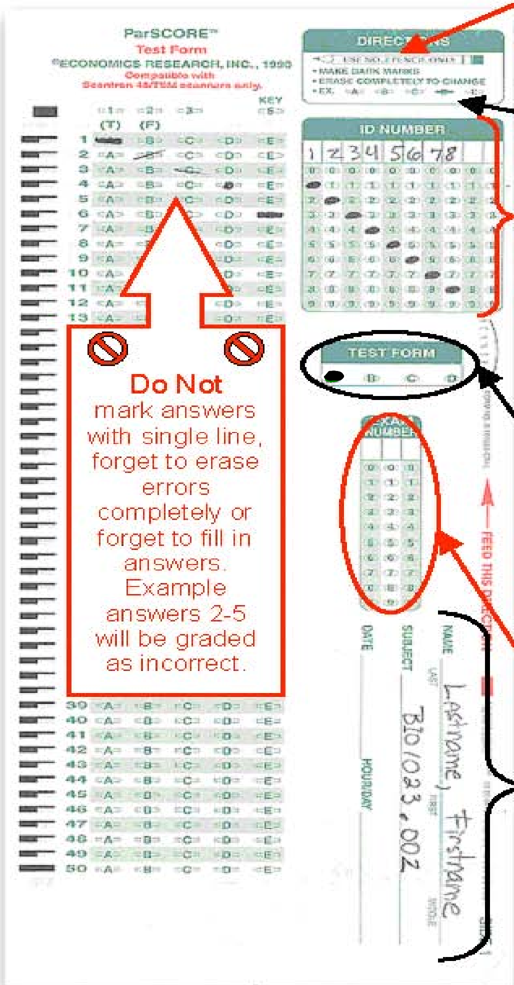
**PRECALCULUS** **Fall 2007** **Exam #1**

Name________________________________________ Banner:_________________________

## Instructions for Students using ParSCORE Test Forms

Required Materials *(available at campus bookstore)*:

- **ParSCORE Test Form – No. X-101864**
- **#2 Pencil**




**Use a #2 Pencil**
**Note**: Marks made with mechanical, recycled, green, and earth friendly pencils as well as pens ***will be marked wrong*** by the scanner.

Fill in the entire rectangle to mark your answer. Example answers 1 and 6 will be graded as correct.

Your ID number is the ***LAST 8 digits of your BANNER ID.*** Drop the first zero on your Banner ID. Example: Student's Banner ID reads **"012345678"**. The ID Number entered on the ParSCORE Test Form is **"12345678"**. *Do not use social security or driver's license number.*

This is Form A

Do not fill in the Exam Number

PRINT your **Name, Course,** and **Section Number** clearly.

| Course = Precal | | 1093.section |
|---|---|---|
| MWF at 9am | => | 1093.004 |
| MWF at 10am | => | 1093.002 |
| MWF at 2pm | => | 1093.003 |

Separate the pages of the exam and use the back of the paper as scratch paper. I'll have a stapler to staple your exam back together. Grades will be available in WebCT as soon as possible.

Cover your work and your Parscore. Don't cheat or appear to be cheating.

If something is illegible then please notify me. If a question is ambiguous then please ask me to clarify.

DON'T GIVE UP! Do your best on every problem. You are not supposed to already know the answer, you are to figure it out using what you know. Use all of your available time. If you finish early, redo the problems to verify correctness. Don't "check your work" - redo it separately without looking at your previous work.

**Circle your answers on this exam and fill in the corresponding bubble on your ParScore.**

#1) For the figure below, suppose A, B, and C represent the area of the squares and x, y, and h are the lengths of the sides of the triangle. According to the figure, which of the following statements is always true.

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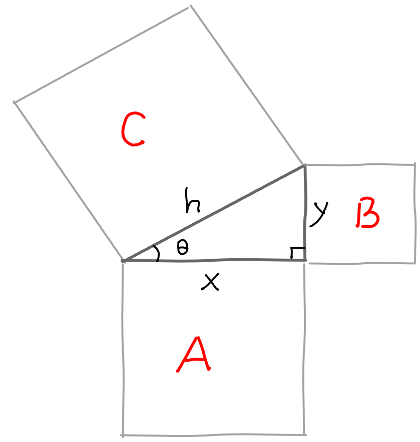
A) $A^2 + B^2 = \cos^2\theta + \sin^2\theta = 1$

B) $\boxed{\dfrac{A}{C} + \dfrac{B}{C} = \dfrac{x^2}{h^2} + \dfrac{y^2}{h^2} = \cos^2\theta + \sin^2\theta = 1}$

