Introduction to Complex Numbers

Solve for x: $x^2+1=0$

Solve for x: $x^2+2x+2=0$

$i := \sqrt{-1}$

IMAGINARY NUMBERS: $\mathbb{I} := \{ yi : y \in \mathbb{R} \}$

EXAMPLES: $\sqrt{-7} = \sqrt{7}\, i \qquad -\sqrt{-3} = -\sqrt{3}\, i$

$\sqrt{-9} = 3i \qquad \sqrt{-8} = 2\sqrt{2}\, i$

COMPLEX NUMBERS: $\mathbb{C} := \{ z = x + yi : x, y \in \mathbb{R} \}$

$$x + yi$$

REAL PART IMAGINARY PART

x AND y ARE BOTH REAL NUMBERS

EXAMPLES

$-2 + 3i \longrightarrow x = \qquad y =$

$\sqrt{2} - i/3 \longrightarrow x = \qquad y =$

$1 + i \longrightarrow x = \qquad y =$

$3 \longrightarrow x = \quad y =$

3 IS A PURELY REAL COMPLEX #

$$\boxed{\mathbb{R} \subset \mathbb{C}}$$

$2i \longrightarrow x = \quad y =$

$2i$ IS A PURELY IMAGINARY COMPLEX #

$$\boxed{\mathbb{I} \subset \mathbb{C}}$$

ADDITION OF COMPLEX NUMBERS

FIND $z_1 + z_2$: $z_1 = 3+2i$ $z_2 = 4-5i$

MULTIPLICATION OF COMPLEX NUMBERS

FIND $z_1 z_2$: $z_1 = 3+2i$ $z_2 = 4-5i$

FIND i^n WHERE $n \in \mathbb{N}$

DIVIDING COMPLEX NUMBERS

FIND z_1/z_2: $z_1 = 3+2i$ $z_2 = 4-5i$

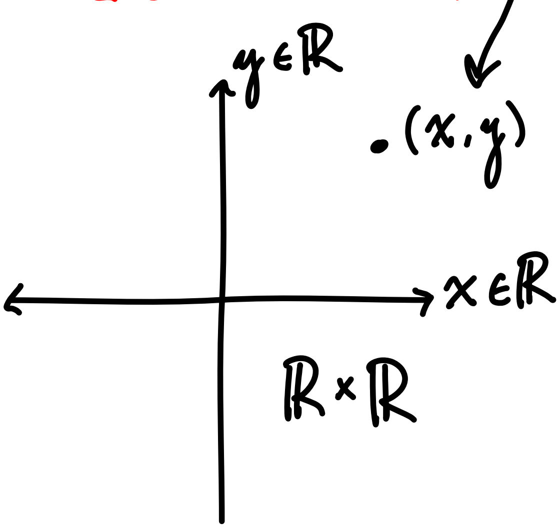
COMPLEX CONJUGATE OF $z = x+yi$ IS $\bar{z} = x-yi$

FIND $z\bar{z}$

Visualizing the ℝeal and ℂomplex Numbers

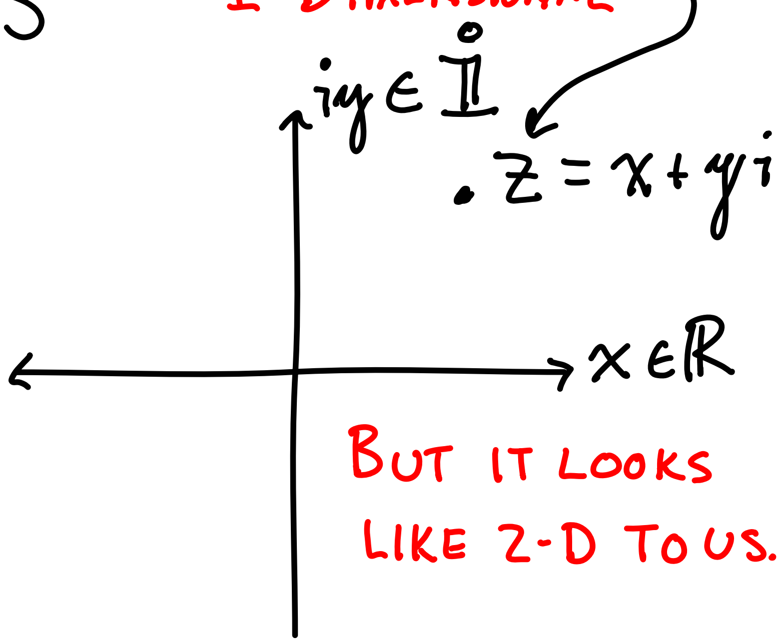
The Cartesian Plane VS The Complex Plane

