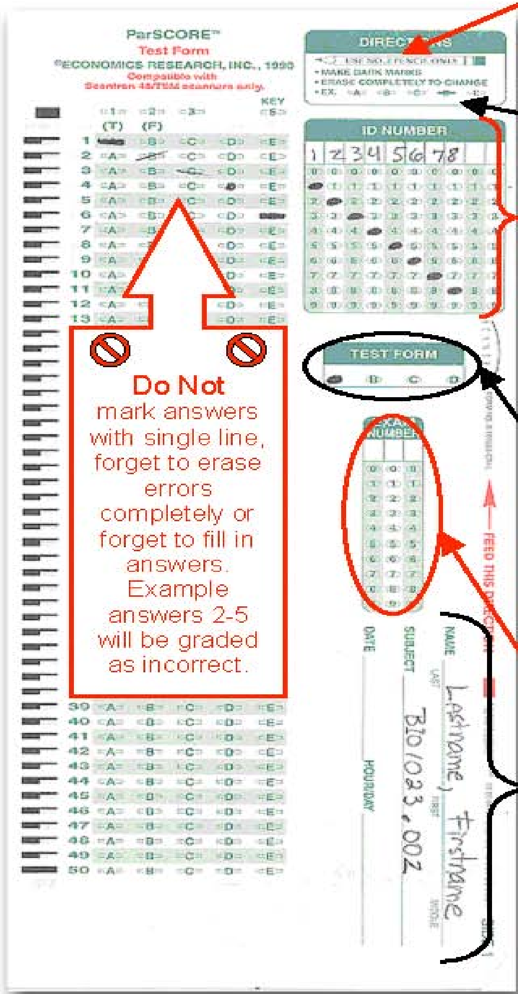
**PRECALCULUS** **Fall 2008** **Exam #1** **Solutions**

Name________________________________ Banner:____________________

## Instructions for Students using ParSCORE Test Forms

Required Materials *(available at campus bookstore)*:

- **ParSCORE Test Form – No. X-101864**
- **#2 Pencil**




**Use a #2 Pencil**
**Note**: Marks made with mechanical, recycled, green, and earth friendly pencils as well as pens ***will be marked wrong*** by the scanner.

Fill in the entire rectangle to mark your answer. Example answers 1 and 6 will be graded as correct.

Your ID number is the ***LAST 8 digits of your BANNER ID.*** Drop the first zero on your Banner ID. Example: Student's Banner ID reads **"012345678"**. The ID Number entered on the ParSCORE Test Form is **"12345678"**. *Do not use social security or driver's license number.*

Form A

Do not fill in the Exam Number

PRINT your **Name, Course,** and **Section Number** clearly.

| Course = Precal | | 1093.section |
|---|---|---|
| TTh at 7pm | => | 1093.001 |
| MWF at 10am | => | 1093.002 |

Separate the pages of the exam and use the back of the paper as scratch paper. I'll have a stapler to staple your exam back together. Grades will be available in WebCT as soon as possible.

Cover your work and your Parscore. Don't cheat or appear to be cheating.

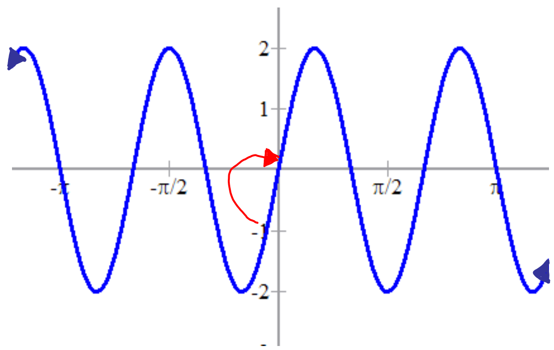
If something is illegible then please notify me. If a question is ambiguous then please ask me to clarify.

