1) If there are 239 beans in a can and 21 cans in a case of beans and 17 cases in a box of beans and you have 501,900 beans, how many full boxes of beans do you have?

a) 4

b) 5

c) 6

d) 7

e) None of the above

$$501{,}900 \text{ beans} \left(\frac{1 \text{ can}}{239 \text{ beans}}\right)\left(\frac{1 \text{ case}}{21 \text{ cans}}\right)\left(\frac{1 \text{ box}}{17 \text{ cases}}\right) = \frac{501{,}900}{239\cdot 21\cdot 17} \text{ boxes}$$

$$\begin{array}{r} 2100 \\ 239\overline{)501{,}900} \\ \underline{478} \\ 239 \\ \underline{239} \\ 0 \end{array}$$

$$\Rightarrow \frac{501{,}900}{239\cdot 21\cdot 17} = \frac{2100}{21\cdot 17} = \frac{100}{17} = 5 \text{ remainder } 15$$

We're 2 cans short of a 6th box $\Rightarrow$ $\boxed{5 \text{ boxes}}$

---

2) Which of the following are completely true?

a) $\sum_{n=0}^{\infty} (-1)^n (2n)! = 0! - 2! + 4! - 6! + \cdots = 1 - 2 + 24 - \cdots$

b) $\{\pi/2 + n\pi \,;\, n \in \mathbb{Z}\} = \{\ldots, -5\pi/2, -3\pi/2, -\pi/2, \pi/2, 3\pi/2, 5\pi/2, \ldots\}$

c) $\pi^\circ < \pi \text{ rad}$

d) All of the above

e) None of the above

---

a) True. Simply plug in increasing values of n for each term in the sum

$$\sum_{n=0}^{\infty} (-1)^n (2n)! = (-1)^0(2\cdot 0)! + (-1)^1(2\cdot 1)! + (-1)^2(2\cdot 2)! + \cdots$$
$$= 0! - 2! + 4! - 6! + \cdots$$
$$= 1 - 2\cdot 1 + 4\cdot 3\cdot 2\cdot 1 - 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 + \cdots$$
$$= 1 - 2 + 24 - \cdots$$

b) True. Again, simply plug in values of n to get each element of the set. This time, n can be any integer (not just positives) and no order is necessary (although we prefer it).

$$\{\pi/2 + n\pi \,;\, n \in \mathbb{Z}\}$$

$$= \{\ldots \pi/2 + (-3)\pi,\ \pi/2 + (-2)\pi,\ \pi/2 + (-1)\pi,\ \pi/2 + (0)\pi,\ \pi/2 + (1)\pi,\ \pi/2 + (2)\pi, \ldots\}$$

$$= \{\ldots, -5\pi/2, -3\pi/2, -\pi/2, \pi/2, 3\pi/2, 5\pi/2, \ldots\}$$

c) True. $\pi^\circ = 3.1415\ldots^\circ < 180^\circ = \pi \text{ rad}$

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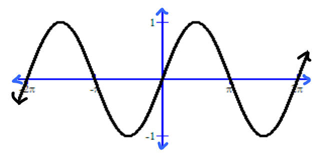
3) The fact that $\sin(x)$ is an odd function can be determined from

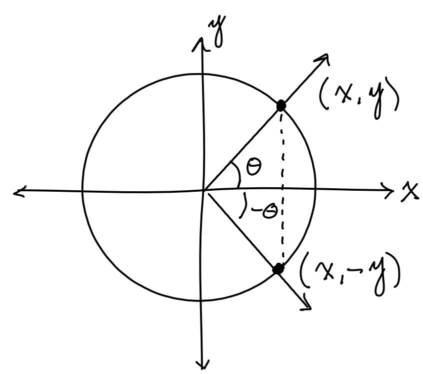
a) the graph of $\sin(x)$



since it is symmetric about the origin.

b) the algebraic representation of $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ since each term has only odd powers of x and therefore $\sin(-x) = -\sin(x)$.

c) the geometry of the unit circle by noticing that given an angle θ, the y coordinate of the corresponding point on the unit circle has the opposite sign of the one for $-\theta$.



