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Fall 2007 **Quiz #11 Solutions** **Precalculus**

Sum Identities

$$\cos(\alpha+\beta) \equiv \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) \equiv \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

#1) Write cos(2x) and sin(2x) in terms of sin(x) and cos(x).

$$\cos(2x) = \cos(x+x) = \cos x\cos x - \sin x \sin x$$

$$= \boxed{\cos^2 x - \sin^2 x \equiv \cos(2x)}$$

$$= (1-\sin^2 x) - \sin^2 x$$

$$= \boxed{1 - 2\sin^2 x \equiv \cos(2x)}$$

$$= \cos^2 x - (1-\cos^2 x)$$

$$= \boxed{2\cos^2 x - 1 \equiv \cos(2x)}$$

Any of the above are correct.

Similarly,

$$\sin(2x) = \sin(x+x)$$

$$= \sin x\cos x + \cos x \sin x$$

$$= \sin x\cos x + \sin x\cos x$$

$$= \boxed{2\sin x\cos x = \sin(2x)}$$

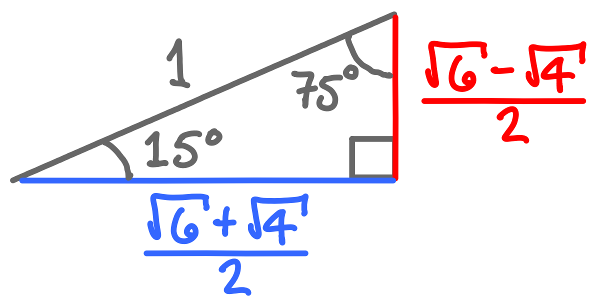
For problem #2 we use the following:

$$\cos(15^\circ) = \cos\left(\frac{\pi}{12}\right) = \cos(45^\circ - 30^\circ)$$

$$= \cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Simalarly,

$$\sin(15^\circ) = \sin(\pi/12) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

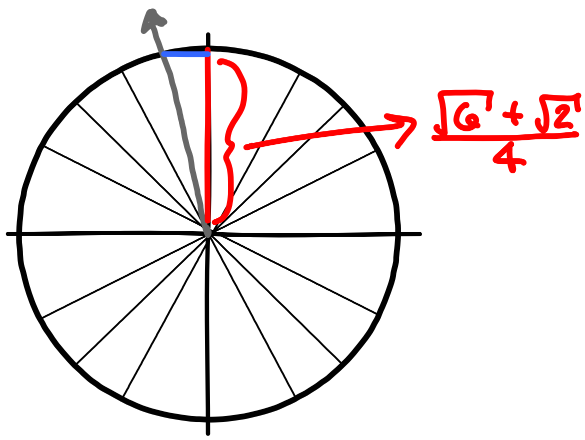


#2) Evaluate the following functions and give exact values:

A) $\sin(105°) = \sin(90° + 15°)$

$= \sin(90°)\cos(15°) + \cos(90°)\sin(15°)$

$= 1\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + 0 \cdot \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$



OR, use $\sin(105°) = \sin(60° + 45°)$

B) $\cos(\pi/12) = \cos(15°) = \frac{\sqrt{6}+\sqrt{2}}{4}$

as was shown on the previous page.

Notice that it is easier to work with degrees than radians when trying to find the appropriate sum or difference to represent a given angle.

$15° = 45° - 30°$ versus $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

c) $\tan(255°)$

- $\sin(255°) = \sin(270° - 15°)$
  $= \sin(270°)\cos(15°) - \cos(270°)\sin(15°)$
  $= (-1)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 0\cdot\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = -\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$

- $\cos(255°) = \cos(270° - 15°)$
  $= \cos(270°)\cos(15°) + \sin(270°)\sin(15°)$
  $= 0\cdot\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + (-1)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = -\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$