DON'T GIVE UP! Do your best on every problem. You are not supposed to already know the answer, you are to figure it out using what you know. Use all of your available time. If you finish early, redo the problems to verify correctness. Don't "check your work" - redo it separately without looking at your previous work to avoid maiking the same mistakes twice.

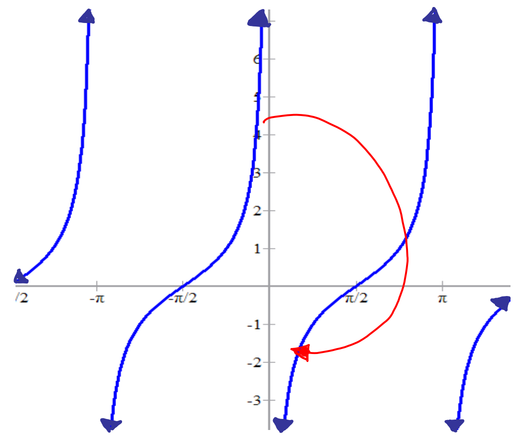
**Circle your answers on this exam and fill in the corresponding bubble on your ParScore.**

#1) Which of the following graphs does not have symmetry about the origin?

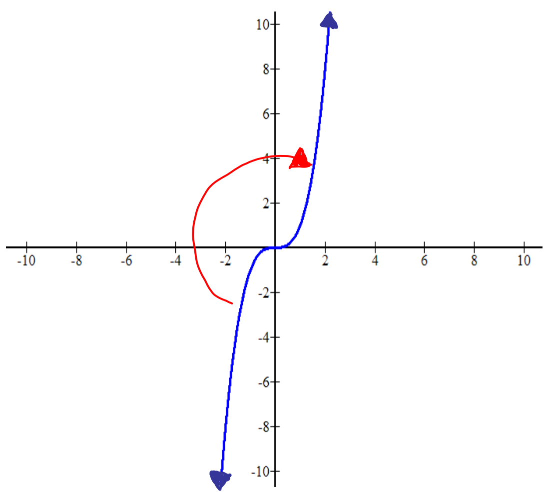
A)




B)




C)




D) All of the above

E) None of the above

They all have symmetry about the origin.

#2) Which of the following is completely true?

A) $\sin(-x) \equiv \sin(x)$

B) $\cos(x) \equiv \sin(x - \pi/2)$

C) $\sin(x) \equiv \cos(x - \pi/2)$

D) All of the above

E) None of the above

A) False. $\sin(-x) \equiv -\sin(x)$

B) False. $\cos(x) \equiv \sin(\pi/2 - x) \neq \sin(x - \pi/2) \equiv -\sin(\pi/2 - x)$

C) True. $\sin(x) \equiv \cos(\pi/2 - x) \equiv \cos[-(x - \pi/2)] \equiv \cos(x - \pi/2)$

↑ since $\cos(-\theta) \equiv \cos(\theta)$

#3) Which of the following pairs of functions have the same graph?

A) $\sin(x-\pi)$ and $\cos(\pi-x)$

B) $\sin^2(x)$ and $\cos^2(x)+1$

$\boxed{\text{C)} \quad -\cos(x) \text{ and } \sin(x-\pi/2)}$ $\quad \sin(x-\pi/2) \equiv -\sin(\pi/2-x) \equiv -\cos(x)$

