2007 SUMMER PRECALCULUS FINAL EXAM

NAME: _____ BANNER _____

No notes, formula sheets, or calculators.
Show all of your work. Label your work clearly so I know what you did for each problem. If you do some work "in your head" give me a brief outline of what your thoughts are. If you make an educated guess, provide me with the motivation behind your answer. Don't rush. When finished, rework the exam to check for errors.

C #1) Assuming the Earth's orbit is circular, what is it's approximate angular speed assuming we are 8 light-minutes away from the sun?

A) $\frac{365\pi}{\text{day}}$ B) $\frac{730\pi}{\text{day}}$ **C) $\frac{2\pi}{365 \text{ days}}$** D) $\frac{730\pi}{8 \text{ light-minutes}}$

E) None of the above

$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$ and $T = 365 \text{ days} \Rightarrow \omega = \frac{1 \text{ rev}}{365 \text{ days}} = \boxed{\frac{2\pi}{365 \text{ days}} = \omega}$

A #2) Convert $30°20'15''$ to units of degrees only.

A) $30° + \frac{1}{3}° + \frac{15}{3600}°$ B) $30.972°$ C) $30.2015°$

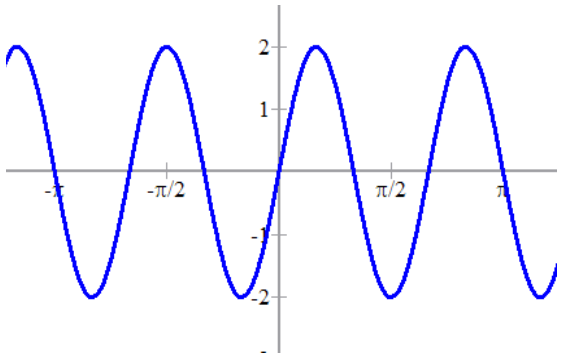
D) $\frac{30.2}{60}° + \frac{30.15}{3600}°$ E) None of the above

$30°20'15'' = 30° + 20'\left(\frac{1°}{60'}\right) + 15''\left(\frac{1'}{60''}\right)\left(\frac{1°}{60'}\right)$

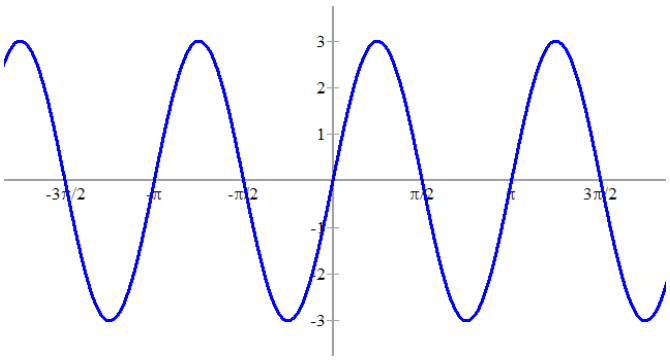
$= \boxed{30° + \frac{1}{3}° + \frac{15}{3600}° = 30°20'15''}$

D #3) Which of the following is the graph of $y = 2\cos(3x - 6\pi)$

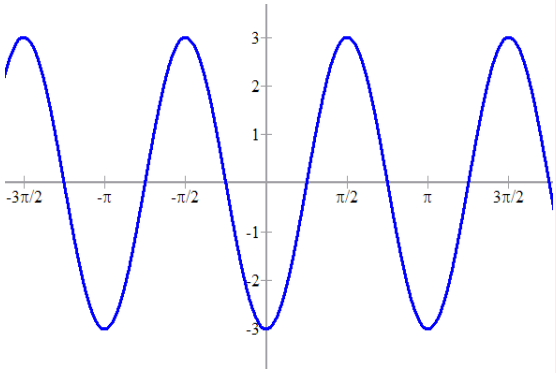
A)