$\boxed{\text{d) All of the above}}$

e) None of the above

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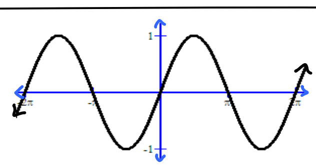
a) True.



Rotate the graph around the origin by 180° degrees and you get the same graph.

b) True. $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$

$$\Rightarrow \sin(-x) = \sum_{n=0}^{\infty} (-1)^n \frac{(-x)^{2n+1}}{(2n+1)!} = -x - \frac{(-x)^3}{3!} + \frac{(-x)^5}{5!} - \ldots$$

$$= -x + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots = -\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right] = -\sin(x)$$

c) True. Look at the given diagram.

4) Which of the following are completely true?

a) $\tan\theta = 1/2 \Rightarrow \sin\theta = 1$ and $\cos\theta = 2$ since $\tan\theta = \sin\theta/\cos\theta$

b) $\sec\theta = 1/\sin\theta$ and it's domain is $\mathbb{R} \setminus \{n\pi, n \in \mathbb{Z}\}$

c) $\cot(-\theta) = \dfrac{\cos(-\theta)}{\sin(-\theta)} = \dfrac{\cos\theta}{-\sin\theta} = -\cot\theta \Rightarrow \cot\theta$ is an even function.

d) All of the above

$\boxed{\text{e) None of the above}}$

---

a) False. First off, $\cos\theta$ can never equal 2. $\cos\theta \in [-1, 1]$.

Remember, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y/r}{x/r} = \dfrac{y}{x}$ There is a hidden radius.

If $\tan\theta = \dfrac{1}{2} = \dfrac{y}{x}$ and $\sin\theta = \dfrac{y}{r}$ and $\cos\theta = \dfrac{x}{r}$

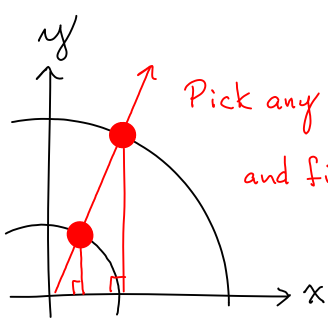
we can choose any x and y we like to represent a point corresponding to the angle theta, so long as their ratio y/x is 1/2. Depending on our choice of y and x, we can find the corresponding radius via the pythagorean theorem.

$\tan\theta = 1/2 = y/x$

Let $y = 1$ and $x = 2$

then $r^2 = x^2 + y^2 = 2^2 + 1^2$

$= 4 + 1 = 5$



Pick any x and y such that $y/x = 1/2$ and find the corresponding radius.

$\Rightarrow r = \pm\sqrt{5}$ (Technically radii are always positive but the minus sign may actually belong to the x or y depending on the quadrant the angle theta lies in.)

$\sin\theta = y/r = \dfrac{1}{\pm\sqrt{5}}$ and $\cos\theta = \dfrac{x}{r} = \dfrac{2}{\pm\sqrt{5}}$

Only knowing that $\tan\theta = 1/2 > 0$ tells us θ is in quadrant 1 or quadrant 3 where $\tan\theta$ is positive. So, both $\sin\theta$ and $\cos\theta$ are positive, or both are negative.

b) False. $\sec\theta = \frac{1}{\cos\theta} \neq \frac{1}{\sin\theta}$.

Although the domain of $\frac{1}{\sin\theta}$ is $\mathbb{R} \setminus \{n\pi, n \in \mathbb{Z}\}$

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c) False. $\cot(-\theta) = -\cot\theta \Rightarrow \cot\theta$ is an odd function.

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5) Which of the following are completely true?

a) $\boxed{\frac{1}{\sec^2\theta} + \frac{1}{\csc^2\theta} - 1 = 0}$

b) $\frac{\cos(1°) + \cos(2°) + \cdots + \cos(359°)}{\sin(1°) + \sin(2°) + \cdots + \sin(359°)} = \tan(1°) + \tan(2°) + \cdots + \tan(359°)$

c) $\cos(20°) + \sin(250°) = \cos(50°) - \sin(220°)$

d) All of the above

e) None of the above

---

a) True. $\frac{1}{\sec^2\theta} + \frac{1}{\csc^2\theta} - 1 = 0 \Leftrightarrow \left(\frac{1}{\sec\theta}\right)^2 + \left(\frac{1}{\csc\theta}\right)^2 = 1 \Leftrightarrow \cos^2\theta + \sin^2\theta = 1$

b) False. $\frac{\cos(1°) + \cos(2°) + \cdots + \cos(359°)}{\sin(1°) + \sin(2°) + \cdots + \sin(359°)} = \frac{-1}{0} =$ undefined

$\tan(1°) + \tan(2°) + \cdots + \tan(359°) =$

We covered problems like this in class (and your homework).