D) $\cos(x)$ and $1/\sin(x)$ $\quad \cos(x) \not\equiv 1/\sin(x) \equiv \csc(x)$

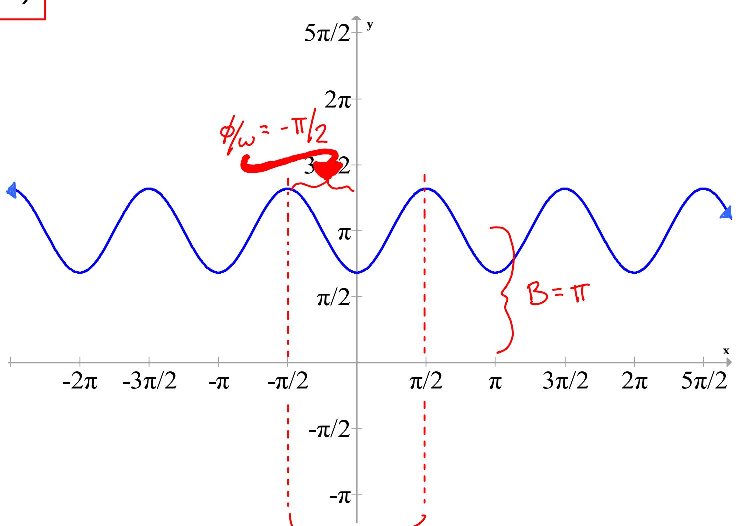
E) None of the above

#4) Which of the following is the graph of $y = \pi + \cos(-2x-\pi)$

$= \pi + \cos[-(2x+\pi)]$

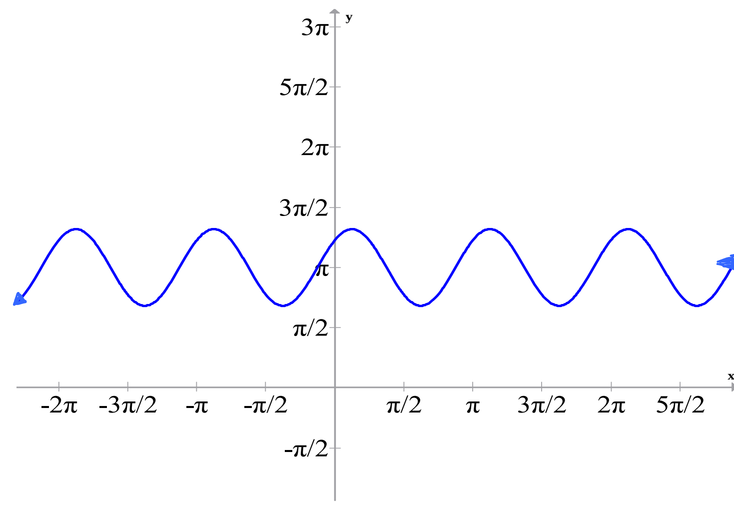
$= \pi + \cos(2x+\pi)$

$\boxed{\text{A)}}$

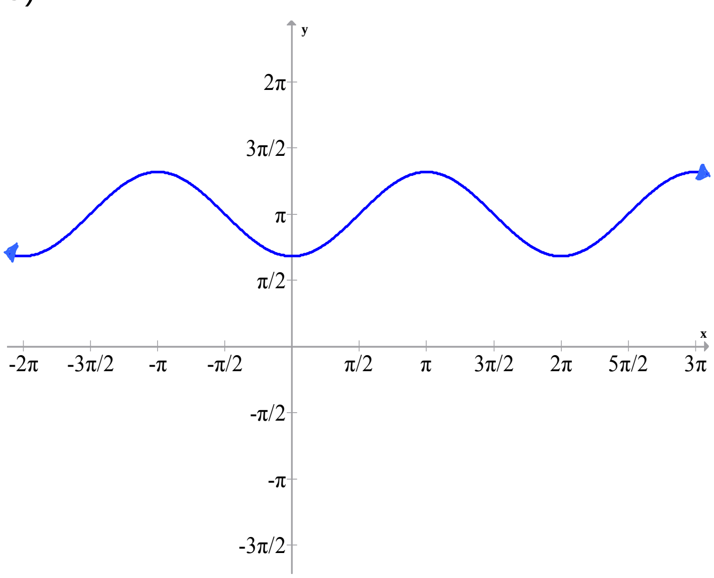