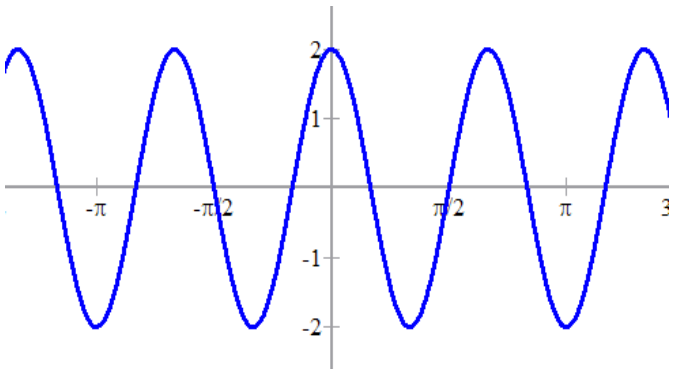
B)



C)

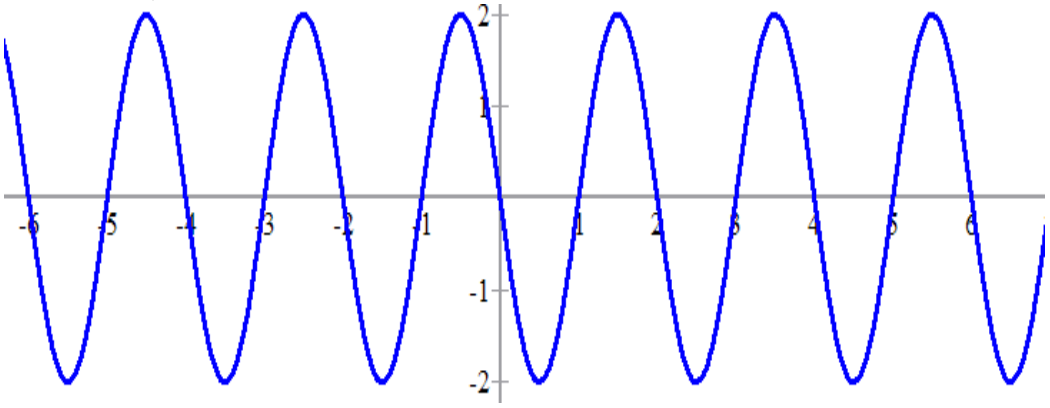


D)



E) None of the above

B #4) This is a graph of which function?



A) $y = 2\cos(\pi x)$

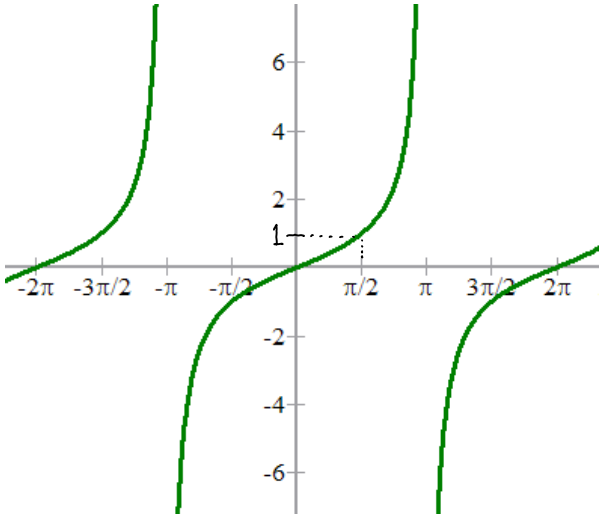
B) $y = 2\sin(\pi x - \pi)$

C) $y = 2\cos(2x - \pi)$

D) $y = 2\sin(\pi x/2 + \pi/2)$

E) None of the above

C #5) This is a graph of which function?



A) $y = \tan(2x)$

B) $y = 2\tan(\pi x)$

C) $y = \tan(x/2 - 2\pi)$

D) $y = \tan(x/2 + \pi)$

E) None of the above

C #6) Which of the following is completly true?

A) $\arcsin[\sin^{-1}(1)]$ is defined.

