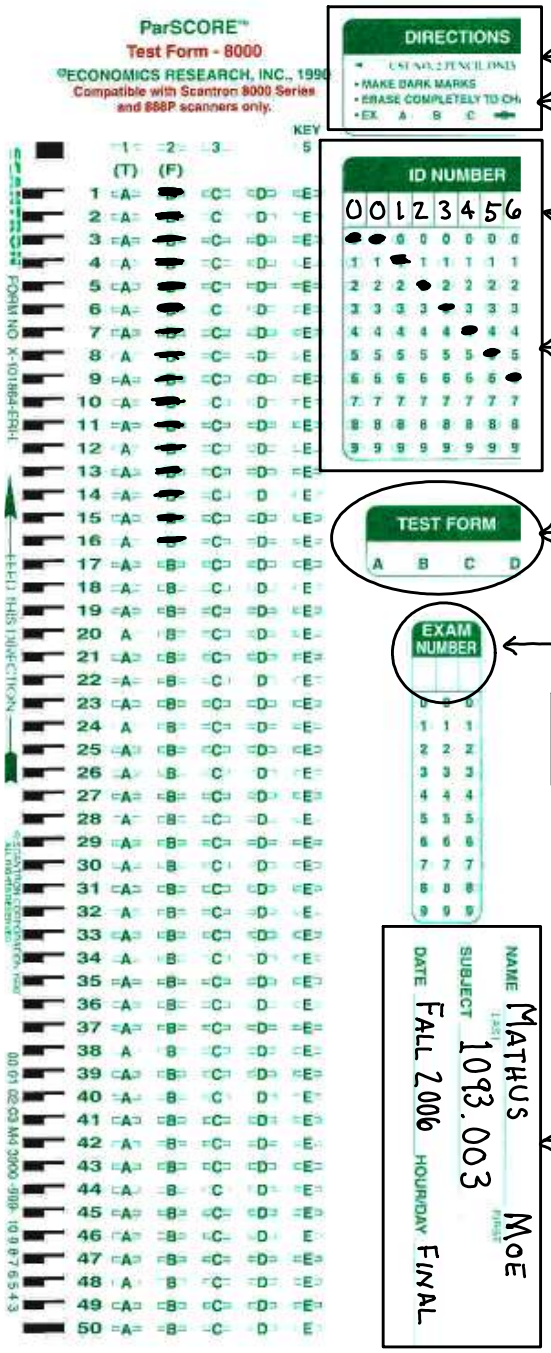
PRECALCULUS **FALL 2006** **FINAL EXAM**

Name_____ Banner:_____

Write your name and Banner here and here.



Use a number 2 pencil.
Make sure you have a 2nd Parscore in case you erase a hole through this one.

Fill in your Banner I.D.
Leave out the @ symbol.
Start on the left.
Fill in the corresponding bubbles.

Fill in your Test Form.
Your Form letter is at the top of the next

Leave blank.

Select **ONE LETTER PER PROBLEM** and **CIRCLE YOUR ANSWERS** on your exam**.**

Print your last and first names legibly.
Print your class number and section that you are enrolled in legibly. If you are taking this exam in a section other than the one you are enrolled in, please hand your exam and Parscore directly to me when you are finished so I can put it with your own section.

MWF 8am = 1093.001
MWF 10am = 1093.002
MWF 2pm = 1093.003
TTH 2pm = 1093.007

Separate the pages of the exam and use the back of the paper as scratch paper. I'll have a stapler to staple your exam back together. You have 2 hours and 45 minutes. Verify your grade once the are released in WebCT as soon as possible. Notify me if there is a problem.

Cover your work and your Parscore. Don't talk or look around the room like an idiot. If something is illegible then please notify me. If a question is ambiguous then please ask me to clarify.

Don't give up. Do your best on every problem. Use all of your available time, your grade is depending on you. If you finish early, redo the problems to verify correctness. Don't "ckeck your work", redo it separately without looking at your previous work.

FINAL EXAM **FORM D** (Fill out the Exam Form section on your Parscore now.)

