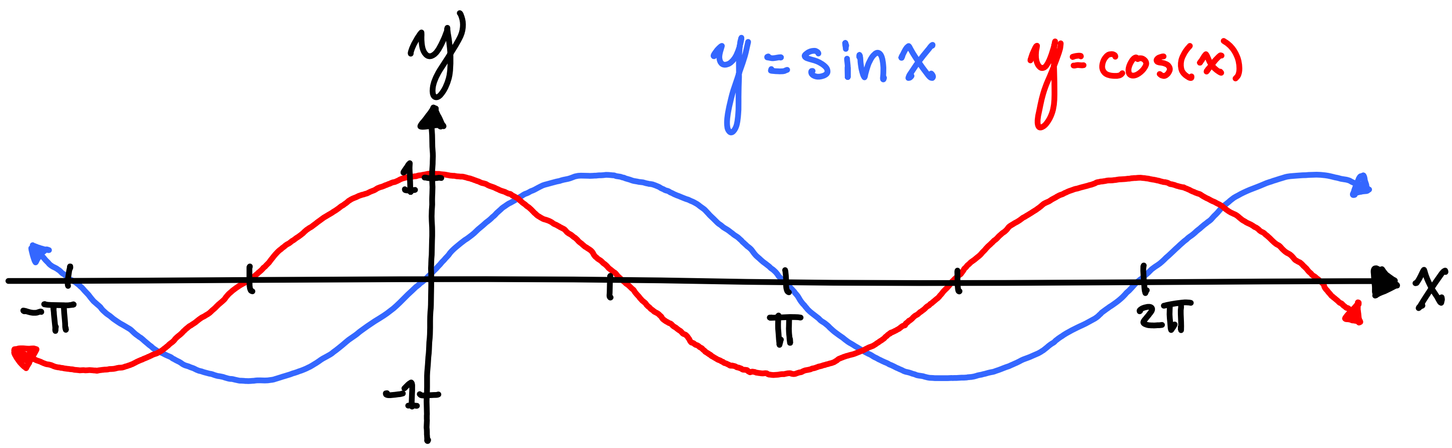




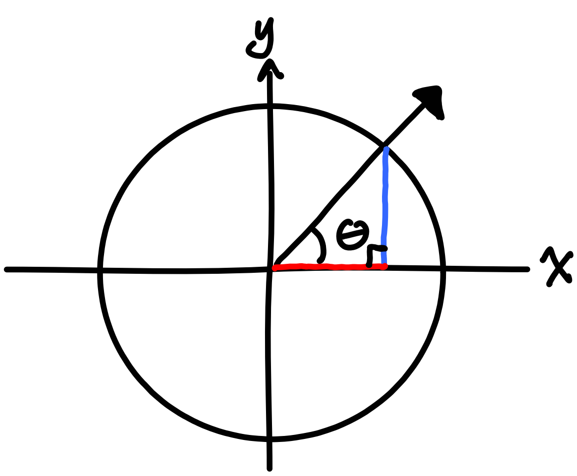
The Basic Graphical Building Blocks







Domain And Range Of Sine And Cosine



	Domain (x)	Range (y)
$\sin\theta$		
$\cos\theta$		

Adjusting Output : $y = A\sin(x) + B$

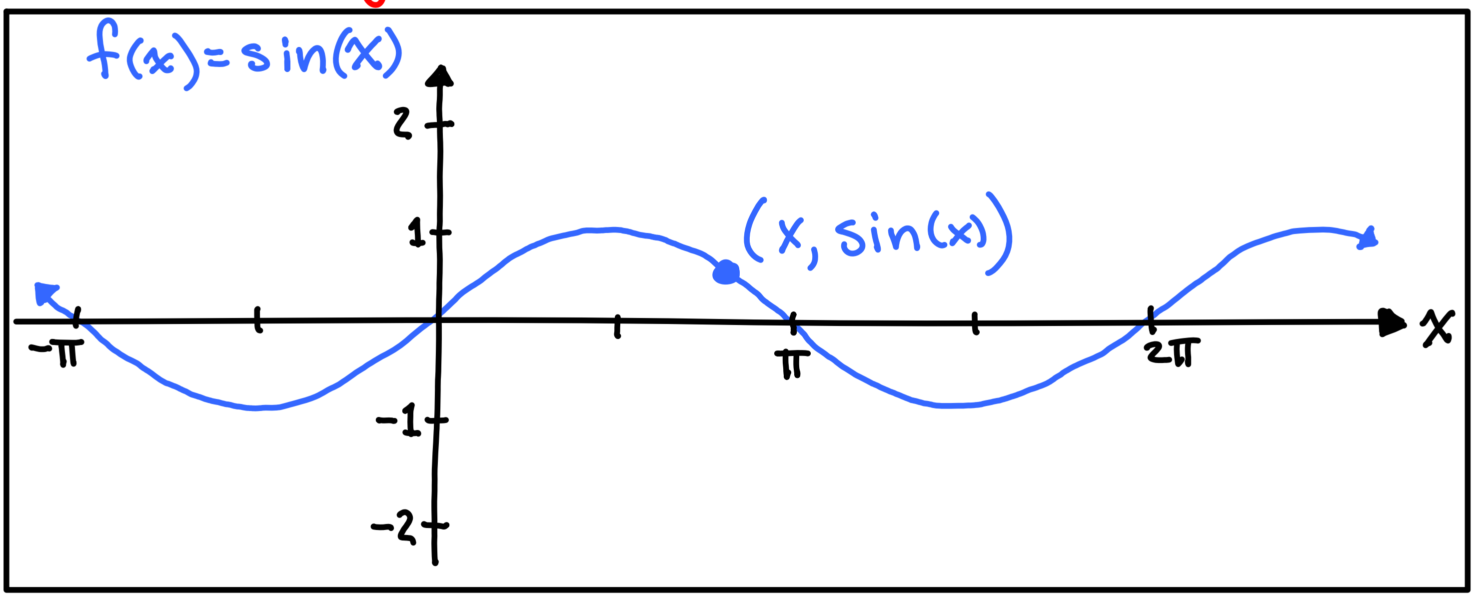
AMPLITUDE MODULATION $y = A\sin(x)$

AMPLITUDE $\equiv |A|$

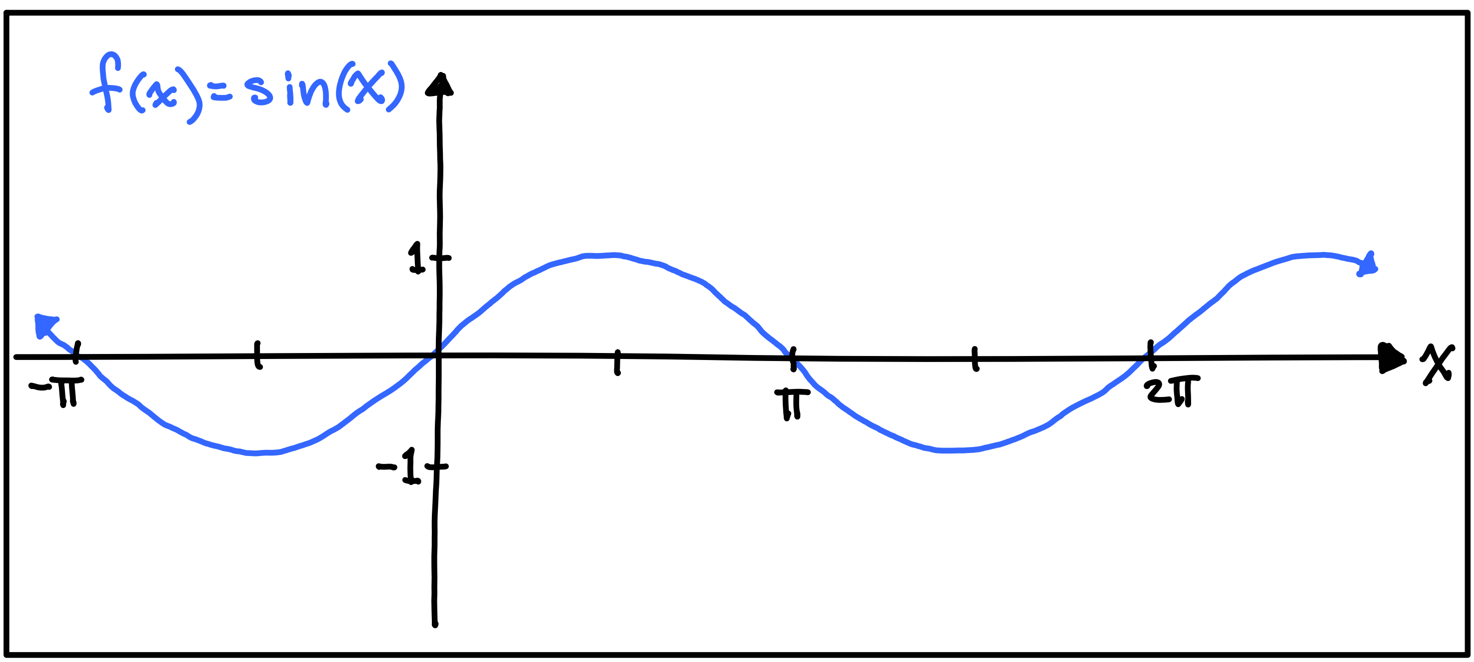
$|A| > 1 \Rightarrow$ VERTICAL STRETCHING