C) $A + B = C = x^2 + y^2 = h^2 = \sin^2 y + \cos^2 x = 1$

D) $C - B = A = h^2 - y^2 = \cos^2\theta - 1 = \sin^2\theta$

E) None of the above

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A) $A^2 + B^2 = \cos^2\theta + \sin^2\theta = 1$ is not completely true since $A^2 + B^2 \neq 1$

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B) $\dfrac{A}{C} + \dfrac{B}{C} = \dfrac{x^2}{h^2} + \dfrac{y^2}{h^2} = \cos^2\theta + \sin^2\theta = 1$ is completely true since

$\cos^2\theta + \sin^2\theta = 1$ is an identity,

and since the areas of the 3 squares are $A = x^2$, $B = y^2$, and $C = h^2$,

we have, by the Pythagorean Theorem,

$$x^2 + y^2 = h^2 \Rightarrow \frac{x^2}{h^2} + \frac{y^2}{h^2} = \frac{h^2}{h^2} = 1 \Leftrightarrow \frac{A}{C} + \frac{B}{C} = 1$$

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C) $A + B = C = x^2 + y^2 = h^2 = \sin^2 y + \cos^2 x = 1$ is not completely true since

$A + B = C = x^2 + y^2 = h^2 \underset{*}{\neq} \sin^2 y + \cos^2 x = 1$ (Note: these are not $\theta$)

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D) $C - B = A = h^2 - y^2 = \cos^2\theta - 1 = \sin^2\theta$ is not completely true.

$\cos\theta = \dfrac{x}{h}$ and $\sin\theta = \dfrac{y}{h} \Rightarrow x^2 = h^2\cos^2\theta$ and $y^2 = h^2\sin^2\theta$ and

$$x^2 + y^2 = h^2 \Leftrightarrow x^2 = h^2 - y^2$$

$$\cos^2\theta + \sin^2\theta = 1 \Leftrightarrow \cos^2\theta - 1 = \sin^2\theta$$

$$A + B = C \Leftrightarrow C - B = A$$

but

$$C - B = A = \overset{x^2+y^2}{h^2} - y^2 = x^2 + y^2 - y^2 = x^2 = h^2\cos^2\theta \underset{*}{\neq} \cos^2\theta - 1 = \sin^2\theta$$

#2) Which of the following pairs of pictures provides a proof of the Pythagorean Theorem?

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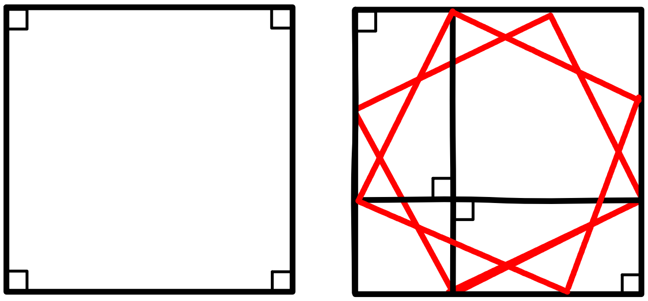
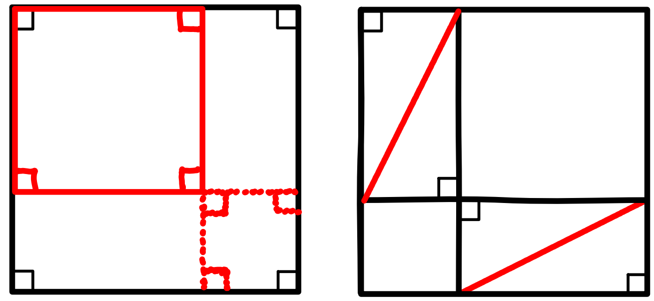
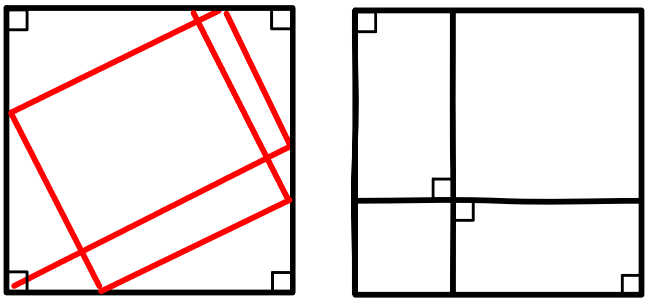
A)