$T = \pi = \frac{2\pi}{2}$

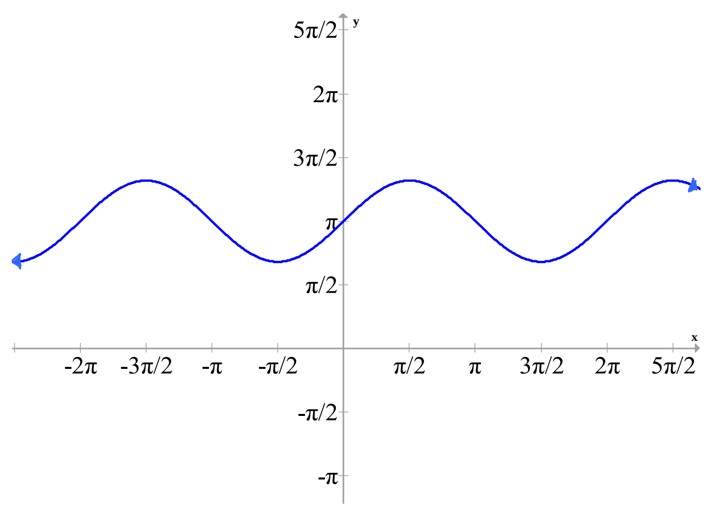
B)




C)




D)

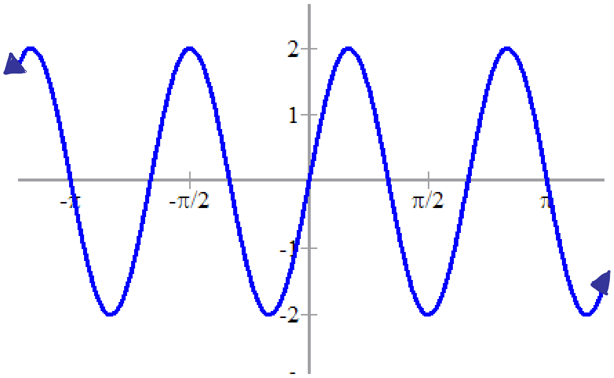



E) None of the above

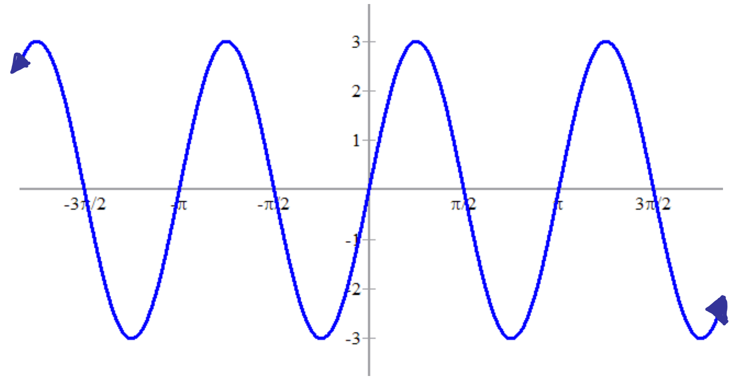
#5) Which of the following is the graph of $y = 1 - 3\sin(2x) - 1 = -3\sin(2x)$

$|A| = 3 \quad \omega = 2 \Rightarrow T = \pi$

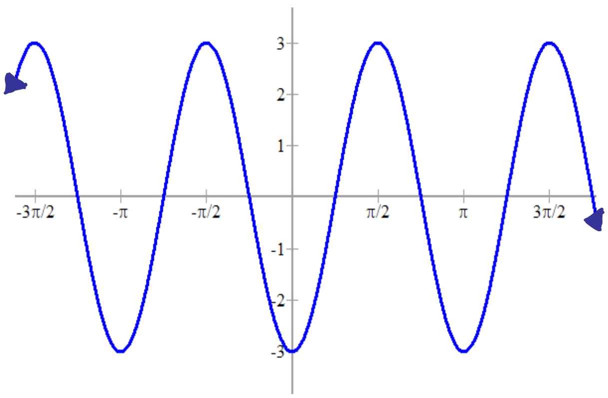
A)