$|A| < 1 \Rightarrow$ VERTICAL COMPRESSION

EXAMPLE: $g(x) = 2\sin(x) = 2f(x)$

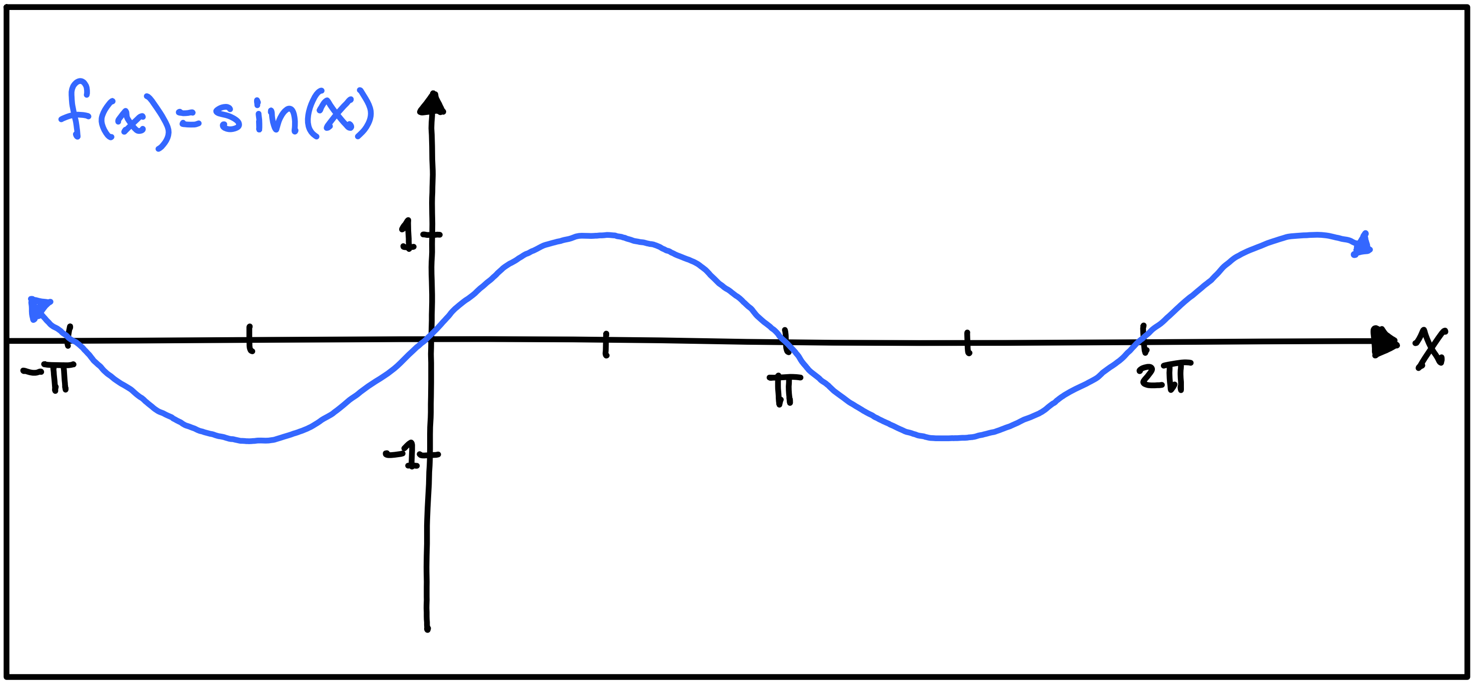


OR JUST RELABEL THE AXIS



FLIP OVER X axis: $y = -\sin(x)$

EXAMPLE: $g(x) = -\sin(x) = -f(x)$

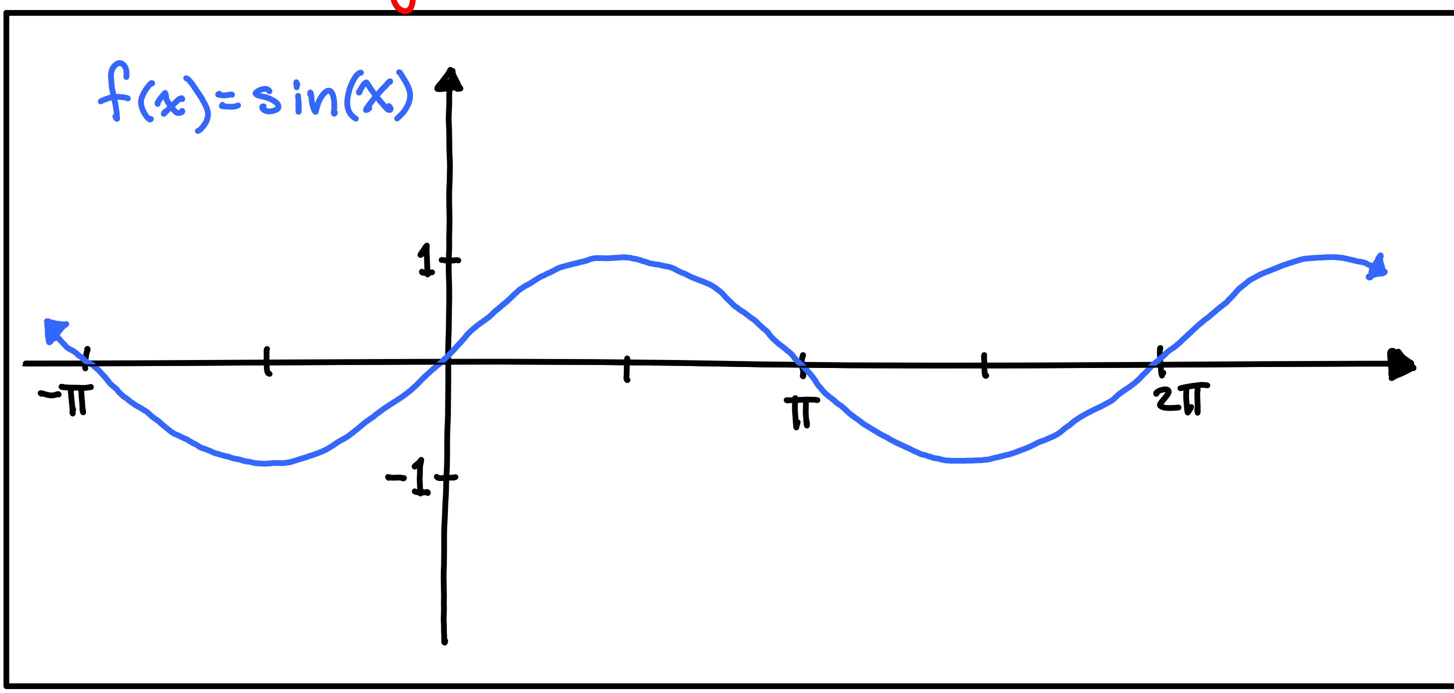


Vertical Shift: $y = \sin(x) + B$