2 Dimensional



VS

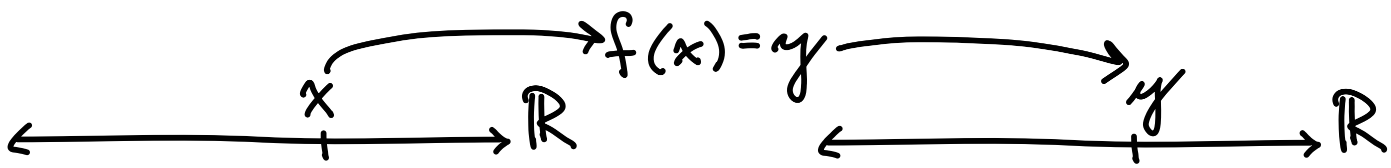
1 Dimensional



But it looks like 2-D to us.

The Cartesian Plane

- 1 point has 2 numbers
- (x, y)
- Used to visualize single variable functions



- Composed of 2 copies of the real numbers

The Complex Plane

- 1 point consists of 1 number
- $z = x + yi$
- Composed of 1 copy of the complex numbers

$\boxed{\text{Composed of 2 copies of the real numbers and products with } \sqrt{-1}}$

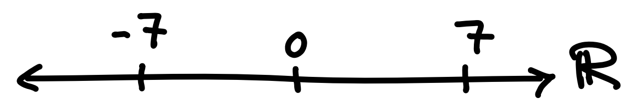
INEQUALITIES

Is $3<7$ or $3>7$?

Is $1+3i < 3+i$ or $3+i < 1+3i$?

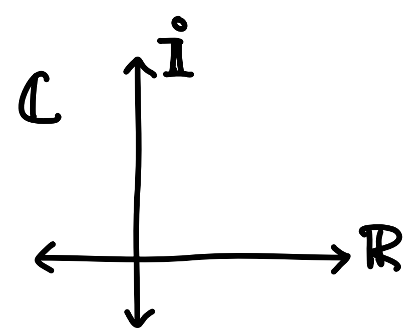
ABSOLUTE VALUE = RADIUS = DISTANCE FROM ORIGIN