$\cos(1°) = -\cos(179°)$, $\cos(2°) = -\cos(178°) \ldots$

all the angles pair together and cancel except for $\cos(180°)$. Draw a picture.

$\sin(1°) = -\sin(359°)$, $\sin(2°) = \sin(358°) \ldots$; $\sin(180°) = 0$

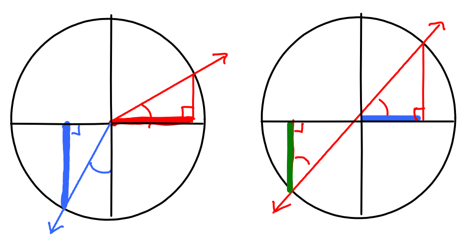
$\tan(1°) = \tan(181°)$, $\tan(2°) = \tan(182°) \ldots$; $\tan(180°) = 0$

Note: Although $\tan(90°)$ and $\tan(270°)$ are undefined,

$\tan(90°) + \tan(270°) = \tan(90°) - \tan(90°) = 0$

c) False. $\cos(20°) + \sin(250°) = 0$

$\cos(50°) - \sin(220°) = 2\cos(50°) \neq 0$



6) Which of the following are completely true?

a) $\sin(2x)$ and $\frac{1}{2}\sin(x)$ have the same graph.

b) $\cos(2x-\pi)$ and $\cos(\pi x-2)$ have the same graph.

c) $\boxed{\cos(4x-2\pi) \text{ and } \cos(4x) \text{ have the same graph.}}$

d) All of the above

e) None of the above

---

a) False. $\sin(2x)$ has period $\frac{2\pi}{2}=\pi$ but $\frac{1}{2}\sin(x)$ has period 2π

b) False. $\cos(2x-\pi)$ has period $\frac{2\pi}{2}=\pi$ but $\cos(\pi x-2)$ has period $\frac{2\pi}{\pi}=2$

c) True. $\cos(4x)$ has period $\frac{2\pi}{4}=\frac{\pi}{2}$. So does $\cos(4x-2\pi)$

$\cos(4x-2\pi)$ is a horizontal shift of $\cos(4x)$ by $\frac{2\pi}{4}=\frac{\pi}{2}$

Shifting a graph horizontally by any multiple of its period will leave it unchanged.

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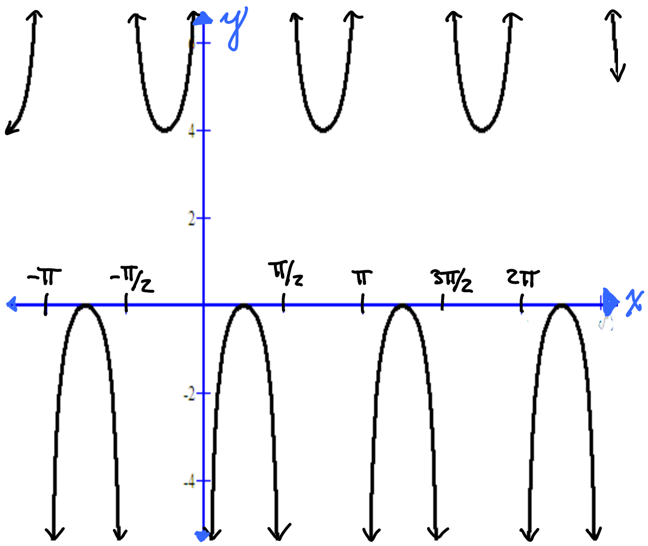
7) Which of the following are completely true?

a) $\sec^2 x - \tan^2 x = \csc^2 x - \cot^2 x$ is an identity

b) $\tan^2 x + 1 = 1/(1-\sin^2 x)$ is an identity

c) 