$B > 0 \Rightarrow$ Shift up

$B < 0 \Rightarrow$ Shift Down

Example: $g(x) = \sin(x) + 1$



ADJUSTING INPUT: $y = \sin(\omega x - \phi)$

FREQUENCY MODULATION: $y = \sin(\omega x)$

OMEGA $\equiv \omega$

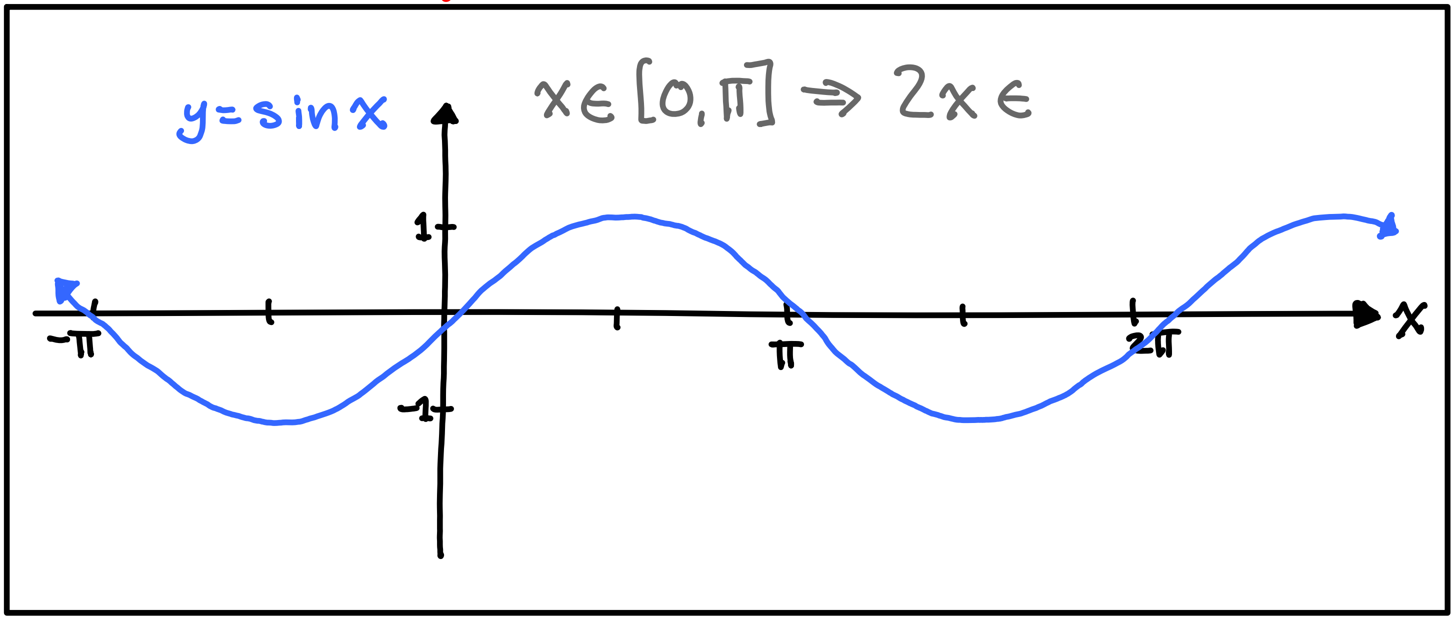
$|\omega| > 1 \Rightarrow$ HORIZONTAL COMPRESSION

