#1 Completely factor the polynomial: $z^6 - 1 = 0$

A) $(z-1)(z+1)(z-i)(z+i)\left[z - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right]\left[z + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right] = 0$

B) $(z-1)(z+1)\left[z - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right]\left[z + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right]\left[z - \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right]\left[z + \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right] = 0$

C) $\boxed{(z-1)(z+1)\left[z - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]\left[z + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]\left[z - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]\left[z + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right] = 0}$

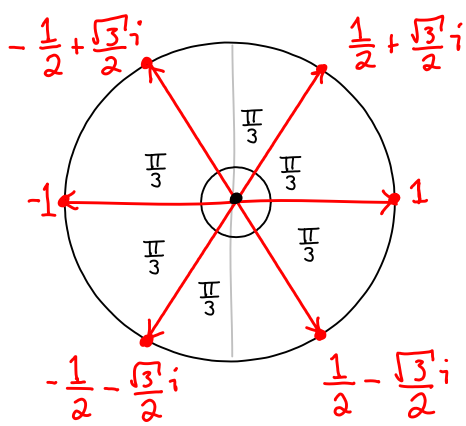
D) $(z-1)(z+1)\left[z - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\right]\left[z + \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\right]\left[z - \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right]\left[z + \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right] = 0$

E) None Of The Above

The Quick Solution

$$z^6 - 1 = 0 \iff z^6 = 1 \iff z = \sqrt[6]{1}$$

has 6 solutions equally spaced around a circle of radius 1 since $1^6 = 1$. A circle has 2π radians, so if the 6 solutions are equally spaced, there must be an angle of $2\pi/6 = \pi/3$ between them. So our solutions must be:



Once you know the solutions you can simply write down the factors. They look like $(z - \text{solution})$, so our factors are: $(z-1)$, $[z-(-1)] = (z+1)$,

$\left[z - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]$, $\left[z - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]$, $\left[z - \left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)\right] = \left[z + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]$,

$\left[z - \left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)\right] = \left[z + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]$. Put them together and we have

$$\boxed{z^6 - 1 = (z-1)(z+1)\left[z - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]\left[z - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]\left[z + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]\left[z + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right] = 0}$$

Notice our solutions came in complex conjugate pairs. This is because the coefficients of the polynomial are all real. $z^6 - 1 = 1 \cdot z^6 - 1$

A Longer Solution

$\sqrt[6]{1} = \sqrt[6]{1[\cos(0)+i\sin(0)]}$ Angles of the solutions will be those angles such that if we add them to themselves 6 times we get an angle of zero or any angle equivalent to zero (any angle pointing towards the number 1).

Mathematically stated $6\theta_K = 0 + 2\pi K \Rightarrow \theta_K = \frac{2\pi K}{6} = \frac{\pi K}{3}$

$$\Rightarrow \theta_0 = 0,\ \theta_1 = \frac{\pi}{3},\ \theta_2 = \frac{2\pi}{3},\ \theta_3 = \pi,\ \theta_4 = \frac{4\pi}{3},\ \text{and}\ \theta_5 = \frac{5\pi}{3}$$

So our solutions are:

$z_0 = \sqrt[6]{1}[\cos(0)+i\sin(0)] = 1$

$z_1 = \sqrt[6]{1}[\cos(\pi/3)+i\sin(\pi/3)] = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$z_2 = \sqrt[6]{1}[\cos(2\pi/3)+i\sin(2\pi/3)] = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$z_3 = \sqrt[6]{1}[\cos(\pi)+i\sin(\pi)] = -1$

$z_4 = \sqrt[6]{1}[\cos(4\pi/3)+i\sin(4\pi/3)] = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$z_5 = \sqrt[6]{1}[\cos(5\pi/3)+i\sin(5\pi/3)] = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

which we can again put in the factor form (z - solution) and multiply the factors together to get our complete solution. What we did above is completely equivalent to using the

N-th Root Formula

$$\boxed{\sqrt[n]{z} = \sqrt[n]{r[\cos\theta + i\sin\theta]} = \sqrt[n]{r}\left[\cos\left(\frac{\theta}{n} + \frac{2\pi K}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi K}{n}\right)\right];\ K = 0, 1, \ldots, n-1}$$

Notation

Writing $r[\cos\theta + i\sin\theta]$ can get tiring. The $\cos\theta + i\sin\theta$ part is often abreviated with $\text{cis}\theta$. This is completely legit, as we can name functions anything we want.

$$\boxed{\text{cis}\theta \equiv \cos\theta + i\sin\theta}$$

The "$\equiv$" symbol means "identitically equal to" or "defined as".