This is a graph of both

$y = 2\csc(2x - \pi) + 2$

and

$y = 2\sec(2x + \pi/2) + 2$

$\boxed{\text{d) All of the above}}$

e) None of the above

---

a) True. $\sec^2 x - \tan^2 x = \dfrac{1}{\cos^2 x} - \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x}(1 - \sin^2 x) = \dfrac{\cos^2 x}{\cos^2 x} = 1$

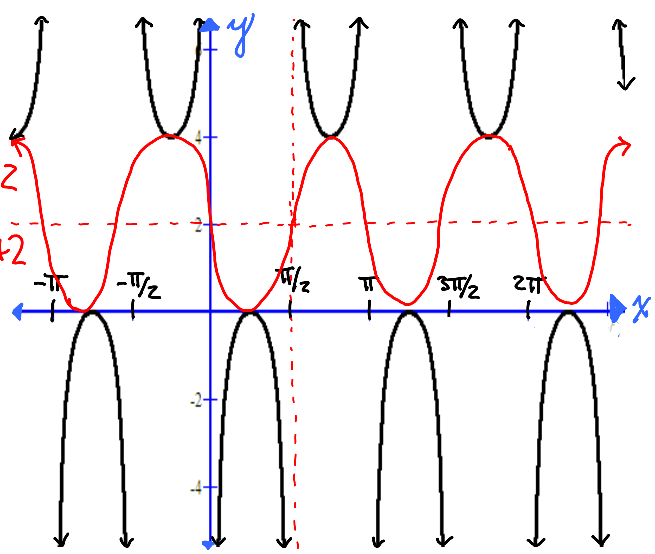
$\csc^2 x - \cot^2 x = \dfrac{1}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x}(1 - \cos^2 x) = \dfrac{\sin^2 x}{\sin^2 x} = 1$

b) True. $\tan^2 x + 1 = \dfrac{\sin^2 x}{\cos^2 x} + 1 = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \dfrac{1}{1 - \sin^2 x}$

c) True.

$2\sin(2x - \pi) + 2$

$2\cos(2x + \pi/2) + 2$



This is a graph of both

$y = 2\csc(2x - \pi) + 2$

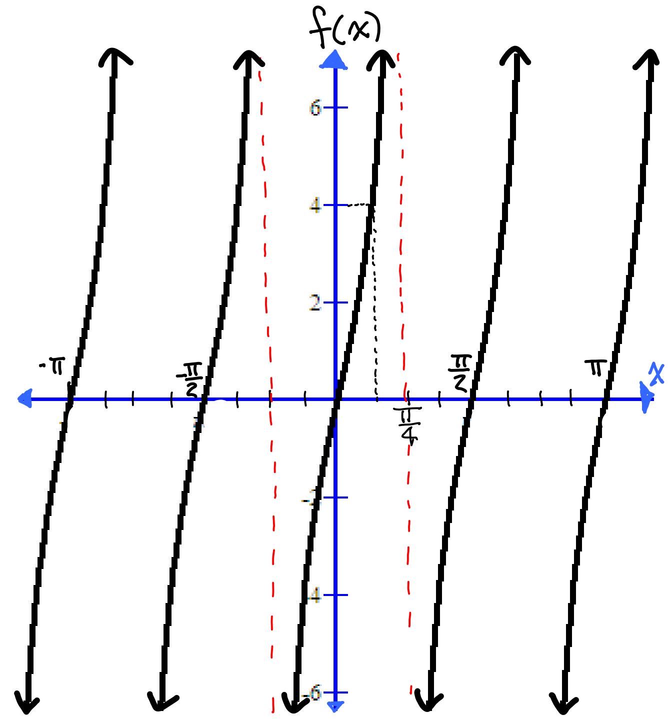
and

$y = 2\sec(2x + \pi/2) + 2$

Look at $2\sin(2x - \pi) + 2$ and $2\cos(2x + \pi/2) + 2$ instead.