B) $\sin[\arcsin(\pi/2)] = \pi/2$

C) $\arccos[\arccos(1)] = \pi/2$

D) $\arccos[\cos(7\pi/6)] = 7\pi/6$

E) None of the above

C) $\arccos[\arccos(1)] = \arccos(0) = \pi/2$

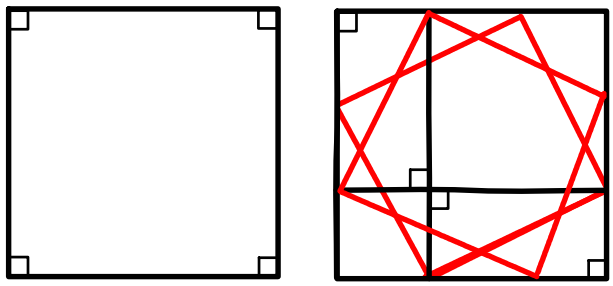
$\arccos(1) = 0$ since $\cos(0) = 1$

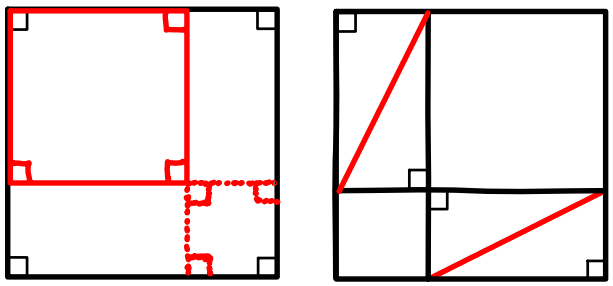
C #7) Which of the following is an identity?

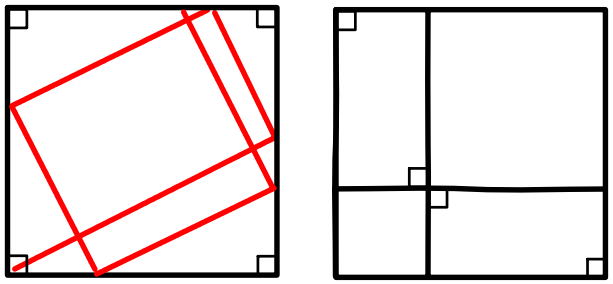
A) $\cos x = 1 - \sin x$ B) $1 + \tan x = \csc x$ C) $\cos^2 x = \dfrac{1}{1 + \tan^2 x}$

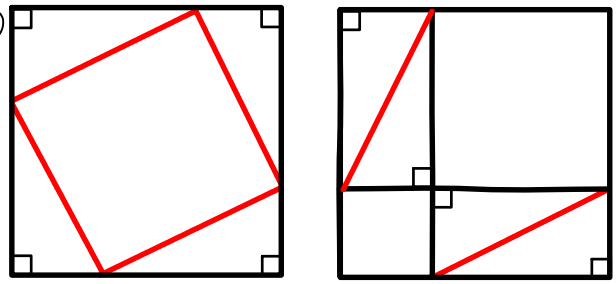
D) All of the above E) None of the above

D #8) Which of the following pairs of pictures provides a proof of the pythagorean theorem?

A) 

B) 

C) 

D) 

E) None of the above

B #9) Solve for x: $3x - 2x^2 = 6$

A) $x = \dfrac{3}{4} \pm \dfrac{\sqrt{39}}{4}$ B) $x = \dfrac{3}{4} \pm \dfrac{\sqrt{-39}}{4}$ C) $x = \pm\dfrac{3}{4} - \dfrac{\sqrt{39}}{4}$

D) $x = 6, 36$ E) None of the above

$3x - 2x^2 = 6 \Leftrightarrow -2x^2 + 3x - 6 = 0$

$\Leftrightarrow x = \dfrac{-3 \pm \sqrt{3^2 - 4(-2)(-6)}}{2(-2)} = \dfrac{-3 \pm \sqrt{9 - 48}}{-4} = \dfrac{3}{4} \pm \dfrac{\sqrt{-39}}{-4} = \dfrac{3}{4} \pm \dfrac{\sqrt{-39}}{4}$

#10) Which of the following is completly true?

A) $\sin(\pi/3) = \sin(2\pi/3)$

B) $\cos(\pi/3) = \cos(7\pi/3)$

C) $\tan(\pi/4) = -\tan(-\pi/4)$

D) All of the above

E) None of the above

A) $\sin(\pi/3) = \sqrt{3}/2 = \sin(2\pi/3)$

B) $\cos(\pi/3) = 1/2 = \cos(7\pi/3)$

C) $\tan(\pi/4) = 1 = -(-1) = -\tan(-\pi/4)$

#11) Which of the following is completly true?

