

Name \_\_\_\_\_ Banner \_\_\_\_\_

## Fall 2007 Quiz #10 Solutions

#1) Prove the identity:  $\frac{1+\tan^2\theta}{1-\tan^2\theta} \equiv \frac{1}{1-2\sin^2\theta}$

$$\frac{1+\tan^2\theta}{1-\tan^2\theta} = \frac{\sec^2\theta}{1-\tan^2\theta} = \frac{1}{\cos^2\theta(1-\tan^2\theta)}$$

$$= \frac{1}{\cos^2\theta\left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right)} = \frac{1}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{1}{(1-\sin^2\theta) - \sin^2\theta} = \frac{1}{1-2\sin^2\theta}$$

#2) Prove the identity:  $-\frac{\sin^3(-\theta)}{1+\cos(-\theta)} \equiv -\frac{\tan(-\theta)}{\sec(-\theta)} + \sin(-\theta)\cos(-\theta)$

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$$\begin{aligned} -\frac{\sin^3(-\theta)}{1+\cos(-\theta)} &= -\frac{\sin^3(-\theta)}{1+\cos(-\theta)} \left( \frac{1-\cos(-\theta)}{1-\cos(-\theta)} \right) \\ &= -\frac{\sin^3(-\theta)}{1-\cos^2(-\theta)} + \frac{\sin^3(-\theta)\cos(-\theta)}{1-\cos^2(-\theta)} \\ &= -\frac{\sin^3(-\theta)}{\sin^2(-\theta)} + \frac{\sin^3(-\theta)\cos(-\theta)}{\sin^2(-\theta)} \\ &= -\sin(-\theta) \left( \frac{\cos(-\theta)}{\cos(-\theta)} \right) + \sin(-\theta)\cos(-\theta) \\ &= -\frac{\sin(-\theta)}{\cos(-\theta)} \cos(-\theta) + \sin(-\theta)\cos(-\theta) \\ &= -\frac{\tan(-\theta)}{\sec(-\theta)} + \sin(-\theta)\cos(-\theta) \end{aligned}$$