$\sin(\theta) = \cos(\theta - \pi/2) \Rightarrow \sin(\underbrace{2x - \pi}_{\theta}) = \cos[(\underbrace{2x - \pi}_{\theta}) - \pi/2] = \cos(2x - 3\pi/2) = \cos(2x - \pi/2)$

since $T = \pi$

8) 

Which of the following is true?

a) $2f(x) = \tan(2x) + 4$

b) $(1/4)f(x) = \tan(4x)$

c) $4f(x) = \tan(2x)$

d) $\boxed{(1/4)f(x) = \tan(2x)}$

e) None of the above

---

a) False $2f(x) = \tan(2x) + 4 \Rightarrow f(x) = \frac{1}{2}\tan(2x) + 2$ would be a shift of $\frac{1}{2}\tan(2x)$ by 2. The graph is tangent-like but not shifted up.

b) False. $(1/4)f(x) = \tan(4x) \Rightarrow f(x) = 4\tan(4x)$ has period $\frac{\pi}{4}$ but the graph has period $\frac{\pi}{2}$.

c) False. $4f(x) = \tan(2x) \Rightarrow f(x) = \frac{1}{4}\tan(2x)$

$\Rightarrow f\left(\frac{\pi}{8}\right) = \frac{1}{4}\tan\left[2\left(\frac{\pi}{8}\right)\right] = \frac{1}{4}\tan\left(\frac{\pi}{4}\right) = \frac{1}{4}$

But the graph says $f\left(\frac{\pi}{8}\right) = 4$

d) True. $(1/4)f(x) = \tan(2x) \Rightarrow f(x) = 4\tan(2x)$ has period $\frac{\pi}{2}$

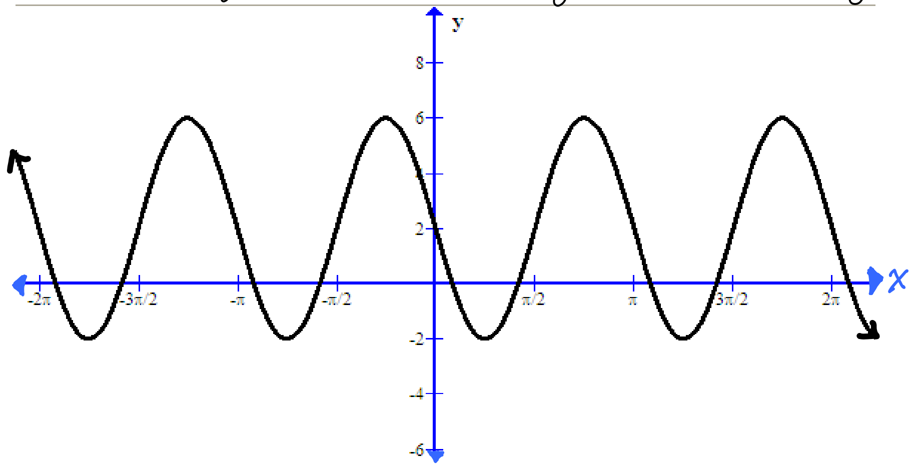
$f\left(\frac{\pi}{8}\right) = 4\tan\left[2\left(\frac{\pi}{8}\right)\right] = 4\tan\left(\frac{\pi}{4}\right) = 4$

These properties both match those of the graph.

Also, the graph is not shifted vertically or horizontally.

We could have just looked at the graph and determined A, B, ω, and ϕ and written its function down.

9) Which of the following is <u>NOT</u> completely true regarding this graph?



a) If treated as a sine graph, its phase shift is $\pi/2$ or $-\pi/2$

b) If treated as a cosine graph, its phase shift is $3\pi/4$ or $-\pi/4$

$\boxed{\text{c) The amplitude is 6 and } T = \pi}$

d) All of the above are completely true.

e) All of the above are <u>NOT</u> completely true.