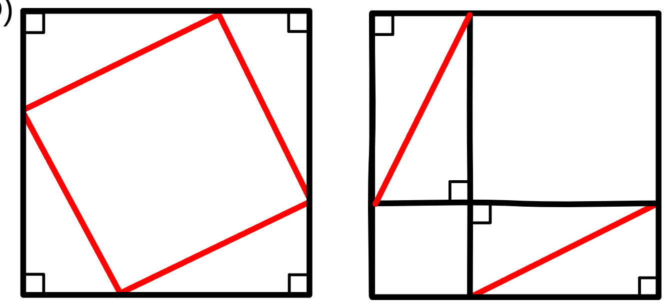
B)



C)



D)



E) None of the above

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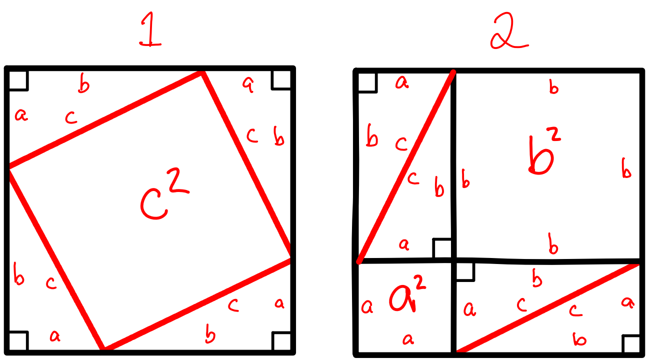
D)




Both squares have the same area $A$.

Both squares have 4 identical triangles with areas $\frac{1}{2}ab$.

Square #1 has a smaller square with area $c^2$.

Square #2 has two smaller squares with areas $a^2$ and $b^2$.

Square #1 has area $A = 4\left(\frac{1}{2}ab\right) + c^2$

Square #2 has area $A = 4\left(\frac{1}{2}ab\right) + a^2 + b^2$

$A = A \Leftrightarrow 4\left(\frac{1}{2}ab\right) + c^2 = 4\left(\frac{1}{2}ab\right) + a^2 + b^2 \Leftrightarrow c^2 = a^2 + b^2$

#3) Which of the following statements is completely true?

A) $\arcsin[\sin^{-1}(1)]$ is defined.

B) $\sin[\arcsin(\pi/2)] = \pi/2$

C) $\arcsin[\sin(\pi/2)] = \sin[\arcsin(\pi/2)]$

D) $\sin^{-1}[\sin(\arcsin(\sin(3\pi/4)))] = \sin[\arcsin(\sin(\sin^{-1}(\pi/4)))]$

E) None of the above

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A) $\arcsin[\sin^{-1}(1)] = \arcsin(\pi/2)$ is not defined since $\pi/2 \notin [-1,1]$

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B) $\sin[\arcsin(\pi/2)] = \pi/2$ is not true since

$\arcsin(\pi/2)$ is undefined because $\pi/2 \notin [-1,1]$ so

$\sin[\arcsin(\pi/2)]$ is undefined as well.

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C) $\arcsin[\sin(\pi/2)] = \sin[\arcsin(\pi/2)]$ is not true since

$\arcsin[\sin(\pi/2)] = \arcsin(1) = \pi/2$ but

$\sin[\arcsin(\pi/2)]$ is undefined because $\pi/2 \notin [-1,1]$

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D) $\sin^{-1}[\sin(\arcsin(\sin(3\pi/4)))] = \sin[\arcsin(\sin(\sin^{-1}(\pi/4)))]$ is true.

For the left hand side, $\sin^{-1}[\sin(\arcsin(\sin(3\pi/4)))] = \sin^{-1}[\sin(\arcsin(\sqrt{2}/2))]$

$= \sin^{-1}[\sin(\pi/4)] = \sin^{-1}(\sqrt{2}/2) = \pi/4$ and

for the right hand side, although $\pi/4$ here would not be interpreted as an angle since it is being plugged into sine inverse rather than sine,

$\sin^{-1}(\pi/4)$ is defined since $\pi/4 \in [-1,1]$.