#2 Find $(-2+\sqrt{3}i)^6$ in polar form.

A) $7^3\left(\cos\left[6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] + i\sin\left[6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]\right)$

B) $7^3\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right]$

C) $7^3\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right]$

D) $\boxed{7^3\left(\cos\left[6\pi - 6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right] + i\sin\left[6\pi - 6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]\right)}$

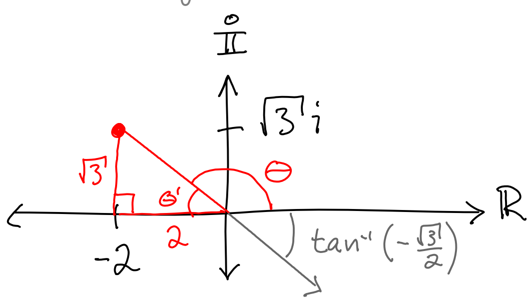
E) None Of The Above

The radius of $-2+\sqrt{3}i$ is $r=\sqrt{x^2+y^2}=\sqrt{(-2)^2+(\sqrt{3})^2}=\sqrt{4+3}=\sqrt{7}$

To see if we can find the angle by using our knowledge of the unit circle, factor out the radius: $-2+\sqrt{3}i=\sqrt{7}\left(-\frac{2}{\sqrt{7}}+\frac{\sqrt{3}}{\sqrt{7}}i\right)$

Unfortunately this is not a recognizable number on the unit circle. We'll have to use tangent. There's a couple of ways to approach it. Both are equivalent.

Here's one way to do it. Draw a triangle. Solve for θ'. Use θ' to get θ.



$$\tan\theta'=\frac{opp}{adj}=\frac{\sqrt{3}}{2}\Rightarrow\theta'=\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta=\pi-\theta'=\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Another way is to use the unit circle definition of tangent.

$\tan\theta=\frac{y}{x}=-\frac{\sqrt{3}}{2}\Rightarrow\theta=\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ Be careful. This is not our angle.

Remember that the range of tangent inverse is $(-\pi/2,\pi/2)$ so this can't be our angle because $-2+\sqrt{3}i$ is in quadrant 2. We must add π to the angle above. It would be better to write

$$\tan\theta'=\frac{y}{x}=-\frac{\sqrt{3}}{2}\Rightarrow\theta'=\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)\Rightarrow\theta=\theta'+\pi=\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\pi$$

So, depending on how we choose to solve for θ, we get 2 different but equal answers.

$\theta=\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)=\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\pi$. This equivalence comes from the fact tangent is an odd function. Pick one. I'll use $\theta=\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

So, $(-2+\sqrt{3}i)^6=\left[\sqrt{7}\left(\cos\left[\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]+i\sin\left[\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]\right)\right]^6$

$$=(\sqrt{7})^6\left(\cos\left[6\left(\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]+i\sin\left[6\left(\pi-\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]\right)$$

$$=\boxed{7^3\left(\cos\left[6\pi-6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]+i\sin\left[6\pi-6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]\right)=(-2+\sqrt{3}i)^6}$$

#3 Which of the following is completely true?

$\boxed{\text{A) } z\bar{z} = r^2, \quad z = x+yi = r(\cos\theta + i\sin\theta) = re^{i\theta}, \quad \sqrt{i} = \pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)}$

B) $z\bar{z} = r, \quad \bar{z} = x+yi \Rightarrow z = x-yi, \quad \sqrt{-i} = -\sqrt{i}$

C) $|z| = \sqrt{z\bar{z}} = r, \quad z = x+yi = r\cos\theta + i\sin\theta, \quad 0 = e^{i\pi}+1$

D) $\overline{2i} = -2i, \quad \bar{z} = z^2, \quad -\bar{2} = -2$

E) NONE OF THE ABOVE

The complex conjugate of a number $z = x+yi$ is denoted with a bar over the variable, $\bar{z} = x - yi$. To get the complex conjugate of a number, just replace i with $-i$ everywhere it appears. Some people use a star $z^* = x - yi$ to denote the complex conjugate.

