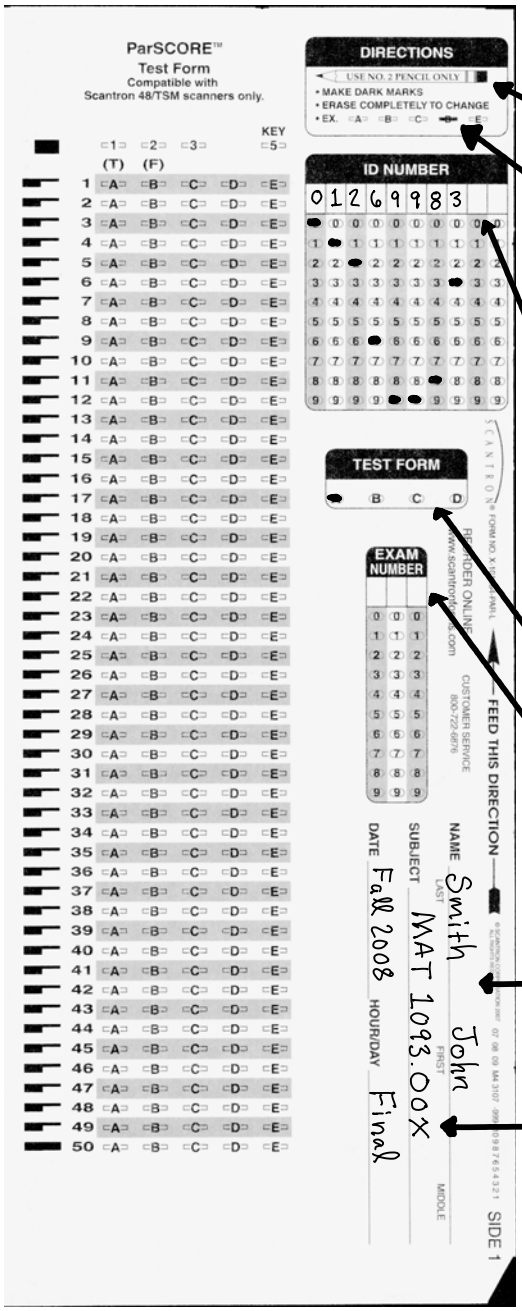
PRECALCULUS **Fall 2008** **Final Exam** **Solutions**

Name_____ Banner:_____



Do write your name in the blank.
Do write your Banner in the blank.

Do use a #2 pencil.
Do **NOT** use anything but a #2 pencil.

Do fill in the entire rectangle to mark your answer.
Do erase errors completely. If your eraser leaves smudges, consider carefully copying your answers to a new parscore without smudges.

Do ignore the @ symbol if it is in your banner I.D.
Do write the **last 8 digits** of your banner I.D. here.
Do start entering digits at the far left.
Do fill in the appropriate oval below each digit.

Do fill in the oval for test form **A**.

Do **NOT** fill in an exam number.

Do write your last name, first name and middle name here.

Do replace the x with the appropriate number for the section that you are enrolled. Look at the chart below if you don't know your section.

Class Days	Time	1093.section
TTR	7pm	1093.001
MWF	10am	1093.002

Do circle your answers on this exam. **Do** fill in the corresponding bubble on your ParScore.

Do **NOT** use a calculator. Do **NOT** use a formula sheet.

Do cover your scratch work. **Do** cover your answers on your exam. **Do** cover your Parscore.

Do **NOT** cheat. Do **NOT** even appear to be cheating.

Do notify me if something is illegible. **Do** ask me to clarify if a question is ambiguous.

Do use the back of the exam pages for scratch work. **Do** feel free to unstaple the pages of the exam.

Grades will be available in WebCT when the Parscore Office finishes grading your exams.
I don't know when this will be so do **NOT** ask me.

#1) Which of the following is completely true?