So, for the right hand side, $\sin[\arcsin(\sin(\sin^{-1}(\pi/4)))] = \pi/4$

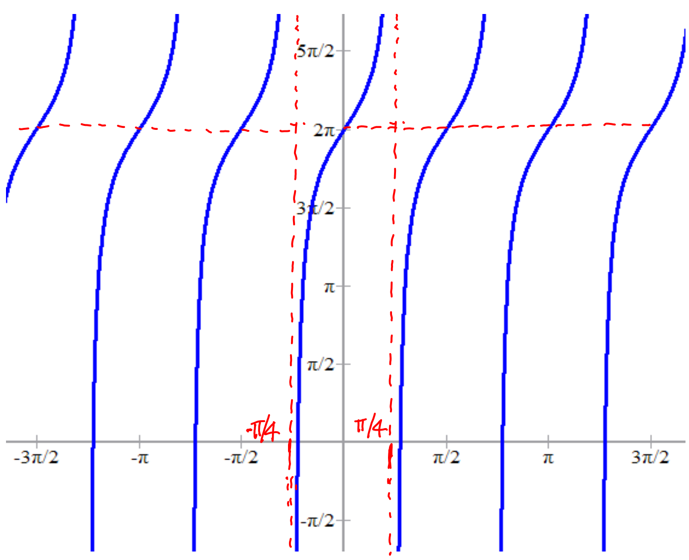
$[-\pi/2, \pi/2]$ $\quad$ $[-\pi/2, \pi/2]$

$\frac{\pi}{4} \in [-1,1]$

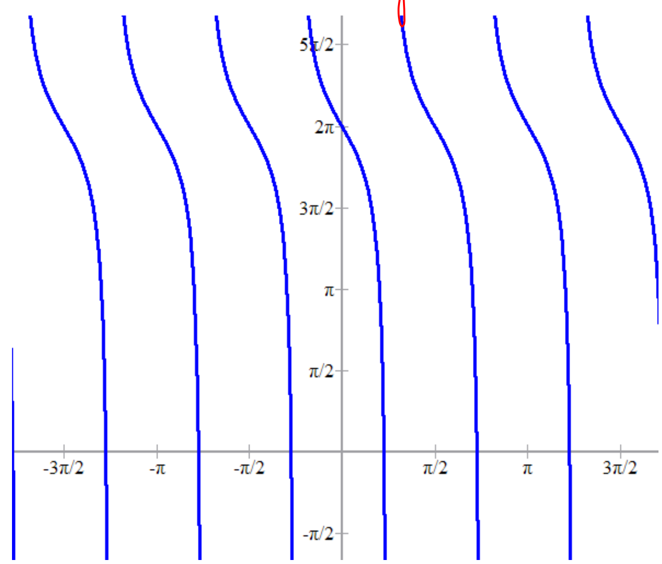
thus $\sin^{-1}[\sin(\arcsin(\sin(3\pi/4)))] = \sin[\arcsin(\sin(\sin^{-1}(\pi/4)))] = \pi/4$