A) • $z\bar{z} = (x+yi)(x-yi) = x^2 - xyi + xyi - y^2i^2 = x^2+y^2 = r^2$

• $z = x+yi = r(\cos\theta + i\sin\theta) = re^{i\theta}$ is true. Remember your basic trig definitions: $\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$ and $\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

Plug these into $x+yi$ to get $r(\cos\theta + i\sin\theta)$. The fact that $r(\cos\theta + i\sin\theta) = re^{i\theta}$ was given to you in class. I did not prove it.

• $\sqrt{i} = \sqrt{1\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right]}$ which has two solutions. The angles of the solutions are equally spaced around a circle of radius 1. Also, since we are looking for a number whose square is i, and we know angles add when multiplying numbers, our solutions must have angles such that if you add it to itself you get an angle that points toward the number i which has an angle of $\pi/2$ or anything equivalent to $\pi/2$. Mathematically,

$$\theta_k + \theta_k = 2\theta_k = \frac{\pi}{2} + 2\pi k \Leftrightarrow \theta_k = \frac{\pi}{4} + \pi k \Rightarrow \theta_0 = \frac{\pi}{4}, \ \theta_1 = \frac{5\pi}{4}$$

So, our solutions are

$$1\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right] = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \quad \text{and} \quad 1\left[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right] = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

This can be done in your head in a matter of seconds. Angles add, radii multiply.

B) • $z\bar{z} = r^2 \neq r$

• $\bar{z} = x + yi \Leftrightarrow z = \bar{\bar{z}} = \overline{x+yi} = x - yi$

• $\sqrt{-i} = \sqrt{1\left[\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right]}$ which has two solutions with radius $r = 1$

and angles such that $2\theta_k = \frac{3\pi}{2} + 2\pi k \Leftrightarrow \theta_k = \frac{3\pi}{4} + \pi k$.

So our solutions are $z_0 = 1\left[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right] = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ and

$z_1 = 1\left[\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right] = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$. Summarized by $\mp\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i$

We found in part A that the solutions to $\sqrt{i}$ were $\pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$. So

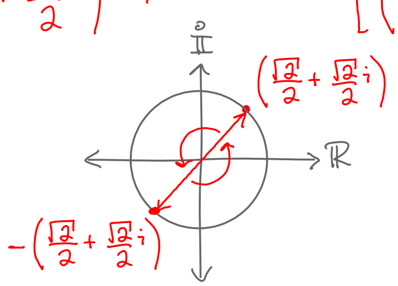
$-\sqrt{i} = -\left[\pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right] = \mp\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \sqrt{i} \neq \mp\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i = \sqrt{-i}$

So, $\sqrt{-i} \neq -\sqrt{i}$.

However, it is interesting to note that $-\sqrt{i}$ shares the same solutions as $\sqrt{i}$. This is no coincidence. It turns out that $-\sqrt{z} = \sqrt{z}$ for any z. Square roots have 2 solutions equally spaced around a circle. This means they must be on opposite sides of the circle. Multiplying by -1 is a rotation by π: $-1 = 1[\cos(\pi) + i\sin(\pi)]$. Angles add when multiplying and -1 has an angle of π. So multiplying two numbers on opposite sides of a circle just makes them switch places.

$x^2 = 4 \Rightarrow x = \pm 2$ and $-1(\pm 2) = \mp 2$ Likewise,

$\sqrt{i} = \pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) \Rightarrow -\sqrt{i} = -1\left[\pm\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right] = \mp\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$



C) • $|z| = \sqrt{x^2+y^2} = \sqrt{(x+yi)(x-yi)} = \sqrt{z\bar{z}} = r$

• $z = x+yi = r\cos\theta + (r\sin\theta)i = r(\cos\theta + i\sin\theta) \neq r\cos\theta + i\sin\theta$

Both $\cos\theta$ and $i\sin\theta$ must be multiplied by r. This comes from the fact that $\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$ and $\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

• $z = r(\cos\theta + i\sin\theta) = re^{i\theta} \Leftrightarrow -1 = 1[\cos(\pi) + i\sin(\pi)] = 1e^{i\pi} \Leftrightarrow 0 = e^{i\pi}+1$

This is considered by some people to be

THE MOST BEAUTIFUL EQUATION

$$\boxed{0 = e^{i\pi}+1}$$

in all of mathematics.