#1 Completely factor the polynomial: $z^6 + 64 = 0.$

A) $(z-2)(z+2)(z-2i)(z+2i)\left[z-(\sqrt{2}+\sqrt{2}i)\right]\left[z+(\sqrt{2}+\sqrt{2}i)\right] = 0$

B) $(z-2)(z+2)\left[z-(\sqrt{2}+\sqrt{2}i)\right]\left[z+(\sqrt{2}+\sqrt{2}i)\right]\left[z-(\sqrt{3}-i)\right]\left[z+(\sqrt{3}-i)\right] = 0$

C) $(z-2i)(z+2i)\left[z-(1+\sqrt{3}i)\right]\left[z+(1+\sqrt{3}i)\right]\left[z-(1-\sqrt{3}i)\right]\left[z+(1-\sqrt{3}i)\right] = 0$

D) $\left[z-(\sqrt{3}+i)\right]\left[z-2i\right]\left[z-(-\sqrt{3}+i)\right]\left[z-(-\sqrt{3}-i)\right]\left[z-(-2i)\right]\left[z-(\sqrt{3}-i)\right] = 0$

E) None of the above.

#2 Find $(-2-\sqrt{3}i)^7$ in polar form.

A) $7^{7/2}\left(\cos\left[7\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]+i\sin\left[7\tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]\right)$

B) $7^{7/2}\left[\cos\left(\frac{7\pi}{2}\right)+i\sin\left(\frac{7\pi}{2}\right)\right]$

C) $7^{7/2}\left[\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right]$

D) $7^{7/2}\left(\cos\left[7\pi-7\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]+i\sin\left[7\pi-7\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]\right)$

E) None of the above.

#3 Which of the following is completely true? Note: "$\Leftrightarrow$" means the equations have the same solutions.

A) $8^{6+3x} = 4 \Leftrightarrow x = -19/9$

B) $\log_4(x^2+2x+1) = 4 \Leftrightarrow \log_2(4x^2+4x+1) = 2$

C) $3^{2x} + 3^{x+1} - 4 = 0 \Leftrightarrow 25^{2x} = 5^{x^2-12}$

D) All of the above.

E) None of the above.

Be careful when solving equations containing logs when you use their properties to combine two or more logs.

Example: $\log_2(x+4) + \log_2(x+6) = 3 \Rightarrow \log_2[(x+4)(x+6)] = 3$

$\Leftrightarrow 2^{\log[(x+4)(x+6)]} = 2^3 \Leftrightarrow x^2+10x+24 = 8 \Leftrightarrow x^2+10x+16 = 0$

$\Leftrightarrow (x+8)(x+2) = 0 \Leftrightarrow x = -8, -2$

Try plugging these solutions in to the original equation and you'll see x = -8 cannot be a solution since log(-8 + 4) is undefined. Sure, log(-8 + 6) is also undefined, but only one log needs to be undefined for the whole thing to fail. Notice, however, the combined version, log[-8 + 4)(-8 + 6)] = log(8), is happily defined. So, be sure to check your solutions in the original equation to verify that they are indeed solutions.

It is true that log(x + 4) + log(x + 6) = log[(x + 4)(x + 6)] is an identity so the equality holds for any value of x... any value such that the equality is defined, that is. For instance,

$$\sin(x) = \sin(x)\frac{\cos(x)}{\cos(x)} = \tan(x)\cos(x)$$

is an identity but, sin(x) takes any value of x but tan(x)cos(x) is undefined for pi/2. So, identities are equal when they are defined.

$$\log(x + 4) + \log(x + 6) = 3 \quad \Rightarrow \quad \log[(x + 4)(x + 6)] = 3$$

I used the "$\Rightarrow$" symbol here instead of "$\Leftrightarrow$" because log[(x + 4)(x + 6)] = 3 has more solutions than log(x + 4) + log(x + 6) = 3 and is therefore a more general statement.

Thus, if log(x + 4) + log(x + 6) = 3 is true, then so is log[(x + 4)(x + 6)] = 3.

However, if log[(x + 4)(x + 6)] = 3 is true, log(x + 4) + log(x + 6) = 3 may not be true as we saw with the case when x = -8.

#4 Which of the following is completely true?