$|\omega| < 1 \Rightarrow$ HORIZONTAL STRETCHING

PERIOD $\equiv T := 2\pi / |\omega|$ for sine + cosine

FREQUENCY $\equiv f := 1/T = \frac{|\omega|}{2\pi}$

Example: $y = \sin(2x)$



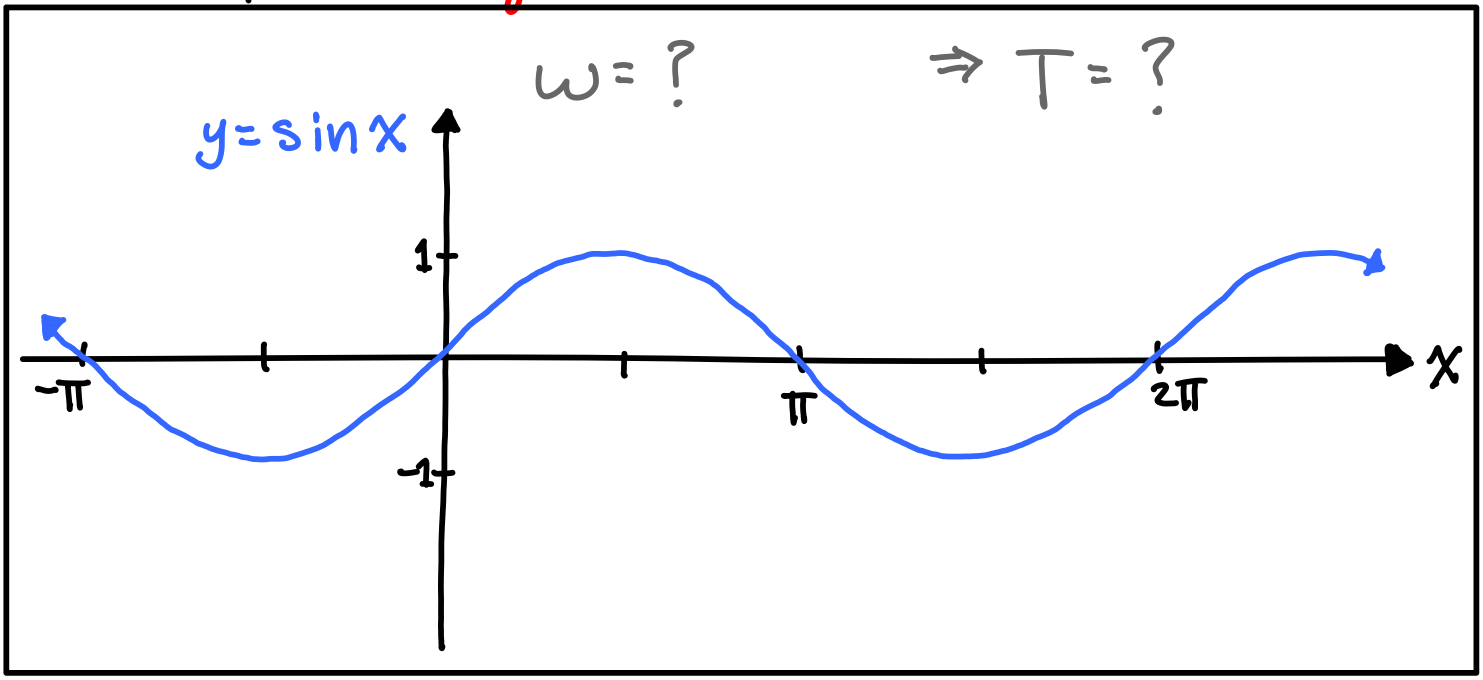
TWICE AS MANY PERIODS $\Rightarrow$ PERIODS HALF AS LONG

Period (T) of $\sin(x)$ or $\cos(x)$ is $T = 2\pi$

Period of $\sin(\omega x)$ or $\cos(\omega x)$ is $\boxed{T = \frac{2\pi}{|\omega|}}$