A) $\arccos[\cos(3\pi/4)] = \arcsin[\sin(3\pi/4)]$

B) $\tan(x+\pi) \equiv \sin(x+2\pi)/\cos(x-2\pi)$

C) $\sin(\theta) \equiv \sec(1/\theta)$

D) All of the above

E) None of the above

A) False.

$\arccos[\cos(3\pi/4)] = \arccos(-\sqrt{2}/2) = -\pi/4$ but

$\arcsin[\sin(3\pi/4)] = \arcsin(\sqrt{2}/2) = \pi/4$

B) True.

$$\tan(x+\pi) \equiv \tan(x) \equiv \frac{\sin(x)}{\cos(x)} \equiv \frac{\sin(x+2\pi)}{\cos(x-2\pi)}$$

C) False. Consider $\theta = \pi$:

$0 = \sin(\pi) \neq \sec(1/\pi)$ thus $\sin(\theta) \neq \sec(1/\theta)$

#2) Which of the following is completely true?

A) $-\sin(-x) \equiv \sin(x)$

B) $\cot(\theta-\pi) \equiv \csc(\theta-2\pi)/\sec(\theta+4\pi)$

C) $\sin(17\pi/4) = \cos(-15\pi/4)$

D) All of the above

E) None of the above

A) True. $-\sin(-x) \equiv --\sin(x) \equiv \sin(x)$

B) True. $\cot(\theta-\pi) \equiv \cot(\theta) \equiv \dfrac{\cos(\theta)}{\sin(\theta)} \equiv \dfrac{\csc(\theta)}{\sec(\theta)} \equiv \dfrac{\csc(\theta-2\pi)}{\sec(\theta+4\pi)}$

C) True. $\sin(17\pi/4) = \sin(17\pi/4-4\pi) = \sin(\pi/4) = \sqrt{2}/2$

$\cos(-15\pi/4) = \cos(-15\pi/4+4\pi) = \cos(\pi/4) = \sqrt{2}/2$

#3) Which of the following is completely true?

A) $\tan(\theta) = 10 \Rightarrow \csc^2(\theta) = 101/100$

B) $mb^2 + c^2b + d^3 = 0 \Rightarrow b = \left(-c^2 \pm \sqrt{c^4 - 4md^3}\right)/2m$

C) $360\pi^\circ = 2\pi^2$

D) All of the above

E) None of the above

A) True.

$\csc^2(\theta) \equiv \cot^2(\theta) + 1 \equiv 1/\tan^2(\theta) + 1 = 1/10^2 + 1 = 1/100 + 1 = 101/100$

B) True. Quadratic Formula: $ax^2+bx+c=0 \Leftrightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Rename $a \mapsto m$, $x \mapsto b$, $b \mapsto c$, $c \mapsto d$ and the Quadratic Formula becomes:

$mb^2 + c^2b + d^3 = 0 \Leftrightarrow b = \left(-c^2 \pm \sqrt{c^4 - 4md^3}\right)/2m$

C) True. $360\pi^\circ = 360\pi^\circ\left(\dfrac{2\pi}{360^\circ}\right) = 2\pi^2$

#4) Which of the following is completely true?

A) $\tan(x)/\sqrt{3} - 1 = 0 \iff (\sin(x) - \sqrt{3}/2)(\cos(x) - 1/2) = 0$

B) $2\sin(x) - \sqrt{3} = 0 \iff 2\cos(x) = 1$

C) $\sin(x)(\cos(x) - 2) = 0 \iff \cos^2(x) - 1 = 0$

D) All of the above

E) None of the above

A) False.

$\tan(x)/\sqrt{3} - 1 = 0 \iff \tan(x) = \sqrt{3} \iff x \in \{\pi/3 + \pi K ; K \in \mathbb{Z}\}$ but

$(\sin(x) - \sqrt{3}/2)(\cos(x) - 1/2) = 0 \iff \begin{cases} \sin(x) = \sqrt{3}/2 \iff x \in \{\pi/3 + 2\pi K, 2\pi/3 + 2\pi K ; K \in \mathbb{Z}\} \\ \cos(x) = 1/2 \iff x \in \{\pm\pi/3 + 2\pi K ; K \in \mathbb{Z}\} \end{cases} \iff x \in \{\pm\pi/3 + 2\pi K, 2\pi/3 + 2\pi K ; K \in \mathbb{Z}\}$

B) False.