A) $2^{3x} = 3^{2x+1} \Leftrightarrow 3^{2x} = 2^{3x+1}$

B) $\log_6(x+3) + \log_6(x+4) = 1 \Leftrightarrow \log_6\left[\frac{1}{x+3}\right] + \log_6\left[\frac{1}{x+4}\right] = -1$

C) $16^{x+2} = 8^{x-3} \Leftrightarrow 27^{x+3} = 81^{x-2}$

D) All of the above.

E) None of the above.

#5 Which of the following is completely true?

A) $\ln\left(\frac{x-1}{x}\right) + \ln\left(\frac{x}{x+1}\right) - \ln(x^2-1) = 1$ has 2 solutions.

B) $\ln(4-x) - \ln(x^2+7x+12) = (1/2)\ln(x^2+1) - 4\ln(1/2) - (1/2)[\ln(x-4) + \ln x]$ has 2 solutions.

C) $2\log(2) + 3\log(x) - (1/2)[\log(x+3) + \log(x-2)] = \log(2-x)$

D) All of the above.

E) None of the above.

#6 Which of the following is completely true? Note: $\cos\theta + i\sin\theta = e^{i\theta}$

A) $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

B) $0 = 1 + e^{i\pi}$ and $1 = e^{i2\pi}$

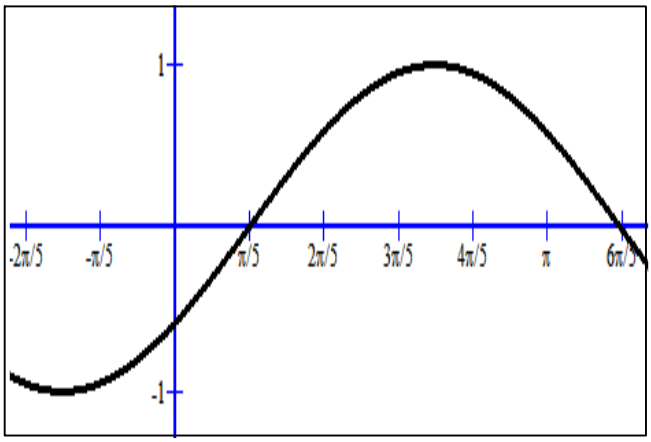
C) If $z = x + iy$ then e^z has radius $r = e^x$ and angle $\theta = y$

D) All of the above.

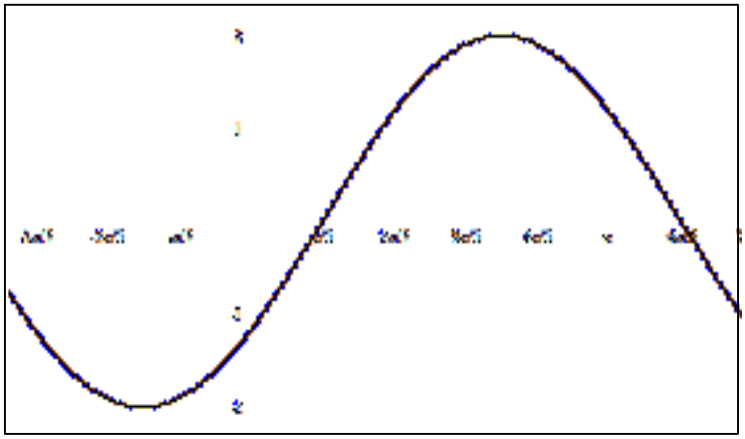
E) None of the above.

#7 Which of the following is completely true?

f(x)



g(x)



A) $f(x) = \sin(2x - \pi/5)$

B) $f(x) = (1/2)\, g(x)$

C) $g(x) = 2\cos(2x - \pi/5)$

D) All of the above.

E) None of the above.

#8 $\sin[2\tan^{-1}(3/5) + 2\sin^{-1}(2/3)] =$

A) $\dfrac{15 + 32\sqrt{5}}{9 \cdot 17}$

B) $\dfrac{17 - 23\sqrt{7}}{42}$

C) $\dfrac{23 + 17\sqrt{3}}{3 \cdot 19}$

D) All of the above.

E) None of the above.

#9 Which of the following is completely true?