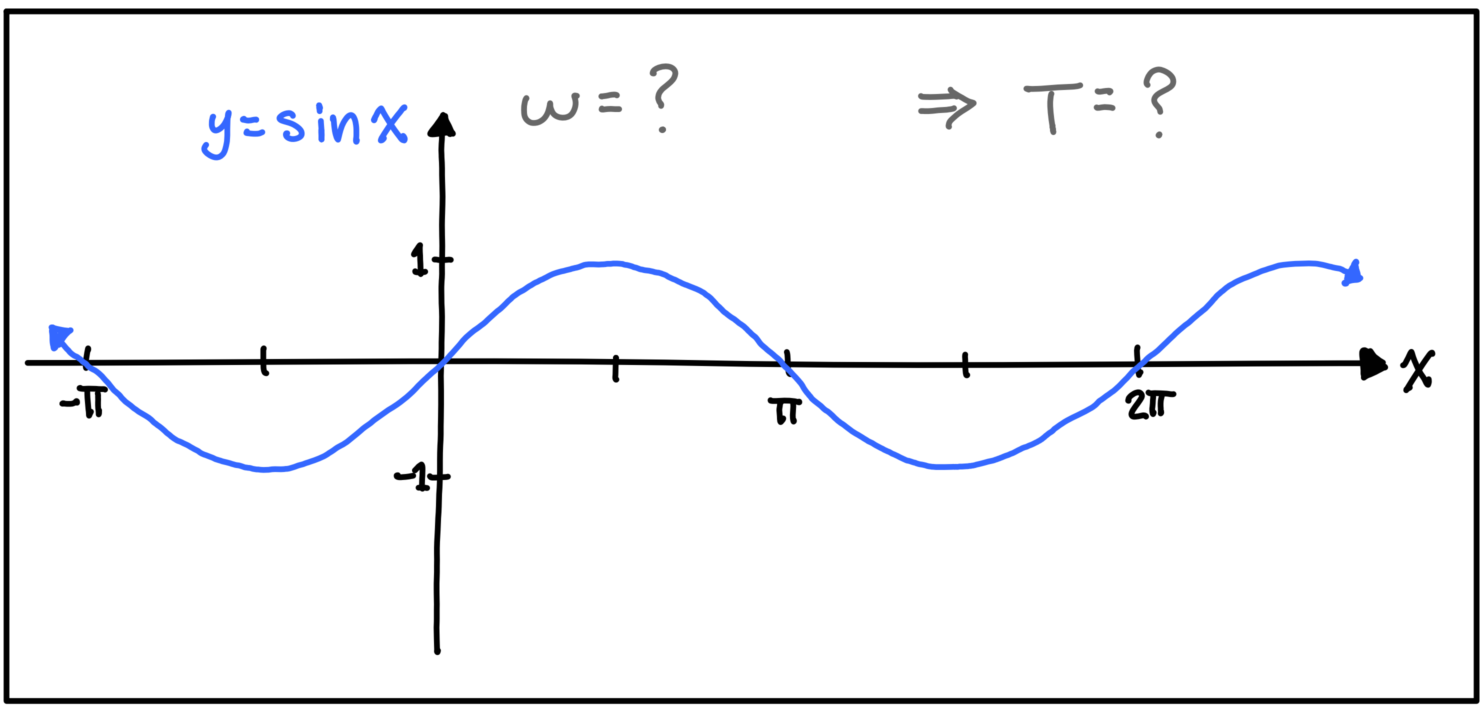
FREQUENCY $\equiv f := 1/T = \frac{|\omega|}{2\pi}$