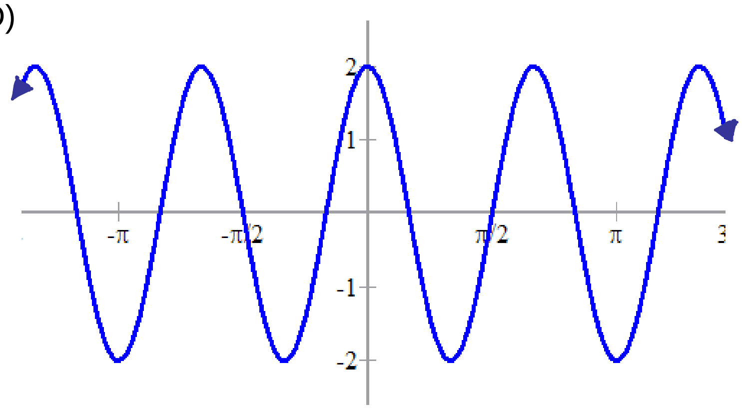

B)




C)




D)




$\boxed{\text{E)}}$ None of the above

#6) Which of the following statements is completely true?

A) $\sin(-76\pi/3) = -\sin(73\pi/6)$

B) $-\cos(17\pi/8) = \cos(-15\pi/8)$

C) $-\tan(32\pi/7) = \tan(-39\pi/7)$

D) All of the above

E) None of the above

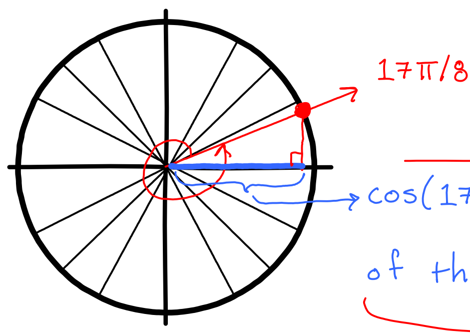
---

A) $\sin(-76\pi/3) = -\sin(76\pi/3) = -\sin(24\pi + 4\pi/3) = -\sin(4\pi/3) = \sqrt{3}/2$

$-\sin(73\pi/6) = -\sin(12\pi + \pi/6) = -\sin(\pi/6) = -1/2 \neq \sqrt{3}/2$

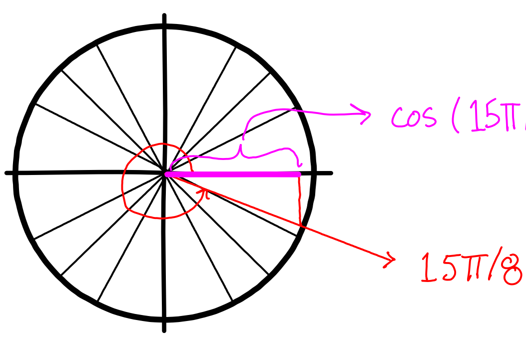
---

B) $-\cos(17\pi/8)$




$\cos(17\pi/8)$ Take the negative of this length to get $-\cos(17\pi/8)$

$\cos(-15\pi/8) = \cos(15\pi/8)$




$\cos(15\pi/8) \longrightarrow \cos(15\pi/8) = \cos(17\pi/8) \neq -\cos(17\pi/8)$

---

$\boxed{\text{C)}}$ $-\tan(32\pi/7) = \tan(-32\pi/7)$

$= \tan(-32\pi/7 - \pi)$ since tangent is $\pi$ periodic

$= \tan(-39\pi/7)$

#7) Assuming the Earth's orbit is circular, what is it's approximate angular speed assuming the Earth is 8 lightminutes away from the sun? (Note that 1 lightminute is a length equal to the distance light travels in 1 minute and also note that there are 365 days in 1 year).