$2\sin(x) - \sqrt{3} = 0 \iff \sin(x) = \sqrt{3}/2 \iff x \in \{\pi/3 + 2\pi K, 2\pi/3 + 2\pi K ; K \in \mathbb{Z}\}$ but

$2\cos(x) = 1 \iff \cos(x) = 1/2 \iff x \in \{\pm\pi/3 + 2\pi K ; K \in \mathbb{Z}\}$

C) True.

$\sin(x)(\cos(x) - 2) = 0 \iff \begin{cases} \sin(x) = 0 \iff x = \pi K ; K \in \mathbb{Z} \\ \cos(x) = 2 \text{ which has no real solutions} \end{cases}$ and

$\cos^2(x) - 1 = 0 \iff \sin^2(x) = 0 \iff \sin(x) = 0 \iff x = \pi K ; K \in \mathbb{Z}$

#5) Which of the following is completely true?

A) $\sin(x^2) = 1 \iff \sin^2(x) = 1$

B) $x\cos(x) = 0 \iff \sin^2(x) - 1 = 0$

C) $\sin(x + \pi/2) = \cos(x - \pi/2) \iff x = \pi/4 + 2\pi K \; ; \; K \in \mathbb{Z}$

D) All of the above

E) None of the above

A) False.

$\sin(x^2) = 1 \iff x^2 = \pi/2 + 2\pi K \iff x = \pm\sqrt{\pi/2 + 2\pi K} \; ; \; K \in \mathbb{Z}$ but

$\sin^2(x) = 1 \iff \sin(x) = \pm 1 \iff x = \pi/2 + \pi K \; ; \; K \in \mathbb{Z}$

B) False.

$x\cos(x) = 0 \iff \begin{cases} x = 0 \\ \cos(x) = 0 \iff x = \pi/2 + \pi K \; ; \; K \in \mathbb{Z} \end{cases}$ but

$\sin^2(x) - 1 = 0 \iff -\cos^2(x) = 0 \iff \cos(x) = 0 \iff x = \pi/2 + \pi K \; ; \; K \in \mathbb{Z}$

C) False.

$\sin(x + \pi/2) = \cos(x - \pi/2) \iff \sin[\pi/2 - (-x)] = \cos[-(\pi/2 - x)]$

$\iff \cos(-x) = \cos(\pi/2 - x)$ since $\sin(\pi/2 - \theta) \equiv \cos(\theta)$ and $\cos(-\theta) \equiv \cos(\theta)$

$\iff \cos(x) = \sin(x)$ since $\cos(\pi/2 - x) \equiv \sin(x)$

$\iff x = \pi/4 + \pi K \neq \pi/4 + 2\pi K \; ; \; K \in \mathbb{Z}$

#6) Which of the following is completely true?

A) $\sin(x) = \sqrt{3}/2 \iff \cos(x) = 1/2$

B) $\tan(x) = 0 \iff \cot(x) = 0$

C) $\sec(x) = 1 \iff \sin(x) = 1$

D) All of the above

E) None of the above

A) False. $\sin(x) = \sqrt{3}/2 \iff x \in \{\pi/3 + 2\pi k, 2\pi/3 + 2\pi k ; k \in \mathbb{Z}\}$ but
$\cos(x) = 1/2 \iff x \in \{\pm\pi/3 + 2\pi k ; k \in \mathbb{Z}\}$

B) False. $\tan(x) = 0 \iff x = \pi k ; k \in \mathbb{Z}$ but
$\cot(x) = 0 \iff x = \pi/2 + \pi k ; k \in \mathbb{Z}$

C) False. $\sec(x) = 1 \iff x = 2\pi k ; k \in \mathbb{Z}$ but
$\sin(x) = 1 \iff x = \pi/2 + 2\pi k ; k \in \mathbb{Z}$

#7) Which of the following is completely true?