D) • $\overline{2i} = -2i$ by the basic definition of the complex conjugate.

• $z = x+yi \Leftrightarrow \bar{z} = \overline{x+yi} = x-yi \Leftrightarrow \bar{\bar{z}} = \overline{x-yi} = x+yi = z \neq z^2$

• $-\bar{2} = -(\overline{2+0i}) = -(2-0i) = -2$

The complex conjugate of any real number is itself since $\bar{x} = \overline{x+0i} = x-0i = x$.

#4 Which of the following is completely true?

$\boxed{\text{A) } \ln[e^{\ln(e)}] = 1}$

C) $\ln(\ln[\ln(e^e)]) = 1$

B) $e^{\ln(e^{\ln(e)})} = 1$

D) ALL OF THE ABOVE

E) NONE OF THE ABOVE

A) $\ln[e^{\overbrace{\ln(e)}^{1}}] = \ln(e^1) = \ln(e^1) = 1$

B) $e^{\ln(e^{\overbrace{\ln(e)}^{1}})} = e^{\underbrace{\ln(e^1)}_{1}} = e^1 = e \neq 1$

C) $\ln(\ln[\underbrace{\ln(e^e)}_{e}]) = \ln[\overbrace{\ln(e)}^{1}] = \ln(1) = 0 \neq 1$

#5 Which of the following is completely true?

A) $\log_{\ln(e^2)}([\ln(e^2)]^2) = 1$

B) $\boxed{\log_x(x^x) = x}$

C) $x^{\log_x(x^x)} = x$

D) All Of The Above

E) None Of The Above

A) $\log_{\underbrace{\ln(e^2)}_{2}}([\underbrace{\ln(e^2)}_{2}]^2) = \log_2(2^2) = 2 \neq 1$

B) By the definition of log, ($\log_a(x) = y \Leftrightarrow a^y = x$) we have

$\log_x(x^x) = x \Leftrightarrow x^x = x^x$

C) $x^{\underbrace{\log_x(x^x)}_{x}} = x^x \neq x$

#6 Which of the following is completely true?

A) If $4^x - 2^x = 0$ then $8^x = 1$

B) $\frac{1}{2}\log_2(x) = \log_4(x)$

C) If $5^{-x} = 3$ then $5^{3x} = 1/27$

D) $\boxed{\text{All Of The Above}}$

E) None Of The Above

A) $4^x - 2^x = 0 \Leftrightarrow (2^2)^x - 2^x = 0 \Leftrightarrow 2^{2x} - 2^x = 0 \Leftrightarrow (2^x)^2 - 2^x = 0$

$\Leftrightarrow 2^x(2^x - 1) = 0 \Leftrightarrow 2^x = 0$ or $2^x - 1 = 0$

2^x can't equal zero for any $x \in \mathbb{R}$ and

$2^x - 1 = 0 \Leftrightarrow 2^x = 1 \Leftrightarrow x = 0$

So $x = 0$ is our only solution. So, $8^x = 8^0 = 1$

B) $\frac{1}{2}\log_2(x) = y \Leftrightarrow \log_2(x) = 2y \Leftrightarrow 2^{2y} = x \Leftrightarrow 4^y = x \Leftrightarrow \log_4(x) = y$

$\frac{1}{2}\log_2(x) = y$ and $\log_4(x) = y \Leftrightarrow \frac{1}{2}\log_2(x) = \log_4(x)$

C) $5^{-x} = 3 \Leftrightarrow \log_5(5^{-x}) = \log_5(3) \Leftrightarrow -x = \log_5(3) \Leftrightarrow x = -\log_5(3)$

$\Rightarrow 5^{3x} = 5^{-3\log_5(3)} = \left(5^{\log_5(3)}\right)^{-3} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

#7 Which of the following is completely true?