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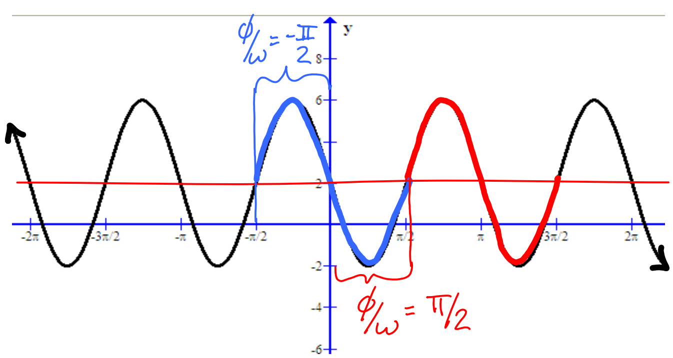
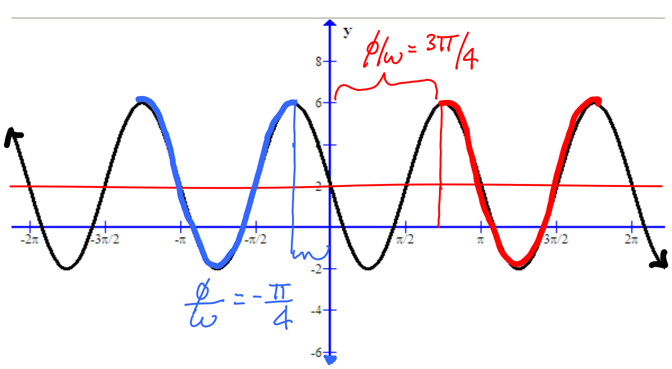
a) True.



b) True.

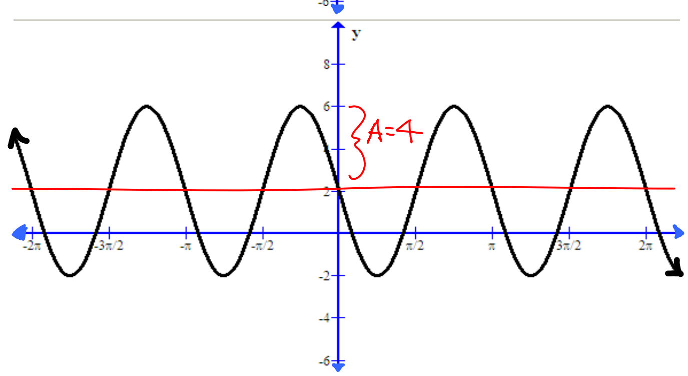


$\Rightarrow$ C is not completely true

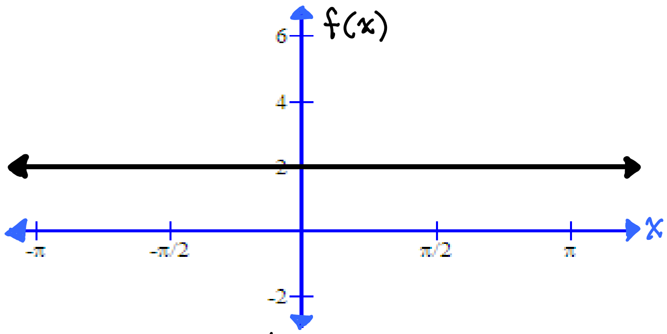
$\Rightarrow$ C is the correct answer.

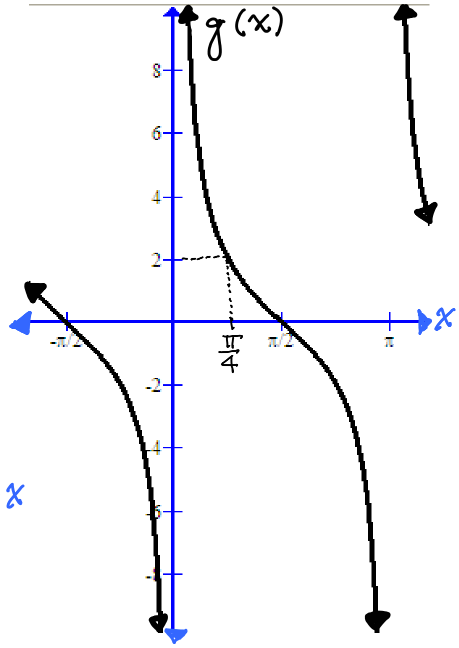
D) "All of the above are completely true." is technically "not completely true" since C is false, but that's not what I meant. If you were confused, you were welcome to ask for clarification.

