Proving Identities

Reciprocal and Quotient Identities

$\sec\theta \equiv \frac{1}{\cos\theta}$ $\qquad$ $\cos\theta \equiv \frac{1}{\sec\theta}$

$\csc\theta \equiv \frac{1}{\sin\theta}$ $\qquad$ $\sin\theta \equiv \frac{1}{\cos\theta}$

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta} \equiv \frac{1}{\cot\theta} \equiv \frac{\sec\theta}{\csc\theta}$$

$$\cot\theta \equiv \frac{\cos\theta}{\sin\theta} \equiv \frac{1}{\tan\theta} \equiv \frac{\csc\theta}{\sec\theta}$$

Pythagorean Identities $x^2 + y^2 = r^2$

$\frac{x^2 + y^2 = r^2}{x^2}$ $\qquad$ $1 + \tan^2\theta \equiv \sec^2\theta$

$\frac{x^2 + y^2 = r^2}{y^2}$ $\qquad$ $\cot^2\theta + 1 \equiv \csc^2\theta$

$\frac{x^2 + y^2 = r^2}{r^2}$ $\qquad$ $\cos^2\theta + \sin^2\theta \equiv 1$

Odd and Even Identities

$\sin(-\theta) \equiv -\sin\theta$

$\cos(-\theta) \equiv \cos\theta$

$\tan(-\theta) \equiv -\tan\theta$

$\cot(-\theta) \equiv -\cot\theta$

$\sec(-\theta) \equiv \sec\theta$

$\csc(-\theta) \equiv -\csc\theta$

Cofunction Identities

$\sin(\theta) \equiv \cos(\pi/2-\theta)$

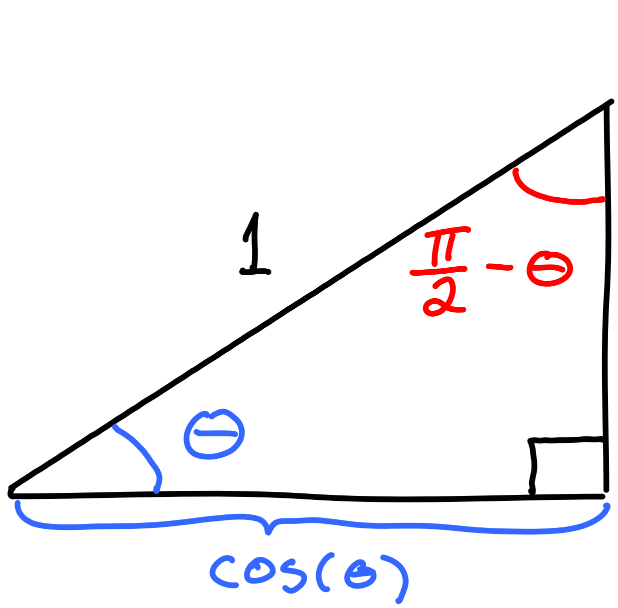
$\cos(\theta) \equiv \sin(\pi/2-\theta)$

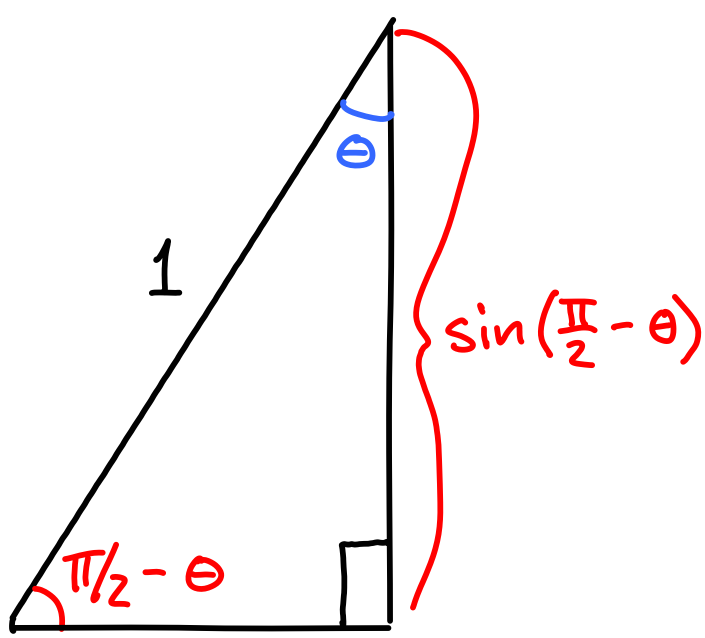
$\tan(\theta) \equiv \cot(\pi/2-\theta)$