A) $\log_5(x^2+x+4)=2 \Leftrightarrow x = \frac{-1 \pm \sqrt{85}}{2}$

B) $a(1-x)^2 + b(1-x) + c = 0 \Leftrightarrow x = \frac{b \pm \sqrt{b^2-4ac}}{2a} + 1$

C) $\log_2(x^2-2x+1) = 2 \Leftrightarrow x = -1, 3$

$\boxed{\text{D) ALL OF THE ABOVE}}$

E) NONE OF THE ABOVE

A) $\log_5(x^2+x+4)=2 \Leftrightarrow 5^{\log_5(x^2+x+4)} = 5^2 \Leftrightarrow x^2+x+4 = 25$

$\Leftrightarrow x^2+x-21=0 \Leftrightarrow x = \frac{-1 \pm \sqrt{1^2-4(1)(-21)}}{2(1)} = \frac{-1 \pm \sqrt{85}}{2}$

B) $a(1-x)^2 + b(1-x) + c = 0 \Leftrightarrow 1-x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Leftrightarrow -x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} - 1 \Leftrightarrow x = \frac{b \pm \sqrt{b^2-4ac}}{2a} + 1$

C) $\log_2(x^2-2x+1) = \log_2[(x-1)^2] = 2 \Leftrightarrow 2^{\log_2[(x-1)^2]} = 2^2$

$\Leftrightarrow (x-1)^2 = 4 \Leftrightarrow x-1 = \pm 2 \Leftrightarrow x = 1 \pm 2 = -1, 3$

OR, $\log_2(x^2-2x+1) = 2 \Leftrightarrow 2^{\log_2(x^2-2x+1)} = 2^2$

$\Leftrightarrow x^2-2x+1 = 4 \Leftrightarrow x^2-2x-3=0 \Leftrightarrow (x-3)(x+1)=0 \Leftrightarrow x=-1,3$

#8 Which of the following is completely true?

A) $\log_5(x^2-2^2)$ and $\log_2[(x-2)^2]$ have the same domain.

$\boxed{\text{B) } \log_3\left(\frac{x+3}{x-3}\right) \text{ and } \log_4\left(\frac{x-3}{x+3}\right) \text{ have the same domain.}}$

C) $\log_6\left(\frac{3-x}{x+3}\right)$ and $\log_2\left(\frac{-x}{3}\right)$ have the same domain.

D) ALL OF THE ABOVE

E) NONE OF THE ABOVE

A) $\log_5(x^2-2^2)$ is defined when $x^2-2^2>0 \Leftrightarrow x^2>4 \Leftrightarrow x>2$ or $x<-2$

$\log_2[(x-2)^2]$ is defined when $(x-2)^2>0 \Leftrightarrow x\neq 0$

$\Rightarrow$ These functions do not have the same domain.

B) $\log_3\left(\frac{x+3}{x-3}\right)$ is defined when $\frac{x+3}{x-3}>0 \Leftrightarrow \begin{Bmatrix} x+3>0 & \text{and} & x-3>0 \\ x+3<0 & \text{and} & x-3<0 \end{Bmatrix}$

$\Leftrightarrow \begin{Bmatrix} x>-3 & \text{and} & x>3 & \Leftrightarrow & x>3 \\ x<-3 & \text{and} & x<3 & \Leftrightarrow & x<-3 \end{Bmatrix} \Rightarrow$ the domain is $(-\infty,-3)\cup(3,\infty)$

The "$\cup$" symbol means "union" and is used to combine sets. Here we are combining the set of all real numbers less than -3 and the set of all real numbers greater than 3.

$\log_4\left(\frac{x-3}{x+3}\right)$ is defined when $\frac{x-3}{x+3}>0 \Leftrightarrow \begin{Bmatrix} x-3>0 & \text{and} & x+3>0 \\ x-3<0 & \text{and} & x+3<0 \end{Bmatrix}$

$\Leftrightarrow \begin{Bmatrix} x>3 & \text{and} & x>-3 \\ x<3 & \text{and} & x<-3 \end{Bmatrix} \Leftrightarrow \begin{matrix} x>3 \\ x<-3 \end{matrix} \Rightarrow$ the domain is $(-\infty,-3)\cup(3,\infty)$

These functions do have the same domain.