$|-7| =$

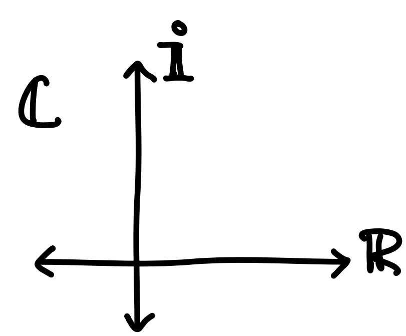


$|7| =$

$|1+3i| =$



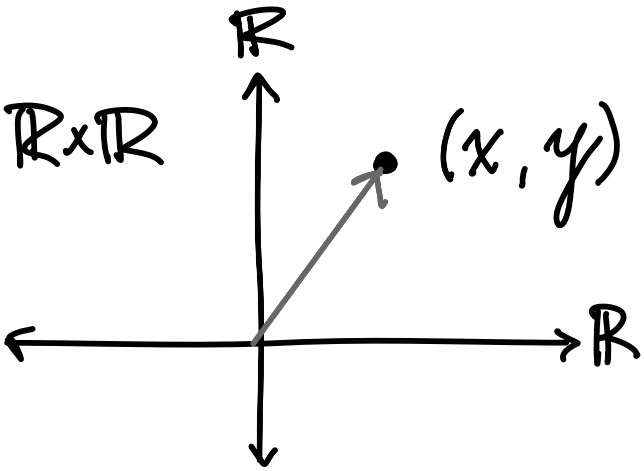
$|3+i| =$

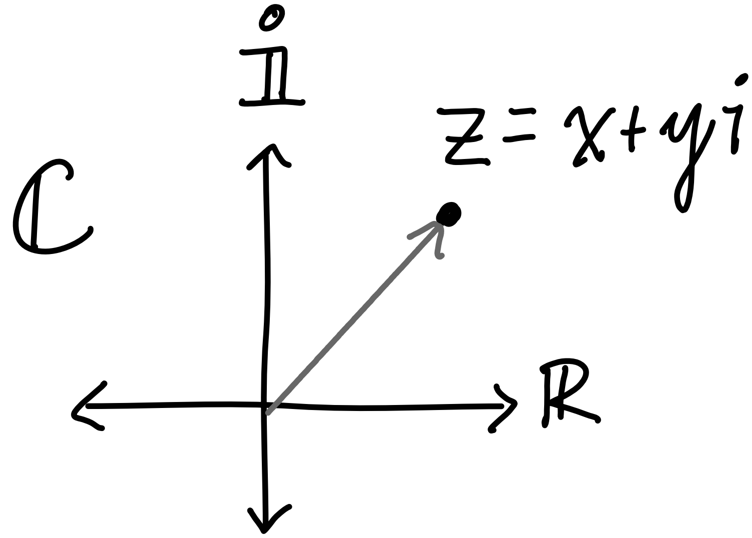


$$r = |z| = \sqrt{x^2+y^2}$$

Cartesian Plane

Complex Plane





$\cos\theta = \frac{x}{r} \Rightarrow$

$\sin\theta = \frac{y}{r} \Rightarrow$

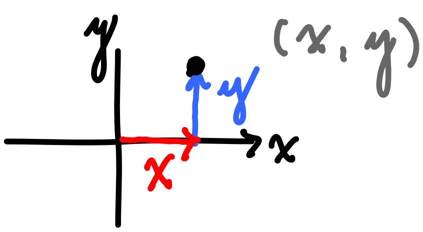
$r =$

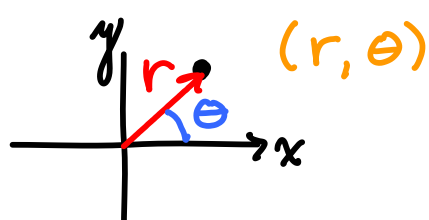
$\theta =$

$\Rightarrow z = x + iy =$

Standard or Cartesian coordinate: (x, y)

Polar coordinate: (r, θ)





Standard Form: $z = x + iy$

$\boxed{\text{Polar Form: } z = r(\cos\theta + i\sin\theta)}$

IF POSITIVE IS FORWARD AND NEGATIVE IS BACKWARD, THEN $+i$ IS ONE TO THE LEFT AND $-i$ IS ONE TO THE RIGHT

IN WHICH DIRECTION IS $1+i$ AND HOW FAR AWAY IS IT?

IN OTHER WORDS, WRITE $1+i$ IN POLAR FORM.

CONVERT $2\sqrt{3}+2i$ TO POLAR FORM

CONVERT $3+2i$ AND $-3-2i$ POLAR FORM

What good is polar form?

If $z_1 = r_1[\cos\theta_1 + i\sin\theta_1]$ and $z_2 = r_2[\cos\theta_2 + i\sin\theta_2]$
Find $z_1 z_2$

$z_1 z_2 =$

When Multiplying Complex Numbers
Angles Add and Radii Multiply

Key to
Entire Section

SHOW THAT ANGLES SUBTRACT AND RADII DIVIDE WHEN DIVIDING COMPLEX NUMBERS

CONVERT THE FOLLOWING TO POLAR FORM AND THEN MULTIPLY THEM. CHECK YOUR ANSWER.

A) $z_1 = -7$ AND $z_2 = 3$

$r_1 =$ $\theta_1 =$

$r_2 =$ $\theta_2 =$

$\Rightarrow \begin{cases} z_1 = \\ z_2 = \end{cases}$

$\Rightarrow z_1 z_2 =$

B) $z_1 = 1 + i \quad z_2 = 1 - i$

$r_1 =$ $\qquad\qquad$ $\theta_1 =$

$r_2 =$ $\qquad\qquad$ $\theta_2 =$

$$\Rightarrow \begin{cases} z_1 = \\ z_2 = \end{cases}$$

$$\Rightarrow z_1 z_2 =$$

FIND i^n WHERE $n \in \mathbb{N}$ USING POLAR FORM

FIND $(1+i)^8$

FIND $(1+i)^3(2-2i)^3(-3+\sqrt{3}i)^4$

FIND 1^3, $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$, AND $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^3$

FIND A FORMULA FOR z^n

SOLVE FOR X: $x^3 - 1 = 0$

SOLVE FOR X: $x^3 + 1 = 0$

SOLVE FOR x: $x^3 - i = 0$

FIND A FORMULA FOR $\sqrt[3]{z}$

SOLVE FOR z_k: $z = z_k^3$

FIND A FORMULA FOR THE n-th ROOTS OF z

SOLVE FOR z_k: $z = z_k^n$

IF THE SOLUTIONS TO A POLYNOMIAL EQUATION ARE 3, $2+i$, AND $1-i$,

WHAT IS THE POLYNOMIAL?

FUNDAMENTAL THEOREM OF ALGEBRA

A POLYNOMIAL OF N-TH DEGREE HAS N-FACTORS

Properties of Roots and Polynomials

Euler's (Oiler's) Number: $e = 2.71\ldots$

$$e = \lim_{n\to\infty}\left(1+1/n\right)^n = 2.71\ldots$$

$$z = x + yi = r[\cos\theta + i\sin\theta] = re^{i\theta}$$

Complex Exponential Notation

$$z = re^{i\theta}$$

Write $z = -1$ as a complex exponential

PROVE $\arctan(1) + \arctan(2) + \arctan(3) = \pi$

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