$\cot(\theta) \equiv \tan(\pi/2-\theta)$

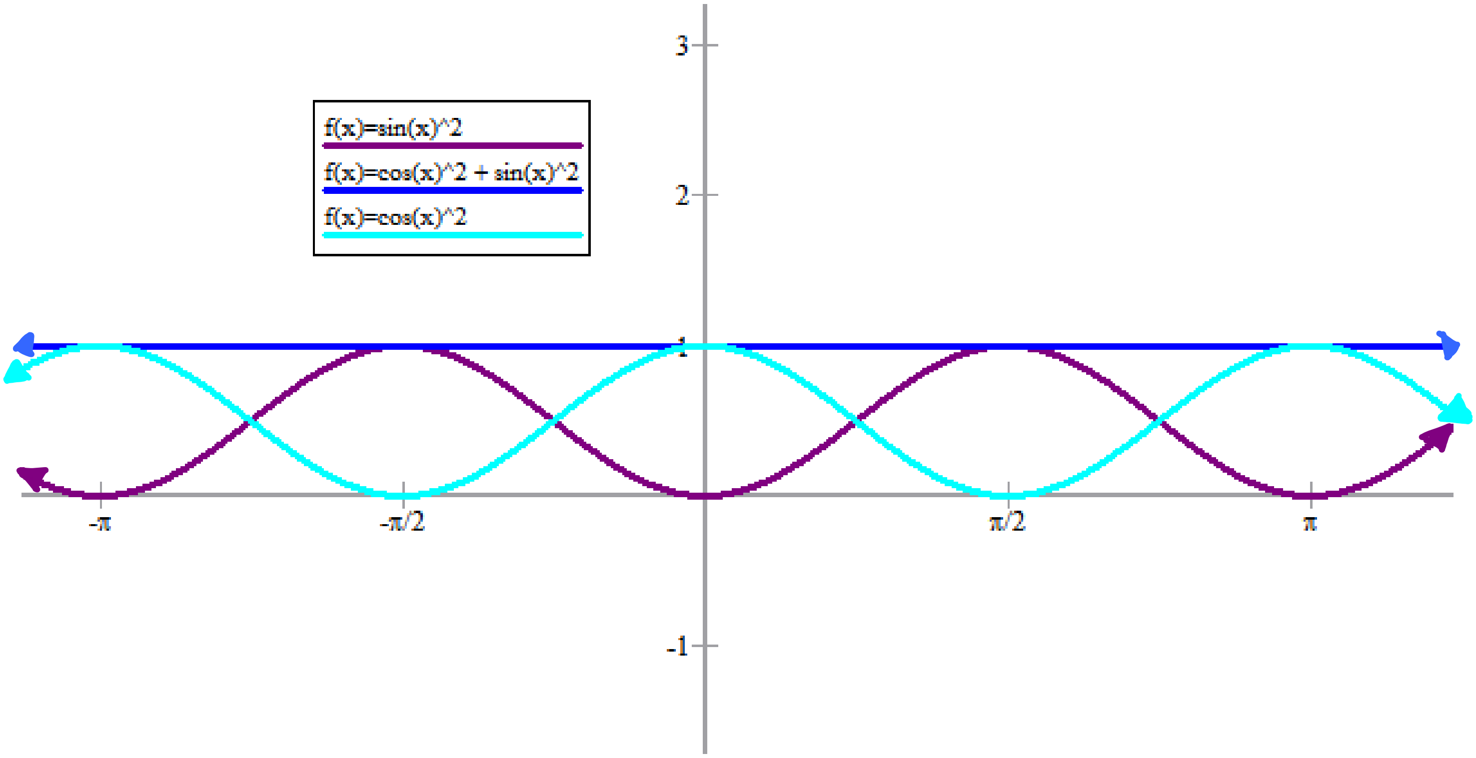
$\sec(\theta) \equiv \csc(\pi/2-\theta)$

$\csc(\theta) \equiv \sec(\pi/2-\theta)$





What Does "Identity" Mean?



Calculus 2: Find the area between the x axis and the graph of $\dfrac{\tan(x)}{\sec(x)}$

$$\frac{\tan(x)}{\sec(x)}$$

The Conjugate Trick

Note: $\cos^2\theta \equiv 1-\sin^2\theta \equiv 1^2-\sin^2\theta \equiv (1-\sin\theta)(1+\sin\theta)$

Prove the identity $\dfrac{\cos\theta}{1+\sin\theta} \equiv \dfrac{1-\sin\theta}{\cos\theta}$

$$\frac{\cos\theta}{1+\sin\theta} \equiv$$

Could't I just cross multiply? Sometimes.

Notation: $\Rightarrow$ $\Leftarrow$ and $\Leftrightarrow$

$A \Rightarrow B$ "A implies B" or "If A then B"

$A \Leftarrow B$ "B implies A" or "If B then A"

$A \Leftrightarrow B$ "A if and only if B"

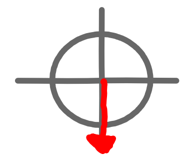
If and only if $\rightarrow$ iff

Alternate Proof of $\dfrac{\cos\theta}{1+\sin\theta} \equiv \dfrac{1-\sin\theta}{\cos\theta}$

