Name____________________Banner____________

Fall 2007 **Quiz #2 Solutions**

Show your work. Use proper notation. Think before you write or give up.
Box your final answers. Write on this paper only. Do easy problems first.

#1) Derive the Quadratic Formula (use proper mathematical notation).
This means, prove that if a, b, and c are constants such that $a \neq 0$,
then $ax^2+bx+c=0 \iff x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$ax^2+bx+c=0$$

Add the right constant to both sides of the equation to "complete the square"

$$\iff x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Just simplifying the right-hand-side

$$\iff x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\iff \underbrace{x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2} = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2-4ac}{4a^2}$$

$$\iff \left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$\iff x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\iff x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \checkmark$$

Alternatively, if you already know the result you could "reverse engineer" the problem:

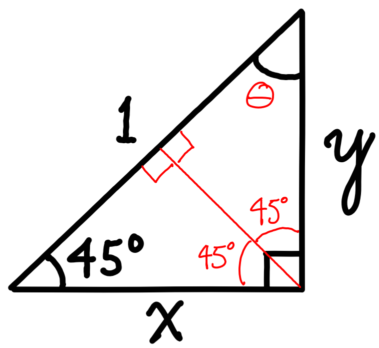
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \iff x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a} \iff \left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$\iff x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \iff x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \iff ax^2 + bx + c = 0$$

Then write the steps in reverse order to make it look like you didn't already know the formula!

#2) Solve the following two triangles (use proper mathematical notation). This means, find all three angles in degrees and find the lengths of all three sides for both triangles.

A)



Although some steps may be trivial, you should write out your solution as formally as possible. These skills are necessary to have mastered when problems become more complicated. Also, being able to communicate your solution is as important as the solution itself.

Einstein's Relativity would not have caught on if he couldn't have explained it.

Since the angle's of a triangle on a flat surface sum to 180°, we have

$\theta + 45° + 90° = 180° \Leftrightarrow \theta = 180° - 90° - 45° = \boxed{45° = \theta}$

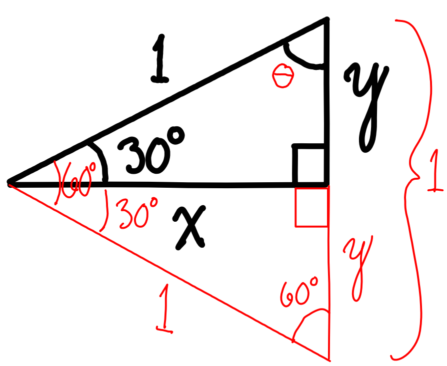
Dividing the triangle into two congruent triangles (both similar to the original) reveals that $x = y$ since they correspond to the same side of congruent triangles.

Also, by the Pythagorean Theorem, $x^2 + y^2 = 1^2 = 1$

combined with $x = y$, we have $x^2 + x^2 = 1 \Leftrightarrow 2x^2 = 1 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\Leftrightarrow \boxed{x = y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}}$

B)



Since the angle's of a triangle on a flat surface sum to 180°, we have

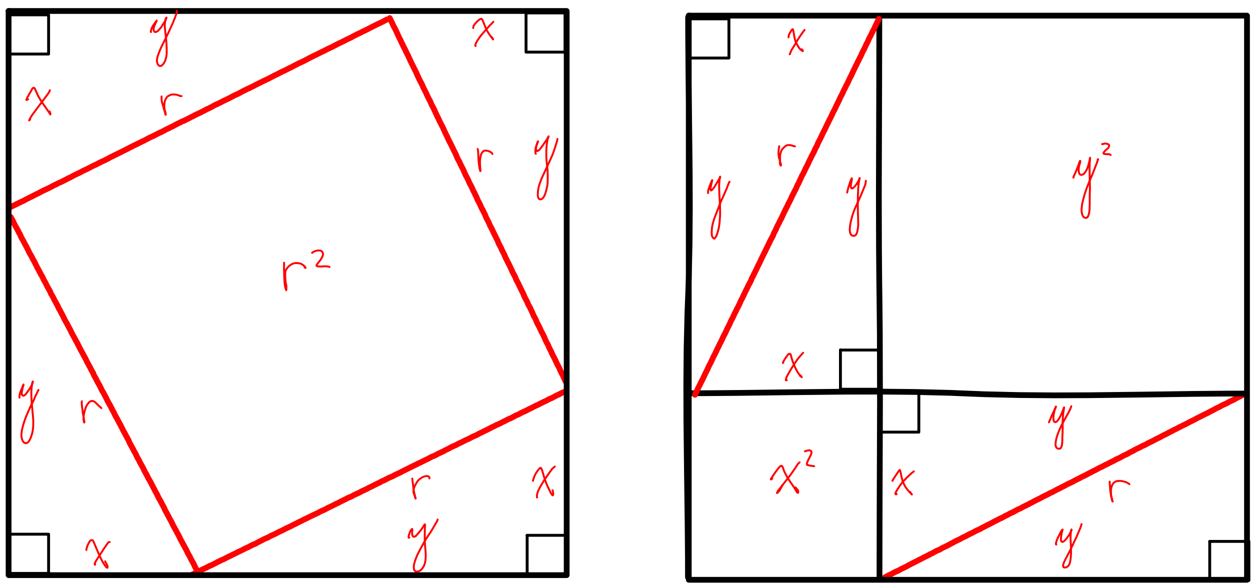
$\theta + 30° + 90° = 180° \Leftrightarrow \theta = 180° - 90° - 30° = \boxed{60° = \theta}$

Duplicating the triangle and combining it with the original produces an equilateral triangle whose extra symmetries can be exploited to solve the triangle. Since all the angles are equal, so must be the sides. Thus, $y + y = 1 \Leftrightarrow \boxed{y = \frac{1}{2}}$

Also, by the Pythagorean Theorem, $x^2 + y^2 = 1^2 = 1 \Leftrightarrow x^2 + \left(\frac{1}{2}\right)^2 = 1 \Leftrightarrow x^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\Leftrightarrow \boxed{x = \frac{\sqrt{3}}{2}}$ where the negative root is neglected since lengths are positive here.

#3) Prove the Pythagorean Theorem. (Extra credit for a proof different from the one given in class)



Both squares above have total area A each.

For the square on the left the area can be represented as $A = 4\left(\frac{1}{2}xy\right) + r^2$

where each of the four triangles has area $\frac{1}{2}xy$

For the square on the right the area can be represented as $A = 4\left(\frac{1}{2}xy\right) + x^2 + y^2$

Since $A = A$, we have

$$A = 4\left(\frac{1}{2}xy\right) + x^2 + y^2 = 4\left(\frac{1}{2}xy\right) + r^2 \iff x^2 + y^2 = r^2$$