Example: $y = \sin\left(\frac{1}{2}x\right)$



HALF AS MANY PERIODS ⇒ PERIODS TWICE AS LONG

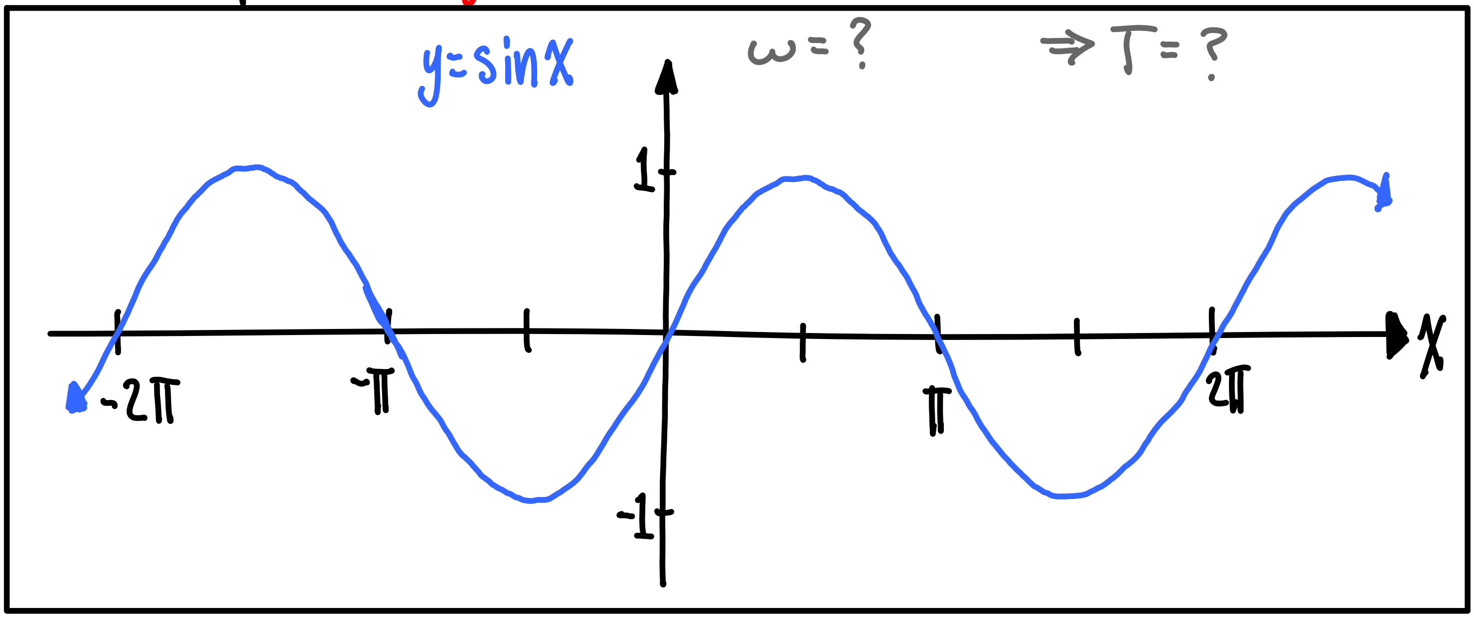
Example: $y = \sin(2\pi x)$



Flip over Y-axis: $y = \sin(-x)$

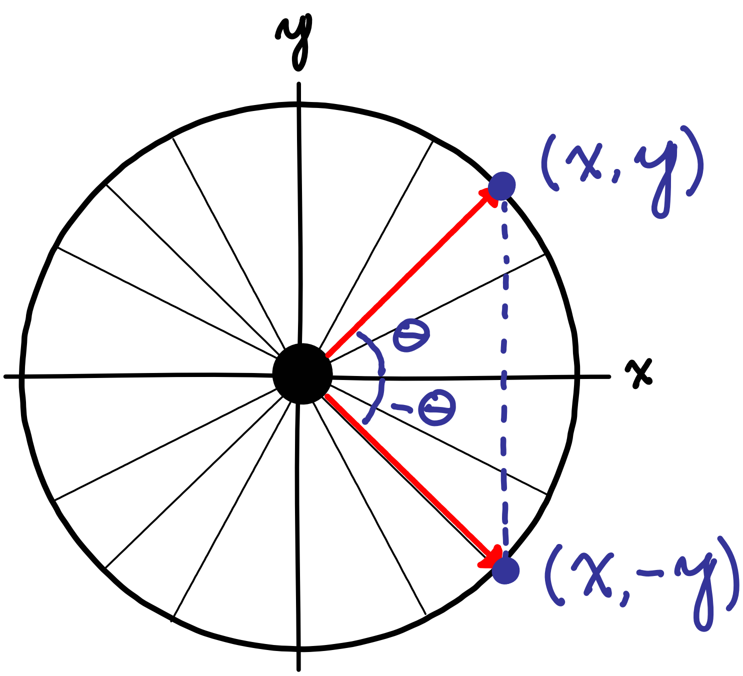
Example: $y = \sin(-2x)$

$\omega = ?$ $\Rightarrow T = ?$



Symmetry: Even or Odd

Symmetry on the unit circle



$\sin(\theta) = y$

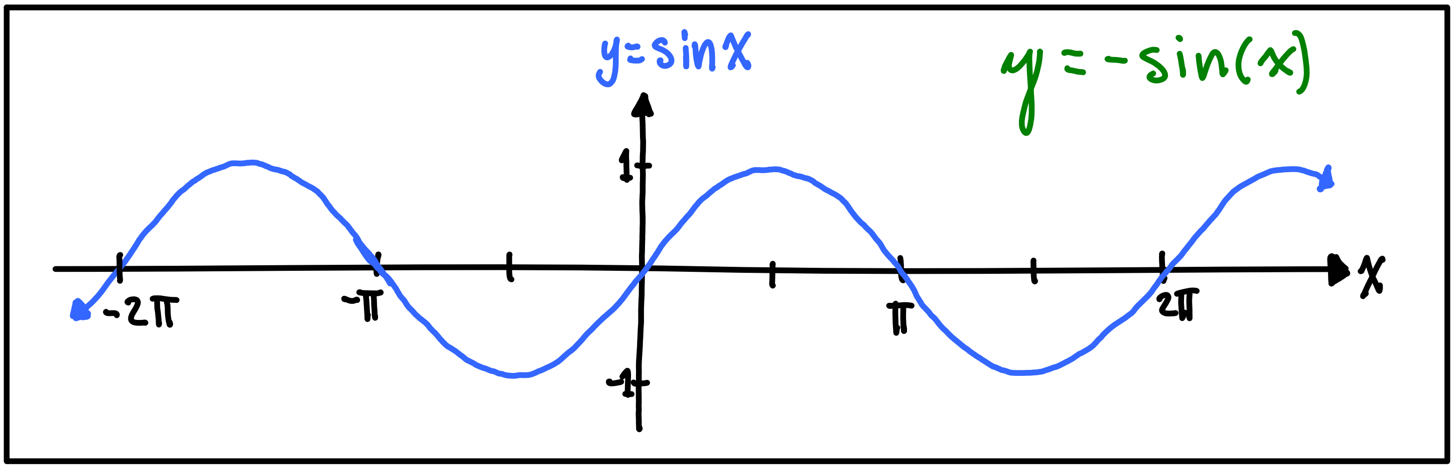
$\sin(-\theta) = -y = -\sin(\theta)$

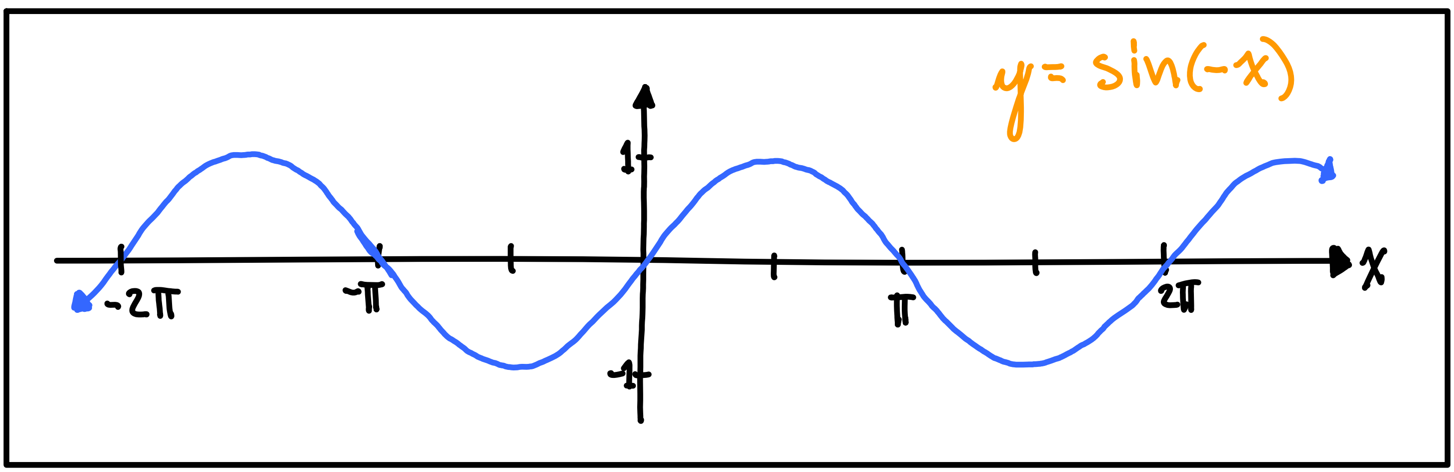
$\cos(\theta) = x = \cos(-\theta)$

$\sin(x)$ is ODD $\Rightarrow$ $\sin(-x) = -\sin(x)$

FLIPPING THE GRAPH OF $\sin(x)$ OVER THE Y-AXIS GIVES THE SAME AS FLIPPING IT OVER THE X-AXIS

FLIP $\sin(x)$ OVER Y AXIS $\equiv$ FLIP $\sin(x)$ OVER X AXIS





ROTATE AN ODD FUNCTION AROUND THE ORIGIN BY 180° AND NOTHING CHANGES.