While standing on the surface of the Earth you may feel like you are not moving, but indeed you are because the Earth is rotating and revolving around the Sun, which itself is orbiting the center of the Milky Way Galaxy. Considering only the rotation of the Earth, you have an angular velocity of w = 1 rev/day. Your linear velocity depends upon where you are on the surface of the Earth.

A) Your maximum linear velocity from Earth's rotation occurs when you are on the Equator.

B) Your maximum angular velocity occurs when you are on the North or South Pole.

C) If you drive North from UTSA keeping your speedometer at 70 mph, your distance from the Earth's axis of rotation becomes smaller as you approach the North Pole causing your total linear velocity (velocity of car across the surface of the Earth plus the velocity of the surface of the Earth with respect to its center) to steadily increase.

D) All of the above.

E) None of the above

#10 Which of the following is Mollweide's Formula? It relates the three sides and three angles of a triangle.
Hint: Start with (a+b)/c and use the Law of Sines and then a Sum-to-Product Formula.

A) $\frac{a+b}{c} = \frac{\cos[(\alpha-\beta)/2]}{\sin(\gamma/2)}$

B) $\frac{a+b}{c} = \frac{\sin[(\alpha-\beta)/2]}{\cos(\gamma/2)}$

C) $\frac{a+b}{c} = \frac{\sin\alpha\cos\beta}{2\cos(\gamma/2)}$

D) All of the above.

E) None of the above.

SUM TO PRODUCT

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

#11 Which of the following is completely true?

A) $\sin\theta - \sqrt{3}\cos\theta = 2 \Leftrightarrow \sqrt{3}\sin\theta + \cos\theta = 2$

B) $2\sin^2\theta - 3\sin\theta = -1 \Leftrightarrow 2\cos^2\theta + \cos\theta = 1$

C) $\cos^2\theta - 2\sin^2\theta - 1 = 0 \Leftrightarrow \sin^2\theta - 2\cos^2\theta - 1 = 0$

D) All of the above.

E) None of the above.

#12 Which of the following is completely true?

A) $(2a\sin\theta\cos\theta)^2 + a^2(\cos^2\theta - \sin^2\theta)^2 = a^2$

B) $(\tan\alpha + \tan\beta)(1 - \cot\alpha\cot\beta) + (\cot\alpha + \cot\beta)(1 - \tan\alpha\tan\beta) = 0$

C) $\dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} = \sec\theta + \tan\theta$

D) All of the above.

E) None of the above.

#13 Which of the following is completely true?

A) $\tan(\sin^{-1}v) = \dfrac{v}{\sqrt{1 - v^2}}$

B) $\sin(\cos^{-1}v) = \sqrt{1 - v^2}$

C) $\cos(\tan^{-1}v) = \dfrac{1}{\sqrt{1 + v^2}}$

D) All of the above.

E) None of the above.

#14 Which of the following is completely true?

A) $\sin(\pi/12) = (\sqrt{6} - \sqrt{2})/4 = \sqrt{2-\sqrt{3}}/2$ and $\cos(\pi/12) = (\sqrt{6} + \sqrt{2})/4 = \sqrt{2+\sqrt{3}}/2$

B) $\sin(\pi/24) = \sqrt{1/2 - (\sqrt{6} + \sqrt{2})/8}$ and $\cos(\pi/24) = \sqrt{1/2 + (\sqrt{6} + \sqrt{2})/8}$

C) $\sin(\pi/16) = \sqrt{\frac{1}{2} - \frac{\sqrt{2+\sqrt{2}}}{4}}$ and $\cos(\pi/16) = \sqrt{\frac{1}{2} + \frac{\sqrt{2+\sqrt{2}}}{4}}$

D) All of the above.

E) None of the above.

#15 Which of the following statements is true concerning the solutions of: $z^2 - (2+5i)z - 3+5i = 0$

A) Both solutions are purely real.

B) Both solutions are not purely real.

C) One solution is purely real and the other is not.

D) One solution is purely imaginary and one is purely real.

E) None of the above

#16 Find the product of the eight 8th roots of unity (1). (Find the roots and multiply them all together.)

A) 1

B) -1

C) i

D) -i

E) None of the above