C) $\log_6\left(\frac{3-x}{x+3}\right)$ is defined when $\frac{3-x}{x+3}>0 \Leftrightarrow \begin{Bmatrix} 3-x>0 & \text{and} & x+3>0 \\ 3-x<0 & \text{and} & x+3<0 \end{Bmatrix}$

$\Leftrightarrow \begin{Bmatrix} x<3 & \text{and} & x>-3 \\ x>3 & \text{and} & x<-3 \end{Bmatrix} \Leftrightarrow \begin{matrix} -3<x<3 \\ \phi \end{matrix} \Leftrightarrow x\in(-3,3)$

The "ϕ" symbol refers to the "null" set. The null set is the set with no elements. In this case there are no real numbers simultaneously less than -3 and greater than 3.

$\log_2\left(\frac{-x}{3}\right)$ is defined when $\frac{-x}{3}>0 \Leftrightarrow x<0 \Rightarrow$ the domain is $(-\infty,0)$.

$\Rightarrow$ These functions do not have the same domain.

#9 Which of the following is completely true?

A) $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

B) You can find the complex conjugate of z by looking at its reflection over the real axis.

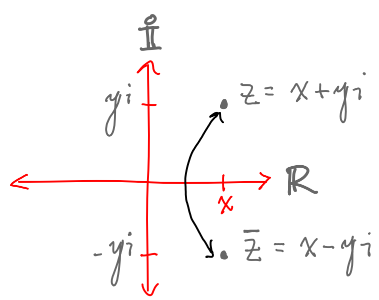
C) The real numbers are Well-Ordered.

D) $\boxed{\text{ALL OF THE ABOVE}}$

E) NONE OF THE ABOVE

A) The "$\subset$" symbol means "subset of." It's like "less than" for sets. The Natural numbers $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ are a subset of the integers $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$ which are a subset of the Rationals $\mathbb{Q} = \{ n/m;\ n, m \in \mathbb{Z}\} = \{\frac{1}{2}, 3/4, -2/7, \text{etc.}\}$ which are a subset of the Real numbers $\mathbb{R}$. The real numbers are composed of the rationals (numbers that can be expressed as an integer over an integer) and the irrationals (numbers that cannot be expressed as an integer over an integer).

B)



You can see from the diagram that the complex conjugate of z, denoted by $\bar{z}$, is on the opposite side of the Real axis and the same distance away from it. This is what "reflection" means. You can also think of it as "flipping" over the Real axis.

C) The real numbers are Well-Ordered. This means you can put them in order from least to greatest and any number is either less than or greater than all the other numbers. Notice this is not the case for the complex numbers. Which is larger, $2-2i$ or $-2+2i$?

#10 Which of the following is completely true?

A) Irrational numbers are numbers that have a square root on the bottom of the fraction.

B) The number i has to do with parallel directions.

C) $x = r\cos\theta$ and $y = r\sin\theta$ are the transformation equations from the real numbers to the complex numbers.

D) ALL OF THE ABOVE

E) $\boxed{\text{NONE OF THE ABOVE}}$

A) Irrational numbers are numbers that cannot be expressed as an integer over an integer in a fraction. Examples include $\sqrt{2}$ and π. Another familiar irrational number is $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$. Not all fractions with square roots in the denominator are irrational.

Consider $\frac{1}{\sqrt{4}} = \frac{1}{2}$ or $\frac{1}{\sqrt{6.25}} = \frac{1}{2.5} = \frac{2}{5}$.

Rational numbers have the property that their decimal expansion eventually repeats itself whereas Irrationals don't.

B) The number i has to do with perpendicular directions. If 10 miles is 10 miles in the direction you are currently facing and -10 miles is taken to mean 10 miles in the opposite direction, then $10i$ miles refers to 10 miles directly to your left and $-10i$ miles refers to 10 miles directly to your right. Take a look at the complex plane to see why.

C) $x = r\cos\theta$ and $y = r\sin\theta$ are the transformation equations from rectangular (or cartesian) coordinates to polar coordinates. They are also used to transform a complex number in rectangular (or standard) form to polar form. They are not used to convert real numbers to complex numbers. That just doesn't make sense. Real numbers are a subset of the complex numbers.