Name______________________Banner____________

Fall 2007 Quiz #1 Solutions

Show your work. Use proper notation. Think before you write or give up.
Box your final answers. Write on this paper only. Do easy problems first.

#1) Solve for x: $x^2 + x = 8 - x \Rightarrow x^2 + 2x - 8 = 0$

$$\Rightarrow (x+4)(x-2) = 0 \Rightarrow \begin{cases} x+4=0 \Rightarrow \boxed{x=-4} \\ \text{or} \\ x-2=0 \Rightarrow \boxed{x=2} \end{cases}$$

#2) Solve for x: $2x^2 + 5x = -3 \Rightarrow 2x^2 + 5x + 3 = 0$

$$\Rightarrow (2x+3)(x+1) = 0 \Rightarrow \begin{cases} 2x+3=0 \Rightarrow \boxed{x=-3/2} \\ \text{or} \\ x+1=0 \Rightarrow \boxed{x=-1} \end{cases}$$

#3) Solve for x and y: $2x + y = 7$ $\quad y + 2x = 5y$ $\quad$ Two Equations Two Unknowns

$2x + y = 7 \Rightarrow y = 7 - 2x$

$y + 2x = 5y \Rightarrow x = 2y = 2(7-2x) = 14 - 4x \Rightarrow 5x = 14 \Rightarrow \boxed{x = \frac{14}{5}}$

$\Rightarrow y = 7 - 2x = 7 - 2\left(\frac{14}{5}\right) = \boxed{7 - \frac{28}{5} = y}$

#4) Expand $(x+y)^6$ and $(x-y)^6$

$$\boxed{(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6}$$

$$\boxed{(x-y)^6 = [x + (-y)]^6 \text{ (replace } y \text{ with } -y) = x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6}$$

Use Pascal's Triangle to find the coefficients for $(a+b)^n$

n=0 1
n=1 1 1
n=2 1 2 1
n=3 1 3 3 1
n=4 1 4 6 4 1
n=5 1 5 10 10 5 1
n=6 1 6 15 20 15 6 1

#5) Evaluate: a) $(3^2)^4$ b) $3^2 \cdot 3^4$ c) $3^{-2} \cdot (3^{-4})^{-1}$

a) $(3^2)^4 = (3\cdot 3)^4 = (3\cdot3)(3\cdot3)(3\cdot3)(3\cdot3) = 3^{2\cdot4} = \boxed{3^8 = (3^2)^4}$

b) $3^2 \cdot 3^4 = (3\cdot3)(3\cdot3\cdot3\cdot3) = 3^{2+4} = 3^6$

c) $3^{-2} \cdot (3^{-4})^{-1} = \dfrac{3^{-2}}{3^{-4}} = 3^{-2}\cdot 3^4 = \dfrac{3^4}{3^2} = \boxed{3^2 = 3^{-2} \cdot (3^{-4})^{-1}}$

or $3^{-2} \cdot (3^{-4})^{-1} = 3^{-2} \cdot 3^{(-4)(-1)} = 3^{-2}\cdot 3^4 = \boxed{3^{4-2} = 3^2}$

#6) Solve for x in terms of c_0, c_1, and c_2: $c_2x^2 + c_1x + c_0 = 0$

Refer to the tutorial on the quadratic formula.

#7) What is the Fundamental Theorem of Algebra (In your own words)?

A polynomial equation of degree n has n factors and therefore n solutions.

Note: "Degree" here refers to the highest power of your variable.

Examples: $6x^2 + x + 9 = 0$ has 2 solutions.

$3x^3 - 1 = 0$ has 3 solutions.

There is more to this theorem but this is the part you need to know.

#8) What is the Fundamental Theorem of Arithematic (In your own words)?

Also known as the Unique Factorization Theorem, states that every natural number (a natural number is a positive integer) greater than 1 can be written as a unique product of prime numbers (prime numbers are those numbers that are divisible only by themselves and one).

Examples: $18 = 2\cdot 3^2$

$27 = 3^3$

$34 = 2\cdot 17$

2, 3, and 17 are all prime numbers

#9) Graph: $f(x) = 1 - x + \frac{x^2}{2}$

$f(-3) = 17/2$

$f(-2) = 5$

$f(-1) = 5/2$

$f(0) = 1$

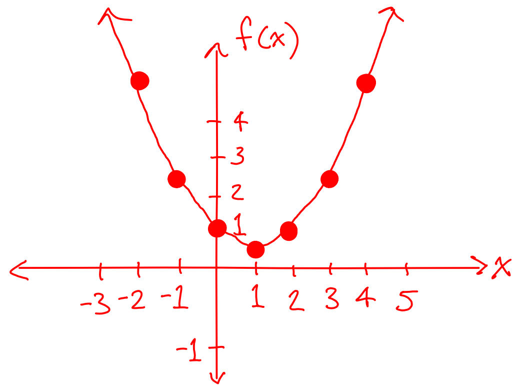
$f(1) = 1/2$

$f(2) = 1$

$f(3) = 5/2$

$f(4) = 5$

$f(5) = 17/2$



#10) Reduce the fraction: $\frac{\sqrt{x^2+2x-y^2-2y}}{\sqrt{x^2-y^2}} = \sqrt{\frac{x^2+2x+1-y^2-2y-1}{(x+y)(x-y)}}$ smart zero $0 = +1-1$

$= \sqrt{\frac{(x+1)^2-(y+1)^2}{(x+y)(x-y)}}$ ← Difference of two squares: $a^2-b^2 = (a+b)(a-b)$

Here $a = x+1$ and $b = y+1$

$= \sqrt{\frac{(x+1+y+1)(x+1-y-1)}{(x+y)(x-y)}} = \sqrt{\frac{(x+y+2)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}} = \sqrt{\frac{x+y+2}{x+y}}$

OR

$\frac{\sqrt{x^2+2x-y^2-2y}}{\sqrt{x^2-y^2}} = \sqrt{\frac{x^2-y^2+2(x-y)}{x^2-y^2}} = \sqrt{\frac{(x+y)(x-y)+2(x-y)}{(x+y)(x-y)}}$

$= \sqrt{\frac{\cancel{(x-y)}(x+y+2)}{\cancel{(x-y)}(x+y)}} = \sqrt{\frac{x+y+2}{x+y}}$