A) $\sec(x) = 2 \iff \csc(x) = 2\sqrt{3}/3$

B) $\cos(x) = 0 \iff \csc(x) = 1$

C) $\tan(x) = 1 \iff \cot(x) = 1$

D) All of the above

E) None of the above

A) False. $\sec(x) = 2 \iff \cos(x) = 1/2 \iff x \in \{\pm\pi/3 + 2\pi K \,;\, K \in \mathbb{Z}\}$ but

$\csc(x) = 2\sqrt{3}/3 \iff \sin(x) = 3/2\sqrt{3} = \sqrt{3}/2 \iff x \in \{\pi/3 + 2\pi K,\ 2\pi/3 + 2\pi K \,;\, K \in \mathbb{Z}\}$

B) False. $\cos(x) = 0 \iff x = \pi/2 + \pi K \,;\, K \in \mathbb{Z}$ but

$\csc(x) = 1 \iff x = \pi/2 + 2\pi K \,;\, K \in \mathbb{Z}$

C) True. $\tan(x) = 1 \iff x = \pi/4 + \pi K \,;\, K \in \mathbb{Z}$ but

$\cot(x) = 1 \iff x = \pi/4 + \pi K \,;\, K \in \mathbb{Z}$

#8) Which of the following is completely true?

A) $\sin(\Theta) = 1 \Leftrightarrow \csc(\Theta) = 1 \Leftrightarrow \Theta = \pi/2 + 2\pi K$ where $K \in \mathbb{Z}$

B) $-\cos(\Theta) = 1/2 \Leftrightarrow \cos(\Theta) = 1/2 \Leftrightarrow \Theta \in \{\pi/3 + \pi K, 2\pi/3 + \pi K ; K \in \mathbb{Z}\}$

C) $\tan(\Theta) = 3/2 \Leftrightarrow \sin(\Theta) = 3 \Leftrightarrow \cos(\Theta) = 2$

D) All of the above

E) None of the above

A) True. $\sin(\Theta) = 1 \Leftrightarrow x = \pi/2 + 2\pi K \; ; K \in \mathbb{Z}$

$\csc(\Theta) = 1 \Leftrightarrow x = \pi/2 + 2\pi K \; ; K \in \mathbb{Z}$

B) False. $-\cos(\Theta) = 1/2 \Leftrightarrow x \in \{\pm 2\pi/3 + 2\pi K ; K \in \mathbb{Z}\} \neq \{\pi/3 + \pi K, 2\pi/3 + \pi K ; K \in \mathbb{Z}\}$

C) False. $1 + \tan^2(\Theta) \equiv \sec^2\Theta \equiv 1/\cos^2(\Theta)$

$$\Leftrightarrow \cos(\Theta) \equiv \pm\sqrt{\frac{1}{1+\tan^2(\Theta)}} = \pm\sqrt{\frac{1}{1+(3/2)^2}} = \pm\sqrt{\frac{1}{4/4+9/4}}$$

$$= \pm\sqrt{\frac{1}{13/4}} = \pm\frac{2}{\sqrt{13}} \neq 2$$

#9) Which of the following is completely true?

A) $2\sin(2x^2) = -\sqrt{3} \iff x \in \{\pm\sqrt{-\pi/6+\pi K}, \pm\sqrt{-\pi/3+\pi K}\,;\, K \in \mathbb{Z}\}$

B) $2\cos(x^2+\pi) = 1 \iff x \in \{\sqrt{\pm\pi/3+(2K-1)\pi}, -\sqrt{\pm\pi/3+(2K-1)\pi}\,;\, K \in \mathbb{Z}\}$

C) $\tan(2x^2+2x+2) = 0 \Rightarrow x = -1/2 \pm \sqrt{1-2(2-\pi K)}/2$ where $K = 1, 2, 3, \ldots$