Therefore, $\tan(255°) = \dfrac{\sin(255°)}{\cos(255°)}$

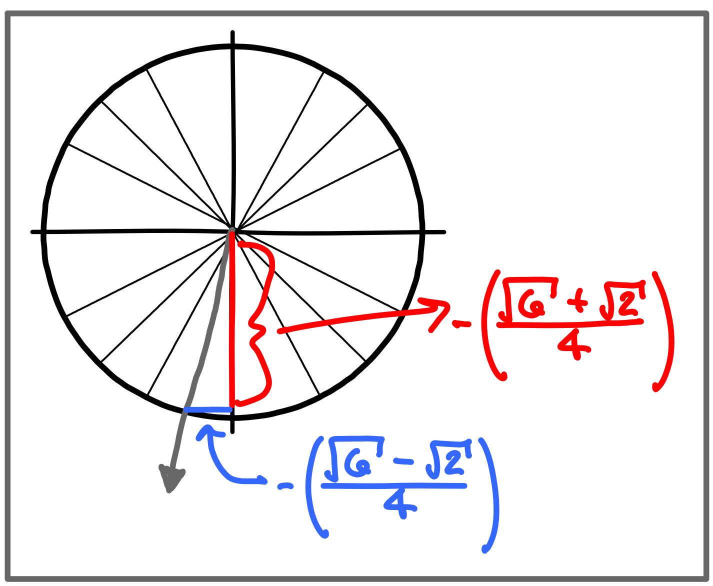
$$= \frac{-\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)}{-\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \tan(255°)}$$

Rationalize if you want:

$$\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$

$$= \frac{6+2\sqrt{6}\sqrt{2}+2}{6-2} = \frac{8+2\sqrt{2\cdot2\cdot3}}{4}$$

$$= \frac{8+4\sqrt{3}}{4} = \boxed{2+\sqrt{3} = \tan(255°)}$$



D) $\sec(23\pi/12) = \sec(23\pi/12 - 2\pi)$

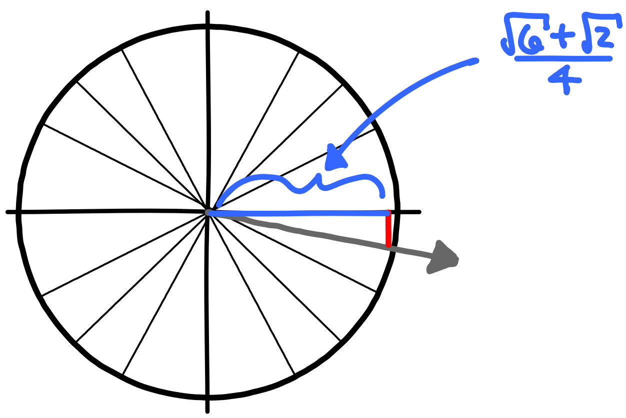
since secant is 2π periodic.

$$= \sec(23\pi/12 - 24\pi/12) = \sec(-\pi/12)$$

$$= \frac{1}{\cos(-\pi/12)} = \frac{1}{\cos(\pi/12)} = \frac{1}{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

since cosine is even

$$= \boxed{\frac{4}{\sqrt{6}+\sqrt{2}} = \sec(23\pi/12)}$$



Again, rationalize if you want:

$$\sec(23\pi/12) = \frac{4}{\sqrt{6}+\sqrt{2}} = \left(\frac{4}{\sqrt{6}+\sqrt{2}}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$

conjugates

$$= \frac{4(\sqrt{6}-\sqrt{2})}{6-2} = \boxed{\sqrt{6}-\sqrt{2} = \sec(23\pi/12)}$$

Extra Credit (25 points)

Find cos(4x) in terms of sin(x) and cos(x).

From #1 we know

$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

and

$\sin(2x) = 2\sin x \cos x$

Therefore,

$\alpha = \beta = 2x$

$$\cos(4x) = \cos(\underbrace{2x}_{\alpha} + \underbrace{2x}_{\beta})$$

$$= \cos(2x)\cos(2x) - \sin(2x)\sin(2x)$$

$$= \cos^2(2x) - \sin^2(2x)$$ which we should have known since $\cos(2x) = \cos^2 x - \sin^2 x$

$$= (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2$$

$$= \cos^4 x - 2\cos^2 x \sin^2 x + \sin^4 x - 4\sin^2 x \cos^2 x$$

$$= \boxed{\cos^4 x - 6\sin^2 x \cos^2 x + \sin^4 x = \cos(4x)}$$

which we could convert to contain only $\cos x$ or only $\sin x$ using the fact that $\sin^2 x + \cos^2 x \equiv 1$