---

A) $365°/1\text{ orbit}$

B) $365\text{ days}/2\pi\text{ lightyears}$

C) $365\text{ days}/\text{orbit}$

D) All of the above

E) None of the above

---

Angular speed, $\omega := \theta/t$ → This is an angle equal to 360° or $2\pi$ radians.

The Earth performs 1 orbit per year,

thus $\omega = \dfrac{1\text{ orbit}}{1\text{ year}} = \dfrac{1\text{ orbit}}{365\text{ days}}$

---

A) False. $365°/1\text{ orbit} = 2\pi/2\pi = 1 \neq \dfrac{1\text{ orbit}}{1\text{ year}}$

---

B) False. 365days/2π lightyears has units of days per lightyear which is not an angle per time so therefore couldn't be a angular speed.

---

C) False. 365 days/orbit has units of days per orbit which is time per angle, not angle per time so therefore couldn't be a angular speed.

---

#8) Which of the following statements is completely true?

---

A) $\sin(x + 3\pi/4) = -\cos(x + \pi/2)$

B) $\cos(x - 6\pi/8) = \cos(3\pi/4 + x)$

C) $\boxed{\pi + \cos(-2x - \pi) = \pi - \sin(2x + \pi/2)}$

D) $\pi + \sin(2x + \pi/2) = \pi + \cos(2x + 3\pi/4)$

E) None of the above

---

For this problem you might consider comparing the graphs of the functions on each side of an equation. If the graphs don't match then the functions are not equal. Or you can approach the problem analytically and use cofunction identities and symmetry.

---

A) False.

cofunction identity: $\sin(\frac{\pi}{2} - \theta) \equiv \cos(\theta)$

symmetry identity: $\cos(-\theta) \equiv \cos(\theta)$

$$\sin(x + 3\pi/4) = \sin\left[\frac{\pi}{2} - \left(-x - \frac{\pi}{2}\right)\right] = \cos\left(-x - \frac{\pi}{2}\right) = \cos\left[-\left(x + \frac{\pi}{2}\right)\right] = \cos(x + \pi/2) \neq -\cos(x + \pi/2)$$

---

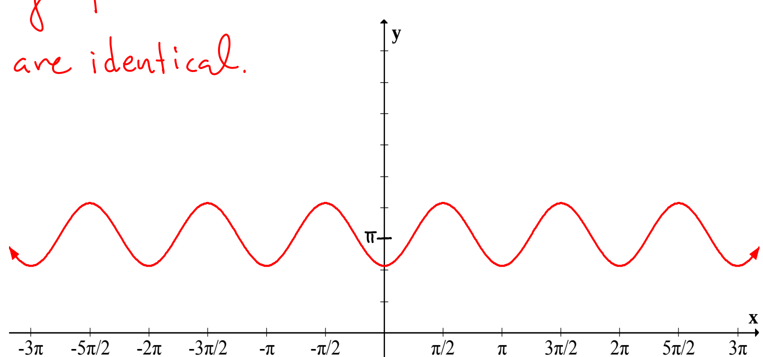
B) False.

cosine is $2\pi$ periodic

$$\cos(x - 6\pi/8) = \cos(x - 6\pi/8 + 2\pi) = \cos(x + 5\pi/4) \neq \cos(3\pi/4 + x)$$

---

C) True. Compare graphs of $\pi + \cos(-2x - \pi)$ and $\pi - \sin(2x + \pi/2)$ and notice they are identical.




---

D) False.

$$\pi + \sin(2x + \pi/2) = \pi + \sin[\pi/2 - (-2x)] = \pi + \cos(-2x) = \pi + \cos(2x)$$

$$\neq \pi + \cos(2x + 3\pi/4)$$

#9) Which of the following statements is completely true?