A) $\sin(2\pi/3) = \cos(-\pi/3)$

B) $\cos(2\pi/3) = \sin(-\pi/3)$

C) $\tan(2\pi/3) = \cot(\pi/3)$

D) All of the above

E) None of the above

A) $\sin(2\pi/3) = \sqrt{3}/2 \neq 1/2 = \cos(-\pi/3)$

B) $\cos(2\pi/3) = -1/2 \neq -\sqrt{3}/2 = \sin(-\pi/3)$

C) $\tan(2\pi/3) = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3} \neq \sqrt{3} = \frac{\sqrt{3}/2}{1/2} = \cot(\pi/3)$

#12) Which of the following is completly true?

A) $\sin(7\pi/6) = \sqrt{3}/2$

B) $\sin(5\pi/6) = -1/2$

C) $\sin(13\pi/6) = 1/2$

D) All of the above

E) None of the above

#13) Which of the following is an identity?

A) $\frac{1+\tan\theta}{1-\tan\theta} = \frac{\cot\theta+1}{\cot\theta-1}$

B) $\sin x = \frac{\tan x}{\cos x}$

C) $\sec x = \cos(90°-x)$

D) All of the above

E) None of the above

#14 Which of the following is an identity?

A) $\cos(x+y) = \cos y \cos x - \sin x \sin y$ B) $\sin(x-y) = \sin y \cos x - \cos y \sin x$

C) $\tan(x+y) = \dfrac{\tan x + \tan y}{1 + \tan x \tan y}$

D) All of the above E) None of the above

#15) Which of the following is an identity?

A) $\sin(4y) = 4\sin(2y)\cos(2y)$ B) $\cos(6q) = 2\cos^2(3q) - 1$

C) $\cos(4p) = 1 - 2\cos^2(2p)$

D) All of the above E) None of the above

#16) $\cos(\tan^{-1}(p)) =$

A) $\dfrac{p}{\sqrt{1+p^2}}$ B) $\pm\dfrac{1}{\sqrt{1+p^2}}$ C) $\dfrac{1}{\sqrt{1+p^2}}$ D) $\dfrac{-p}{\sqrt{1+p^2}}$ E) None of the above

$\cos(\tan^{-1}(p)) = \cos\theta$ where $\theta = \tan^{-1}(p) \Rightarrow \begin{cases} \theta \in (-\pi/2, \pi/2) \Rightarrow \cos\theta > 0 \\ \tan\theta = p \end{cases}$

$1 + \tan^2\theta = \sec^2\theta = 1/\cos^2\theta \Rightarrow \cos\theta = \sqrt{\dfrac{1}{1+\tan^2\theta}} = \dfrac{1}{\sqrt{1+p^2}}$

#17) $\cos(\sin^{-1}(1/2)) =$

A) $-\sqrt{3}/2$ B) $1/2$ C) $\sqrt{3}/2$ D) Undefined E) None of the above

$\cos(\sin^{-1}(1/2)) = \cos(\pi/6) = \sqrt{3}/2$

#18) If $f(z) = \sin(2/z)$ then

A) $f(-z) = f(z)$ B) $f(2/z) = \sin z$ C) $f(z) = \pi/2$

D) All of the above E) None of the above

#19) Suppose $f(y) = \sin\left(\frac{1}{y}\right)$. Which of the following is true for $f(y)$:

(a) f is defined for all $y = 0$,

(b) f is defined for all y such that $-\infty < y \leq 0$ and $0 < y < \infty$,

(c) f is undefined for $y = \cos\left[n\pi + \frac{\pi}{2}\right]$ if $n \in \mathbb{Z}$,

(d) $f(y)$ is undefined for all y such that $\theta = 0$,

(e) None of the above.

#20) Suppose A, B, C represent the area of the squares and x, y, h are the lengths of the sides of a triangle. According to the figure which statement is always true.

(a) $A^2 + B^2 = \cos^2\theta + \sin^2\theta = 1$

(b) $\frac{A}{C} + \frac{B}{C} = \frac{x^2}{h^2} + \frac{y^2}{h^2} = \cos^2\theta + \sin^2\theta = 1$

(c) $A + B = C = x^2 + y^2 = h^2 = \sin^2 y + \cos^2 x = 1$

(d) $C - B = A = h^2 - y^2 = \cos^2\theta - 1 = \sin^2\theta$

(e) None of the above