Double Angle and Half Angle Identities

Don't Forget

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

OR IF YOU WANT TO BE CREATIVE

$$\cos(\text{Tom} \pm \text{Jerry}) = \cos(\text{Tom})\cos(\text{Jerry}) \mp \sin(\text{Tom})\sin(\text{Jerry})$$
$$\sin(\text{Cheech} \pm \text{Chong}) = \sin(\text{Cheech})\cos(\text{Chong}) \pm \cos(\text{Cheech})\sin(\text{Chong})$$

PRACTICE USING DIFFERENT NOTATIONS!

WRITE $\sin(2\theta)$ IN TERMS OF $\cos\theta$ AND $\sin\theta$

WRITE $\sin\theta$ IN TERMS OF $\cos\left(\frac{\theta}{2}\right)$ AND $\sin\left(\frac{\theta}{2}\right)$

Try to do it just by thinking about the previous problem.

WRITE $\cos(2\theta)$

a) IN TERMS OF $\cos\theta$ AND $\sin\theta$

b) IN TERMS OF $\cos\theta$ ONLY

c) IN TERMS OF $\sin\theta$ ONLY

DOUBLE ANGLE IDENTITIES

JUST THE SPECIAL CASE OF SUM IDENTITY WHEN $\alpha = \beta = \theta$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - \sin^2\theta \end{cases}$$

OR ARE THEY HALF ANGLE IDENTITIES?

AFTERALL, θ IS HALF OF 2θ

USE THE DOUBLE ANGLE IDENTITIES TO DERIVE "POWER REDUCING" IDENTITIES

a) Find $\sin^2\theta$ in terms of $\cos(2\theta)$

b) Find $\cos^2\theta$ in terms of $\cos(2\theta)$

NOW DERIVE THE HALFANGLE IDENTITIES

POWER REDUCING $\Leftrightarrow$ HALF ANGLE

$$\sin^2(\theta) = \frac{1-\cos(2\theta)}{2} \Leftrightarrow \sin^2(\theta/2) = \frac{1-\cos\theta}{2}$$

$$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2} \Leftrightarrow \cos^2(\theta/2) = \frac{1+\cos\theta}{2}$$

NOTICE YOU CAN TRADE EXPONENTS FOR MULTIPLE ANGLES

WRITE $\sin^4\theta$ IN TERMS OF COSINE AND/OR SINE WITHOUT EXPONENTS.

WRITE $\cos(4\theta)$ AS A 4th DEGREE POLYNOMIAL IN $\cos\theta$

REMEMBER WE FOUND THAT

$$\sin(15°) = \frac{\sqrt{6}-\sqrt{2}}{4} \quad \text{AND} \quad \cos(15°) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

USING THE SUM AND DIFFERENCE IDENTITIES

FIND $\cos(15°)$ USING THE HALF ANGLE IDENTITY AND SHOW YOUR RESULT MATCHES THE RESULT WE FOUND PREVIOUSLY USING THE SUM AND DIFFERENCE IDENTITIES.

WRITE $\tan(2\theta)$ IN TERMS OF $\tan\theta$ ONLY

WRITE $\tan(\theta/2)$ IN TERMS OF $\cos\theta$ USING THE HALF ANGLE IDENTITIES FOR SINE AND COSINE.

REMEMBER THAT BOTH THE HALF ANGLE IDENTITIES CAME FROM THE DOUBLE ANGLE IDENTITY FOR COSINE

WRITE TAN$(\theta/2)$ IN TERMS OF SINθ AND COSθ WITHOUT THE AMBIGUOUS $\pm$

A) USING BOTH, THE SINE AND COSINE DOUBLE ANGLE IDENTITIES

B) USING THE FIRST VERSION OF TAN$(\theta/2)$ AND THE PYTHAGOREAN THEOREM

C) USE YOUR IDENTITY FROM PART A AND THE CONJUGATE TRICK TO GET A 3RD FORM.

TANGENT HALF ANGLE IDENTITIES

$$\boxed{\tan(\theta/2) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}}$$

EVALUATE $\cos\left[2\tan^{-1}\left(-3/4\right)\right]$

IF $\tan\theta = \frac{2}{3}$ AND $\theta \in [\pi, 3\pi/2)$, FIND $\sin(\theta/2)$ AND $\cos(2\theta)$

IF $\cos\theta = x/r$ AND $\sin\theta = y/r$ FIND

A) $\cos(2\theta)$ AND $\sin(2\theta)$

B) $\cos(\theta/2)$ AND $\sin(\theta/2)$

C) $\tan(2\theta)$ AND $\tan(\theta/2)$

IN TERMS OF x, y, AND r

PROVE $\frac{1}{8}\left[1-\cos(4\theta)\right] \equiv \sin^2\theta\cos^2\theta$

Evaluate $\cos(\pi/24)$ AND $\cos(\pi/48)$