ODD $\Leftrightarrow$ SYMMETRY ABOUT THE ORIGIN

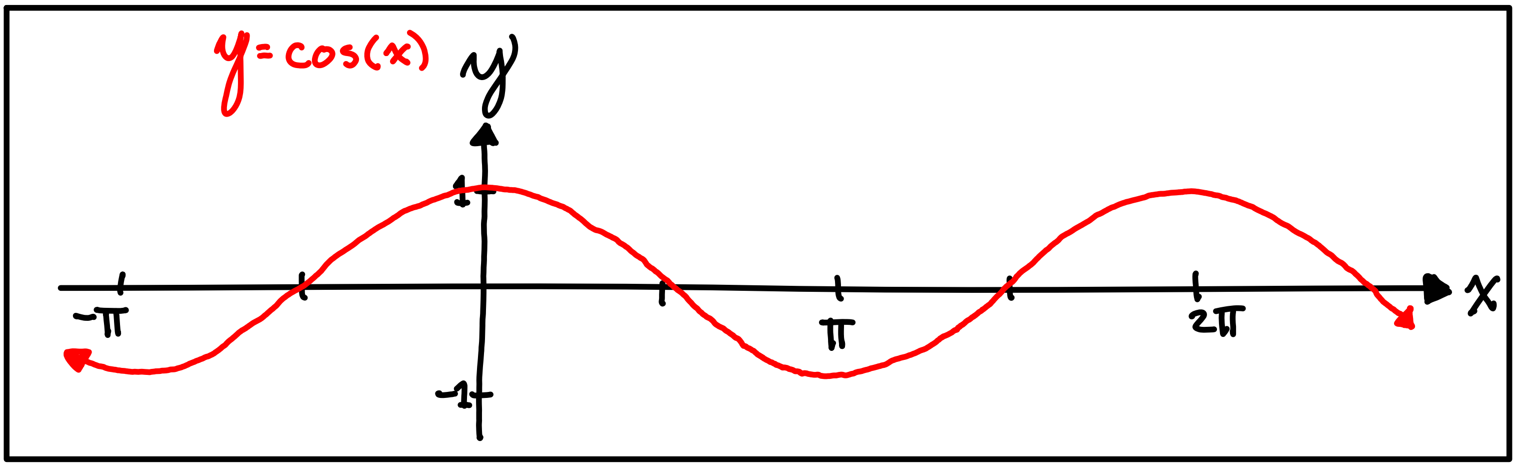
FLIP AN ODD GRAPH OVER ANY AXIS TWICE DOES NOTHING