#4) Which of the following is the graph of $y = 2\pi + \tan(2x - \pi)$

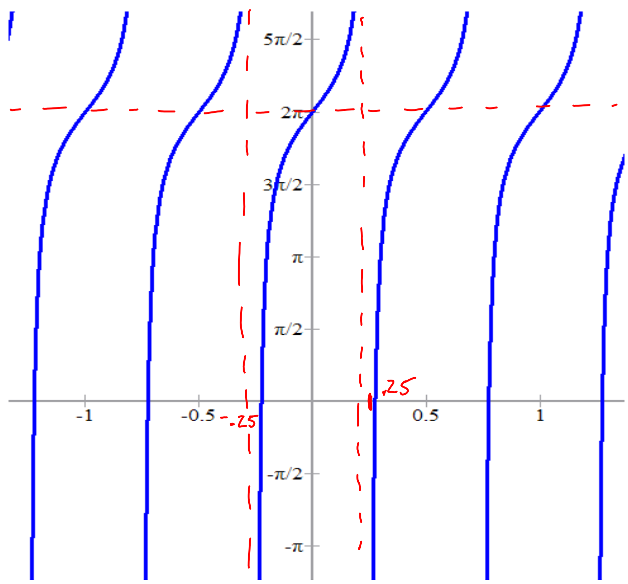
A) $T = \pi/2$ $B = 2\pi$



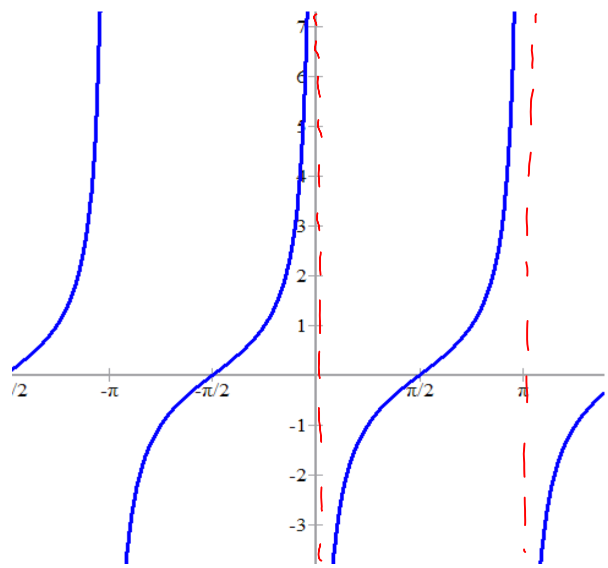

B) Looks like cotangent variation




C) $T = 1/2$, $B = 2\pi$

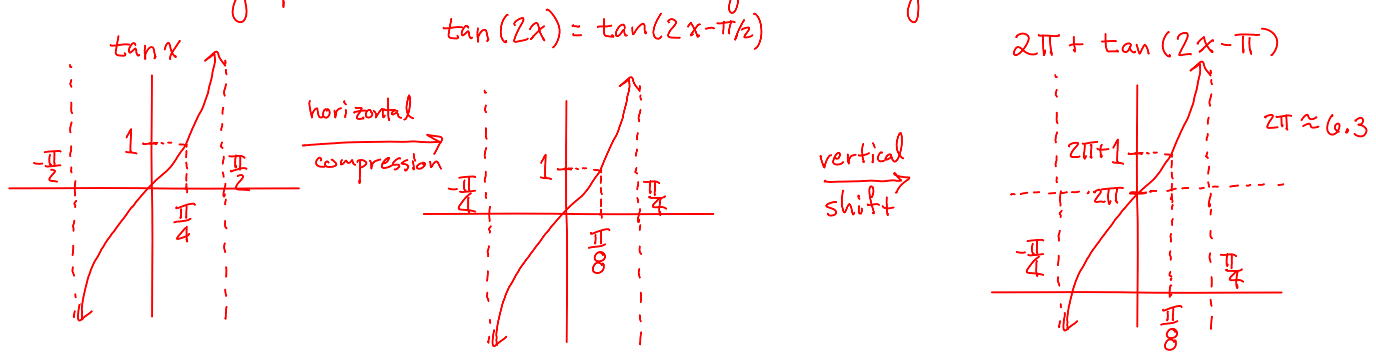



D) $T = \pi$, $\phi/\omega = \pi/2$



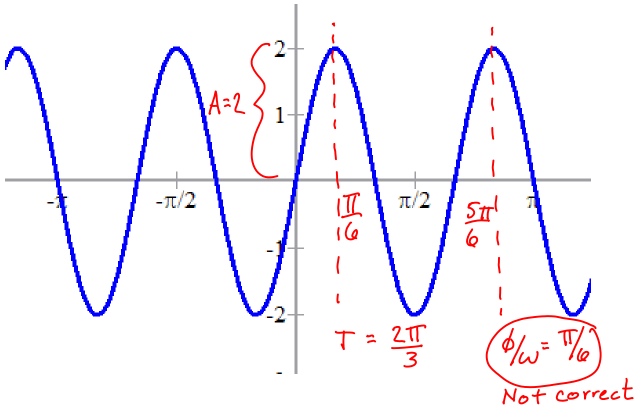

E) None of the above

$y = 2\pi + \tan(2x - \pi)$ has vertical shift $2\pi$, phase shift $\phi/\omega = \pi/2$ and period $T = \pi/\omega = \pi/2$. Also, $\tan(2x-\pi) = \tan(2x)$ because tangent is $\pi$ periodic. thus from the graph of $\tan x$ we can find the graph of $y = 2\pi + \tan(2x - \pi)$

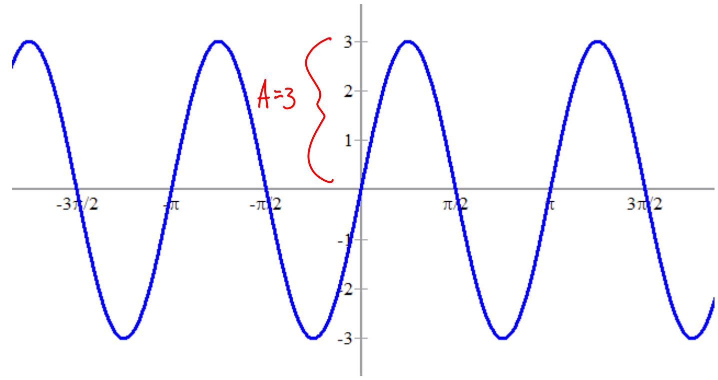


#5) Which of the following is the graph of $y = 2\cos(3x - 6\pi)$

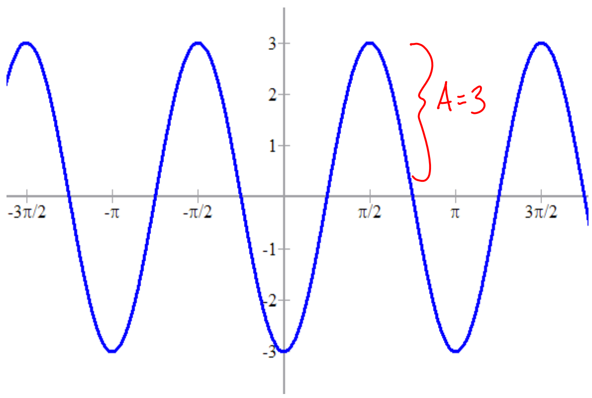
A)