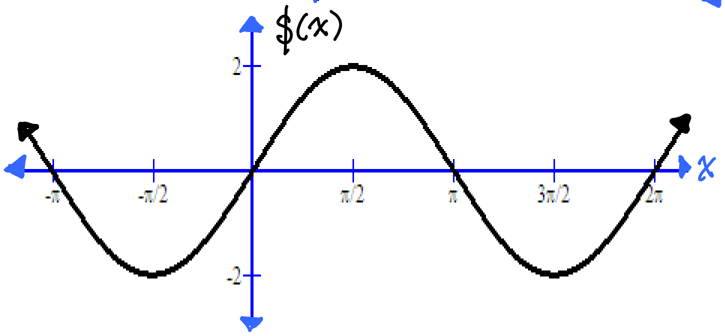
c) False.

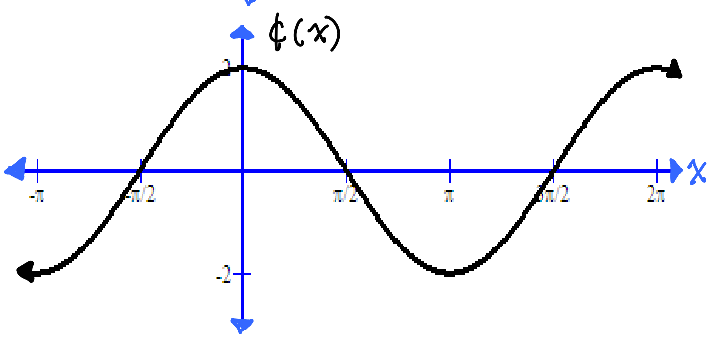


10) Which of the following are completely true about the following graphs?









a) $f(x) = 2(\sin^2 x + \cos^2 x)$ and $g(x) = 2\tan(x)$

b) $\$(x - \pi/2) = ¢(x)$ and $g(x + \pi/2) = -2\tan(x)$

$\boxed{\text{c) } f(x) = (1/2)(\$^2(x) + ¢^2(x))}$

d) All of the above

e) None of the above

From the graphs we can see that $f(x) = 2$, $\$(x) = 2\sin(x)$, $¢(x) = 2\cos(x)$ and $g(x) = 2\cot(x)$.

a) False. $2(\sin^2 x + \cos^2 x) = 2 = f(x)$ but $g(x) = 2\cot(x) \neq 2\tan(x)$.

b) False. $g(x + \pi/2) = 2\cot(x + \pi/2) = 2\cot(x - \pi/2) = -2\cot(\pi/2 - x) = -2\tan(x)$ but
$\$(x - \pi/2) = 2\sin(x - \pi/2) = -2\sin(\pi/2 - x) = -2\cos(x) = -¢(x) \neq ¢(x)$

c) True. $(1/2)(\$^2(x) + ¢^2(x)) = (1/2)[(2\sin x)^2 + (2\cos x)^2]$
$= (1/2)4(\sin^2 x + \cos^2 x) = 2 = f(x)$

11) EXTRA CREDIT (10 Points): If $g(x) = 4x^2$ and $f(x) = 4x^2 - 12x + 9$ then

a) The graph of $f(x)$ is a graph of $g(x)$, except shifted to the right by 3/2.

b) The graph of $f(x)$ is a graph of $g(x)$, except shifted to the right by $12x$ and shifted up by 9.

c) The graph of $\frac{1}{g(x)}$ has a vertical asymptote at $x = g(0)$ but $\frac{1}{f(x)}$ has a vertical asymptote at $x = g(3/2)$

d) All of the above

e) None of the above

---

a) True. $g(x - 3/2) = 4(x - 3/2)^2 = 4x^2 - 12x + 9 = f(x)$

b) False. $f(x)$ is $g(x)$ shifted to the right, so shifting $g(x)$ up can't possibly give you $f(x)$.

c) False. $g(x) = 4x^2 \Rightarrow g(0) = 0$.

So $\frac{1}{g(g(0))} = \frac{1}{g(0)} = \frac{1}{0}$ = undefined $\Rightarrow$ vertical asymptote. But,

$g(3/2) = 4(3/2)^2 = 9$

$\Rightarrow \frac{1}{f(g(3/2))} = \frac{1}{f(9)} = \frac{1}{4(9)^2 - 12 \cdot 9 + 9} = \frac{1}{324 - 108 + 9} = \frac{1}{225}$ is defined

and therefore there is no vertical asymptote at $\frac{1}{f(g(3/2))}$.