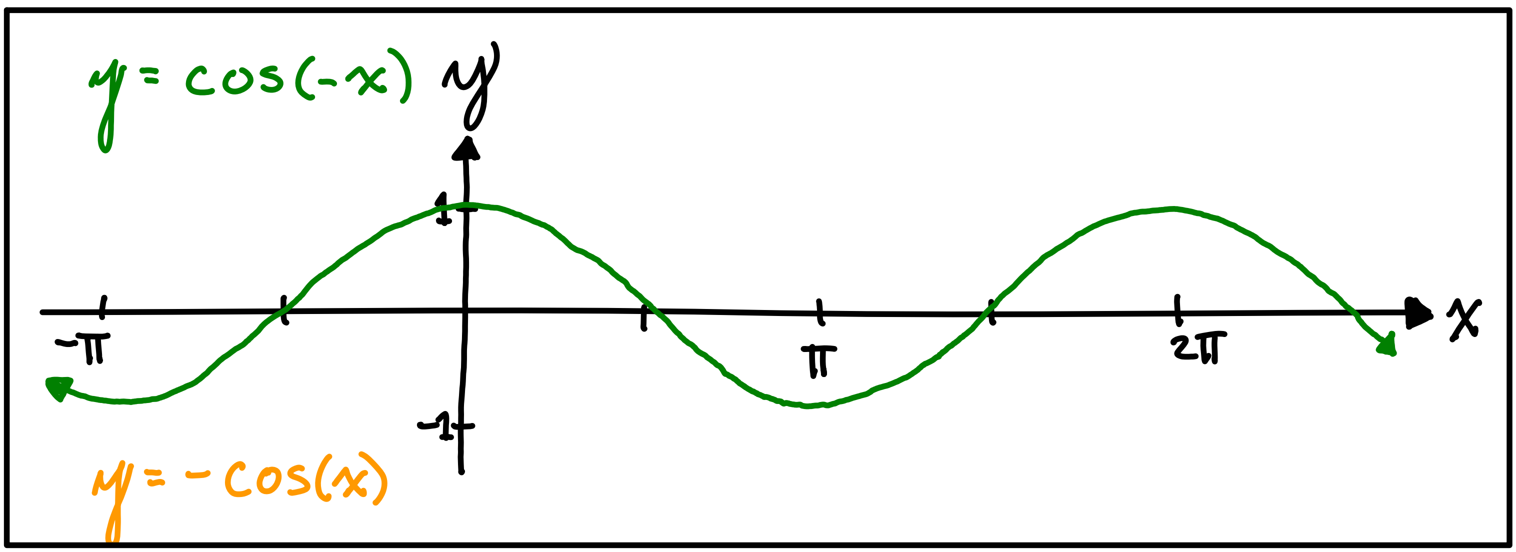
$$\sin(-x) = -\sin(x)$$

$$\Leftrightarrow \quad -\sin(-x) = --\sin(x) = \sin(x)$$

$$\Leftrightarrow \quad -\sin(-x) = \sin(--x) = \sin(x)$$

$\cos(x)$ IS EVEN $\Rightarrow$ $\cos(-x) = \cos(x)$





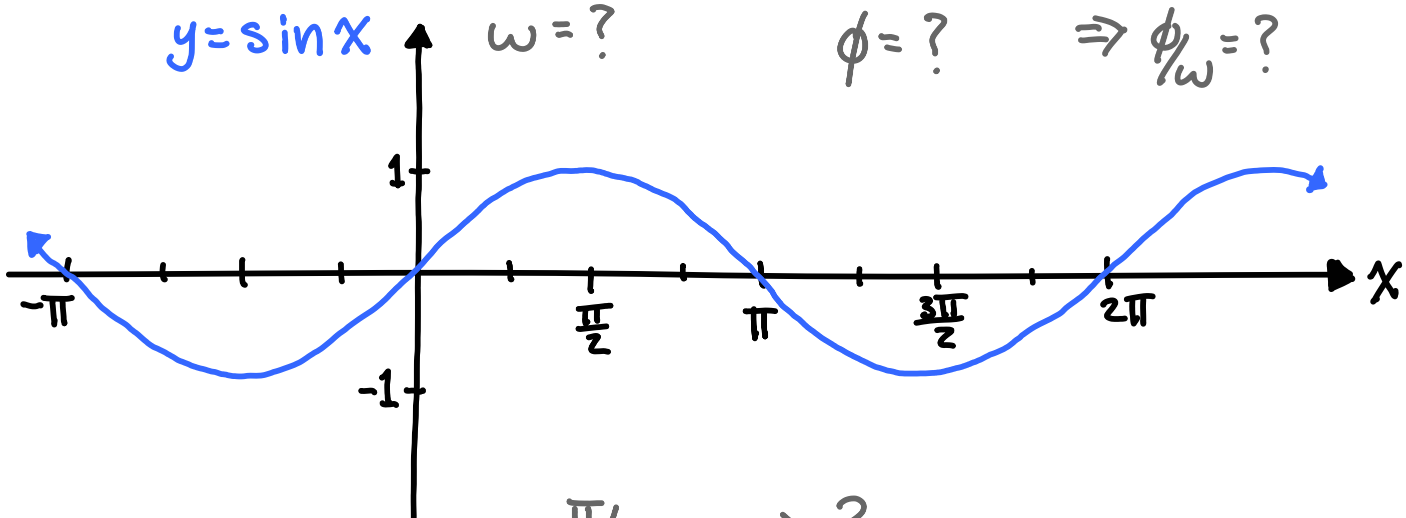
FLIPPING $\cos(x)$ OVER THE Y-AXIS DOES NOTHING: $\cos(-x) = \cos(x)$

EVEN $\Rightarrow$ SYMMETRY ABOUT THE Y-AXIS.

Horizontal Shifts (Phase Shifts)

Example: $y = \sin(x - \frac{\pi}{4})$

$y = \sin x$ $\quad \omega = ?$ $\quad \phi = ?$ $\quad \Rightarrow \phi/\omega = ?$



$\sin(0) = 0$ $\quad x - \pi/4 = 0 \Rightarrow ?$

$\sin(0) = 0$: What do we plug in for x for $\sin(x - \pi/4)$ to be zero?

$y = \sin(\omega x - \phi)$ $\quad \phi \rightarrow$ Phi (Fee or Fie)

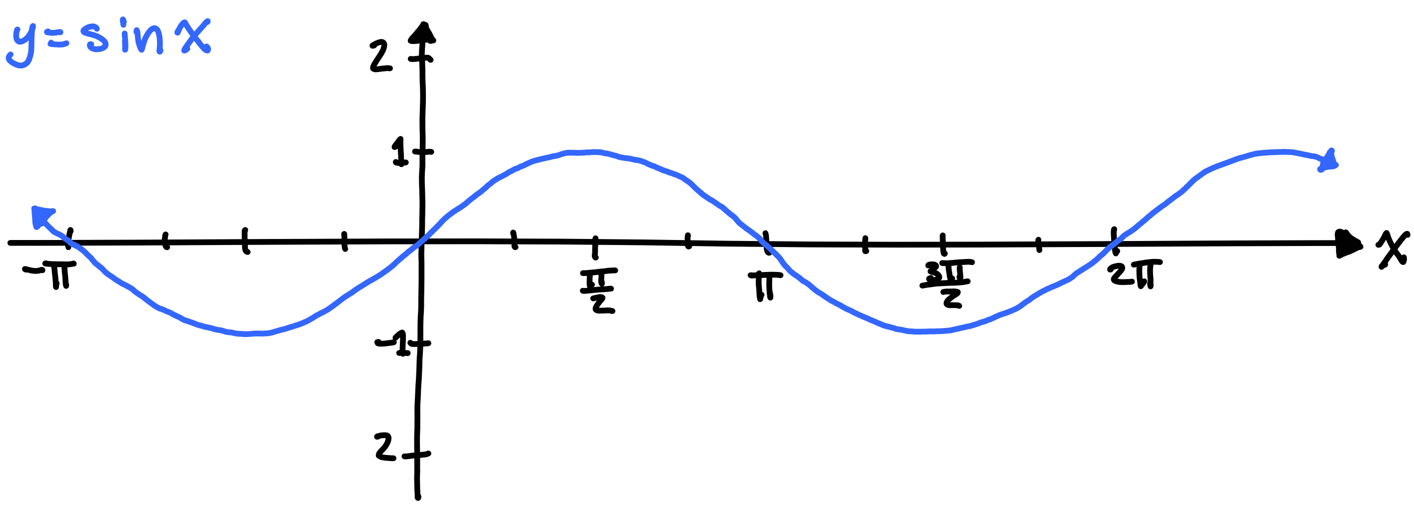
Subtract from $x \Rightarrow$ Shift right

Add to $x \Rightarrow$ Shift left

$$\text{Phase Shift} \equiv \phi/\omega$$

Example: $y = \sin\left(2x - \frac{\pi}{2}\right)$

$\omega = ?$ $\phi = ?$ PHASE SHIFT $= ?$



PHASE SHIFT EXPLANATION

BASIC ALGEBRA: FACTORING AND SMART ONE

$$\omega x - \phi = \omega x - \left(\frac{\omega}{\omega}\right)\phi = \omega\left(x - \phi/\omega\right)$$

ϕ/ω IS SUBTRACTED FROM x, NOT JUST ϕ.

Vertical stretch or compression;
reflection about x-axis if negative

Vertical shift

$$y = A f\left[\omega\left(x - \underbrace{\varphi/\omega}_{\text{Horizontal shift}}\right)\right] + B$$

Horizontal stretch or compression;
reflection about y-axis if negative