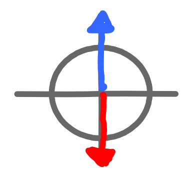
$$\sin^2\theta + \cos^2\theta \equiv 1 \qquad \theta \in \mathbb{R}$$

$$\Leftrightarrow \quad \cos^2\theta \equiv 1 - \sin^2\theta \equiv (1-\sin\theta)(1+\sin\theta)$$

$$\Rightarrow \quad \frac{\cos^2\theta}{1+\sin\theta} \equiv 1 - \sin\theta \qquad \bigstar\ \theta \neq -\frac{\pi}{2} + 2n\pi$$



$$\Rightarrow \quad \frac{\cos\theta}{1+\sin\theta} \equiv \frac{1-\sin\theta}{\cos\theta} \qquad \bigstar\ \theta \neq \frac{\pi}{2} + n\pi$$



PROVE $\dfrac{1+\sin\theta}{\sin\theta} + \dfrac{\cot\theta - \frac{1}{\sec\theta}}{\cos\theta} \equiv 2\csc\theta$

$$\frac{1+\sin\theta}{\sin\theta} + \frac{\cot\theta - \frac{1}{\sec\theta}}{\cos\theta} \equiv$$

PROVE $\csc\theta\tan\theta \equiv \sec\theta$

$\csc\theta\tan\theta \equiv$

PROVE $-\sin^2(-\theta) - \cos^2(-\theta) \equiv -1$

$-\sin^2(-\theta) - \cos^2(-\theta) \equiv$

PROVE $$\frac{\cos^2(-\theta)-\sin^2(-\theta)}{\sin(-\theta)-\cos(-\theta)} \equiv \sin\theta-\cos\theta$$

$$\frac{\cos^2(-\theta)-\sin^2(-\theta)}{\sin(-\theta)-\cos(-\theta)} \equiv$$

PROVE $$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \equiv 2\csc\theta$$

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \equiv$$

PROVE $\dfrac{\tan\theta + \cot\theta}{\sec\theta \csc\theta} \equiv 1$

$\dfrac{\tan\theta + \cot\theta}{\sec\theta \csc\theta} \equiv$

Prove $\sin(\tan^{-1}(t)) \equiv \dfrac{t}{\sqrt{1+t^2}}$

$\sin(\tan^{-1}(t)) \equiv$

STRATEGY

- KEEP IN MIND YOUR TARGET FORM
- DON'T SUBSTITUTE OR FACTOR "JUST BECAUSE YOU CAN."
- HOW MANY DIFFERENT KINDS OF TRIG FUNCTIONS ARE ON EACH SIDE OF THE IDENTITY?

  YOU ONLY NEED 2 TRIG FUNCTIONS.
- HOW MANY TERMS ARE ON EACH SIDE OF THE IDENTITY?

  FIND WAYS TO CONVERT BETWEEN SUMS AND PRODUCTS.
- IS THERE A FRACTION ON EACH SIDE OF THE IDENTITY?
- WHAT DOES THE LEFT HAND SIDE HAVE/NOT HAVE IN COMMON WITH THE RIGHT HAND SIDE?
- REDUCING THE BIG SIDE TO THE SMALLER SIDE IS OFTEN EASIER.
- WILL CONVERTING TO SINES AND COSINES HELP?
- USE "$\Rightarrow$" and "$\Leftrightarrow$" for a less restricted approach to proving identities.

PROVE $$\frac{1+\tan\theta}{1-\tan\theta} \equiv \frac{\cot\theta+1}{\cot\theta-1}$$

PROVE $$\frac{\sin^3\theta + \cos^3\theta}{1 - 2\cos^2\theta} \equiv \frac{\sec\theta - \sin\theta}{\tan\theta - 1}$$

NOTE: $1 - 2\cos^2\theta \equiv 1 - \cos^2\theta - \cos^2\theta$

$$\frac{\sin^3\theta + \cos^3\theta}{1 - 2\cos^2\theta}$$

$$\frac{\sec\theta - \sin\theta}{\tan\theta - 1}$$

PROVE $\dfrac{\cot\theta}{1-\tan\theta}+\dfrac{\tan\theta}{1-\cot\theta}\equiv 1+\tan\theta+\cot\theta$

$\dfrac{\cot\theta}{1-\tan\theta}+\dfrac{\tan\theta}{1-\cot\theta}$ $\quad$ $1+\tan\theta+\cot\theta$

NOTE ON NOTATION

LEARN TO DEVELOPE YOUR OWN NOTATION WHEN IT IS USEFUL.

SOMETIMES I WRITE $\$(x)$ FOR $\sin(x)$ AND $¢(x)$ FOR $\cos(x)$ JUST TO SAVE TIME WRITING.

OF COURSE I HAVE TO STATE THIS AT THE BEGINNING OF MY WORK.

$$\$(x) := \sin(x) \quad \text{AND} \quad ¢(x) := \cos(x)$$

THE ":=" SYMBOL IS OFTEN USED TO MEAN "IS DEFINED TO BE."

DON'T FORGET THAT YOU CAN USE THE DEFINITIONS BASED ON A CIRCLE,

$\tan\theta = y/x \qquad \cos\theta = x/r$ etc.

THIS MAY REDUCE WRITING AND PROVIDE EXTRA PERSPECTIVE.

IF THIS IS OVER YOUR HEAD THEN REVIEW YOUR ALGEBRA