A) $\cos(17\pi/8) = \sin(19\pi/8)$

B) $\tan(\frac{\pi}{2} - x) \equiv \cot(x)$

C) $-\sin(\frac{5\pi}{2} - x) \equiv -\cos(x)$

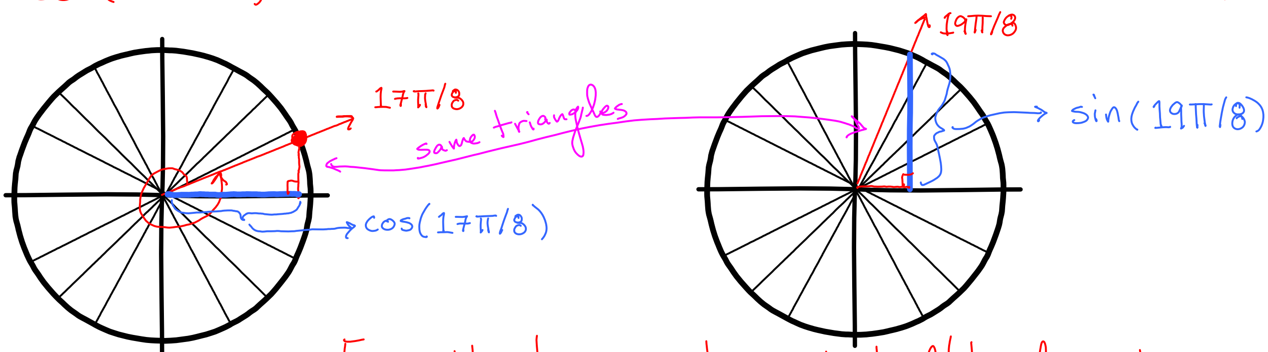
D) All of the above

E) None of the above

---

A) True. Compare the following diagrams.

$\cos(17\pi/8) = \cos(2\pi + \pi/8)$ $\sin(19\pi/8) = \sin(2\pi + 3\pi/8)$




From the diagrams above, it should be clear that

$\cos(17\pi/8) = \sin(19\pi/8)$

---

B) True. $\tan(\pi/2 - x) = \cot(x)$ is a cofunction identity

---

C) True.

$$-\sin(\tfrac{5\pi}{2} - x) = -\sin(\tfrac{5\pi}{2} - x - 2\pi)$$ because sine is $2\pi$ periodic

$$= -\sin(\tfrac{\pi}{2} - x)$$

$$= -\cos(x)$$

cofunction identity

#10) Which of the following statements is completely true?

A) $\cos(2) > \sin(1)$

B) $\cos(\theta+\pi/2) \equiv \cos(\theta-\pi/2)$

C) $\dfrac{\pi/4}{\tan(1)} = \dfrac{1}{\cot(\pi/4)}$

D) All of the above

E) $\boxed{\text{None of the above}}$

A) Consider $\pi = 3.14... = 3 + 0.14...$

So, $\pi/3 = 3.14.../3 \approx 1$ but $\pi/3 > 1$ by $0.14.../3$

Therefore,

$$\left.\begin{array}{l}\sin(1) \approx \sin(\pi/3) > 0 \\ \cos(2) \approx \cos(2\pi/3) < 0\end{array}\right\} \Rightarrow \cos(2) < \sin(1) \Rightarrow \text{Option A is false.}$$

B) Consider $\theta = \pi/2$, then

$\cos(\theta+\pi/2) = \cos(\pi/2+\pi/2) = \cos(\pi) = -1$ but

$\cos(\theta-\pi/2) = \cos(\pi/2-\pi/2) = \cos(0) = 1$

So, $\cos(\theta+\pi/2) \not\equiv \cos(\theta-\pi/2)$ $\Rightarrow$ Option B is false.

C) $\dfrac{1}{\cot(\pi/4)} = \dfrac{1}{1} = 1$ but $\tan(1) \neq \pi/4$ so $\dfrac{\pi/4}{\tan(1)} \neq 1 = \dfrac{1}{\cot(\pi/4)}$

$\Rightarrow$ Option C is false.

Creative Commons Copyright by-nc-sa 3.0 2008 MathematicalRadical.org Revision 20081028