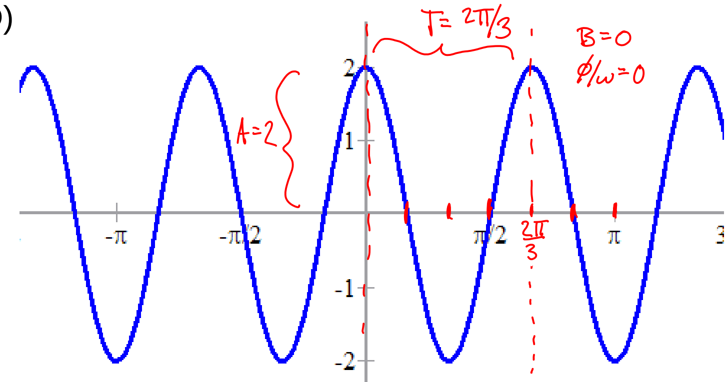

B)




C)




D)




$y = 2\cos(3x + 2n\pi)$ ; $n \in \mathbb{Z}$

E) None of the above

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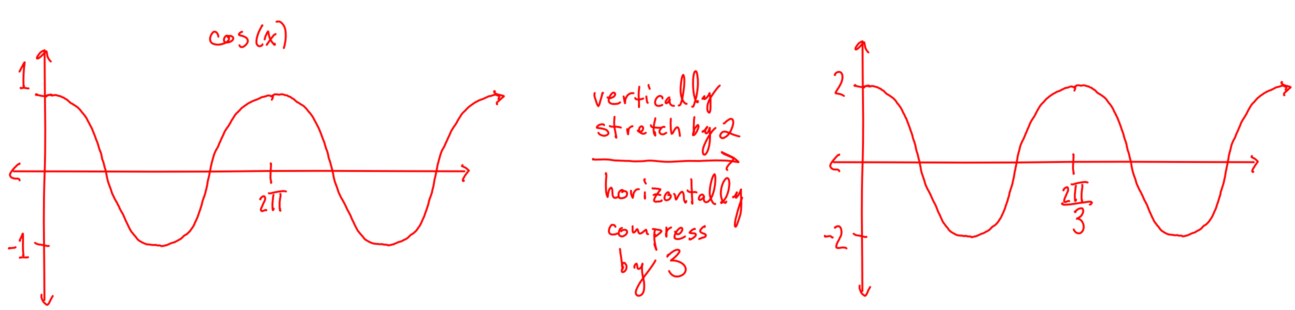
$y = 2\cos(3x - 6\pi) = 2\cos(3x)$ since cosine is $2\pi$ periodic. So, our function has Amplitude $A = 2$, Period $T = 2\pi/\omega = 2\pi/3$, no vertical shift, and phase shift $\phi/\omega = 6\pi/3$ which amounts to no shift at all since $6\pi/3 = 3(2\pi/3) = 3T$

From the graph of cosine we can get our desired graph:



#6) Which of the following statements is completely true?

A) $\sin(-76\pi/3) = -\sin(40\pi/3)$

B) $\cos(17\pi/8) = \cos(-15\pi/8)$

C) $\tan(32\pi/7) = \tan(39\pi/7)$

D) All of the above

E) None of the above

A) $\sin(-76\pi/3) = -\sin(40\pi/3)$ is completely true since

$\sin(-76\pi/3) = -\sin(76\pi/3)$ (because sine is odd)

$= -\sin\left(\frac{4\pi}{3} + \frac{72\pi}{3}\right) = -\sin\left(\frac{4\pi}{3} + 24\pi\right) = -\sin\left(\frac{4\pi}{3} + 12\cdot 2\pi\right)$

$= -\sin\left(\frac{4\pi}{3}\right)$ (because sine is $2\pi$ periodic)

$= -\sin\left(\frac{4\pi}{3} + 6\cdot 2\pi\right) = -\sin\left(\frac{4\pi}{3} + 12\pi\right) = -\sin\left(\frac{4\pi}{3} + \frac{36\pi}{3}\right) = -\sin\left(\frac{40\pi}{3}\right)$

B) $\cos(17\pi/8) = \cos(-15\pi/8)$ is completely true since

$\cos(17\pi/8) = \cos(17\pi/8 - 4\pi)$ (since cosine is $2\pi$ periodic)

$= \cos(17\pi/8 - 32\pi/8) = \cos(-15\pi/8)$

C) $\tan(32\pi/7) = \tan(39\pi/7)$ is completely true since

$\tan\left(\frac{32\pi}{7}\right) = \tan\left(\frac{4\pi}{7} + \frac{28\pi}{7}\right) = \tan\left(\frac{4\pi}{7} + 4\pi\right) = \tan\left(\frac{4\pi}{7}\right)$ (since tangent is $\pi$ periodic)

$= \tan\left(\frac{4\pi}{7} + 5\pi\right) = \tan\left(\frac{4\pi}{7} + \frac{35\pi}{7}\right) = \tan\left(\frac{39\pi}{7}\right)$

#7) Assuming the Earth's orbit is circular, what is it's approximate angular speed assuming the Earth is 8 lightminutes away from the sun? (Note that 1 lightminute is a length equal to the distance light travels in 1 minute and also note that there are 365 days in 1 year).

A) $365\pi/\text{day}$

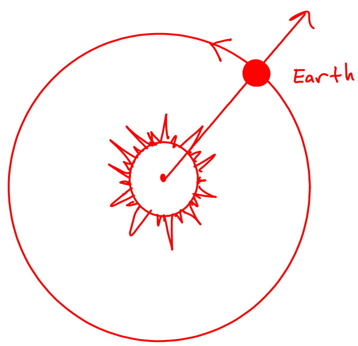
B) $16\pi$ lightminutes / 365 days

has units of length/time but angular speed has units of angle/time

C) $\boxed{2\pi/365 \text{ days}}$

D) All of the above

E) None of the above




angular speed: $\omega = \dfrac{\theta}{\text{time}}$

It should be common knowledge that it takes 1 year for the Earth to complete an orbit about the sun and that 1 year = 365 days. Thus,

$$\omega = \frac{\theta}{\text{time}} = \frac{1 \text{ revolution}}{1 \text{ year}} = \frac{2\pi}{365 \text{ days}}$$

#8) Which of the following statements is completely true if tan(x) = 3/2 and sin(x) > 0?

A) $\cos(x) = -2/\sqrt{13}$ and $\sin(x) = 3/\sqrt{13}$

B) $\csc(x) = \sqrt{13}/2$ and $\sec(x) = \sqrt{13}/3$

C) $\cos(x) = \pm 2/\sqrt{13}$ and $\sin(x) = \pm 3/\sqrt{13}$

D) $\boxed{\cos(x) = 2/\sqrt{13} \text{ and } \sin(x) = 3/\sqrt{13}}$

E) None of the above

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If $\tan(x) = 3/2$ and $\sin(x) > 0$

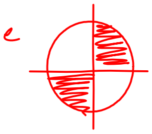
then from the Pythagorean, Reciprocal, and Quotient Identities we have

$$x^2 + y^2 = r^2 \Rightarrow \begin{cases} \dfrac{x^2}{r^2} + \dfrac{y^2}{r^2} = \dfrac{r^2}{r^2} \Rightarrow \cos^2\theta + \sin^2\theta \equiv 1 \\[2ex] \dfrac{x^2}{x^2} + \dfrac{y^2}{x^2} = \dfrac{r^2}{x^2} \Rightarrow 1 + \tan^2\theta \equiv \dfrac{1}{\cos^2\theta} \equiv \sec^2\theta \\[2ex] \dfrac{x^2}{y^2} + \dfrac{y^2}{y^2} = \dfrac{r^2}{y^2} \Rightarrow \cot^2\theta + 1 \equiv \dfrac{1}{\sin^2\theta} \equiv \csc^2\theta \Leftrightarrow \dfrac{1}{\tan^2\theta} + 1 \equiv \dfrac{1}{\sin^2\theta} \Leftrightarrow \sin^2\theta \equiv \dfrac{1}{\dfrac{1}{\tan^2\theta} + 1} \end{cases}$$

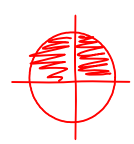
Therefore, $\sin^2 x \equiv \dfrac{1}{\dfrac{1}{\tan^2 x} + 1} = \dfrac{1}{\dfrac{1}{(3/2)^2} + 1} = \dfrac{1}{\dfrac{1}{9/4} + 1} = \dfrac{1}{\dfrac{4}{9} + 1} = \dfrac{1}{\dfrac{13}{9}} = \dfrac{9}{13} \Rightarrow \sin(x) = \pm\sqrt{\dfrac{9}{13}} = \pm\dfrac{3}{\sqrt{13}}$

but

$\tan(x) = 3/2 > 0 \Rightarrow$ { if $x$ is an angle, it must be in quadrants 1 or 3



$\sin(x) > 0 \Rightarrow$ { if $x$ is an angle, it must be in quadrants 1 or 2



These are both satisfied when $x$ is an angle in quadrant 1

and, in quadrant 1, sine, cosine, tangent, cotangent, secant, and cosecant are all positive.

Thus, $\boxed{\sin(x) = 3/\sqrt{13}}$ and since $\cos^2\theta + \sin^2\theta \equiv 1 \Leftrightarrow \cos^2\theta \equiv 1 - \sin^2\theta$, we have,

$\cos(x) = \sqrt{1 - \sin^2(x)} = \sqrt{1 - 9/13} = \sqrt{4/13} = \boxed{2/\sqrt{13} = \cos(x)}$.

Therefore $\sec(x) \equiv \dfrac{1}{\cos(x)} = \boxed{\dfrac{\sqrt{13}}{2} = \sec(x)}$, $\csc(x) \equiv \dfrac{1}{\sin(x)} = \boxed{\dfrac{\sqrt{13}}{3} = \csc(x)}$, and

$\cot(x) \equiv \dfrac{1}{\tan(x)} = \boxed{\dfrac{2}{3} = \cot(x)}$

#9) Solve for x: $2\sin^2 x - 3\sin x = -1$

A) $x \in \{\pi/2 + 2k\pi,\ \pi/6 + 2k\pi,\ 5\pi/6 + 2k\pi;\ k \in \mathbb{Z}\}$

B) $x \in \{(\pi/6)^2 + \pi k;\ k \in \mathbb{Z}\}$

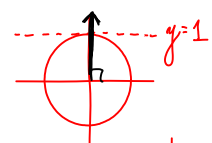
C) $x \in \{\pm\pi/6 + 2\pi k,\ \pi/2 + 2k\pi;\ k \in \mathbb{Z}\}$

D) $x = \pi/6, 5\pi/6$

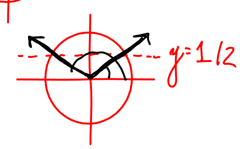
E) None of the above

$$2\sin^2 x - 3\sin x = -1 \Leftrightarrow \sin^2(x) - \frac{3}{2}\sin(x) = -\frac{1}{2} \Leftrightarrow \sin^2(x) - \frac{3}{2}\sin(x) + \frac{1}{2} = 0$$

$$\Leftrightarrow \left(\sin(x) - 1\right)\left(\sin(x) - \frac{1}{2}\right) = 0 \Leftrightarrow \begin{cases} \sin(x) - 1 = 0 \Leftrightarrow \sin(x) = 1 \\ \text{or} \\ \sin(x) - \frac{1}{2} = 0 \Leftrightarrow \sin(x) = 1/2 \end{cases}$$




$$\text{and} \begin{cases} \sin(x) = 1 \Leftrightarrow x \in \{\pi/2 + 2\pi k;\ k \in \mathbb{Z}\} \\ \sin(x) = 1/2 \Leftrightarrow x \in \{\pi/6 + 2\pi k,\ 5\pi/6 + 2\pi k;\ k \in \mathbb{Z}\} \end{cases}$$




$$\Leftrightarrow \boxed{x \in \{\pi/2 + 2\pi k,\ \pi/6 + 2\pi k,\ 5\pi/6 + 2\pi k;\ k \in \mathbb{Z}\}}$$

#10) Which of the following statements is completely true?

A) $\pi^\circ > 3$ radians

B) $\cos(\theta + \pi) \equiv \cos(\theta - \pi)$

C) $\tan(x/y) \equiv \cot(y/x)$

D) All of the above

E) None of the above

A) $\pi^\circ > 3$ radians is not true since $\pi^\circ = 3.14...^\circ$ and 3 radians $\approx \pi$ rad $= 180^\circ$

B) $\cos(\theta + \pi) \equiv \cos(\theta - \pi)$ is completely true since cosine is $2\pi$ periodic

$$\cos(\theta + \pi) \equiv \cos[(\theta + \pi) - 2\pi] \equiv \cos(\theta - \pi)$$

C) $\tan(x/y) \equiv \cot(y/x)$ is not true. Although, it is true that

$$\tan(x/y) \equiv \frac{1}{\cot(x/y)} \quad \text{and} \quad \cot(y/x) = \frac{1}{\tan(y/x)}$$