D) All of the above

E) None of the above

A) True.

$$2\sin(2x^2) = -\sqrt{3} \iff \sin(2x^2) = -\sqrt{3}/2 \iff 2x^2 = \begin{cases} -\pi/3 + 2\pi K \\ -2\pi/3 + 2\pi K \end{cases};\ K \in \mathbb{Z}$$

$$\iff \boxed{x = \begin{cases} \pm\sqrt{-\pi/6+\pi K} \\ \pm\sqrt{-\pi/3+\pi K} \end{cases};\ K \in \mathbb{Z}}$$

B) True.

$$2\cos(x^2+\pi) = 1 \iff \cos(x^2+\pi) = 1/2 \iff x^2+\pi = \pm\pi/3 + 2\pi K\ ;\ K \in \mathbb{Z}$$

$$\iff x = \begin{cases} \sqrt{\pm\pi/3-\pi+2\pi K} \\ -\sqrt{\pm\pi/3-\pi+2\pi K} \end{cases};\ K \in \mathbb{Z} \iff \boxed{x = \begin{cases} \sqrt{\pm\pi/3+(2K-1)\pi} \\ -\sqrt{\pm\pi/3+(2K-1)\pi} \end{cases};\ K \in \mathbb{Z}}$$

C) True.

$$\tan(2x^2+2x+2) = 0 \Rightarrow 2x^2+2x+2 = \pi K\ ;\ K \in \mathbb{Z}$$

$$\Rightarrow 2x^2+2x+2-\pi K = 0\ ;\ K \in \mathbb{Z}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{2^2-4(2)(2-\pi K)}}{2(2)}\ ;\ K \in \mathbb{Z} \qquad \text{Quadratic Formula}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{2^2-2^4+2^3\pi K}}{2^2} = \frac{-2 \pm 2\sqrt{1-2^2+2\pi K}}{2^2} = -\frac{1}{2} \pm \frac{\sqrt{1-2(2-\pi K)}}{2}\ ;\ K \in \mathbb{Z}$$

but $1-2(2-\pi K) \geq 0$ when $K = 1, 2, 3, \ldots$

$$\Rightarrow \boxed{x = -\frac{1}{2} \pm \frac{\sqrt{1-2(2-\pi K)}}{2}\ ;\ K = 1, 2, 3, \ldots}$$

#10) Which of the following is completely true?

A) $\sec^2(2x) + \csc^2(2x) \equiv \sec^2(2x)\csc^2(2x)$

B) $\cos^2(2x) + 2\cos^3(x)\sin^2(x) \equiv 1$

C) $2 - 2\cos^2(x) \equiv 1 + \cos(2x)$

D) All of the above

E) None of the above

A) True. $\sec^2(2x) + \csc^2(2x) \equiv \frac{1}{\cos^2(2x)} + \frac{1}{\sin^2(2x)}$

$\equiv \frac{1}{\cos^2(2x)} \cdot \frac{\sin^2(2x)}{\sin^2(2x)} + \frac{1}{\sin^2(2x)} \frac{\cos^2(2x)}{\cos^2(2x)}$

$\equiv \frac{\sin^2(2x) + \cos^2(2x)}{\sin^2(2x)\cos^2(2x)} \equiv \frac{1}{\sin^2(2x)\cos^2(2x)} \equiv \frac{1}{\sin^2(2x)} \frac{1}{\cos^2(2x)}$

$\equiv \sec^2(2x)\csc^2(2x)$

B) False. $\cos^2(2x) + 2\cos^3(x)\sin^2(x) \equiv \cos^2(2x) + [2\cos(x)\sin(x)]^2/2$

$\equiv \cos^2(2x) + \sin^2(2x)/2 \not\equiv 1$

since $1 \equiv \cos^2(2x) + \sin^2(2x) \not\equiv \cos^2(2x) + \sin^2(2x)/2$

C) False. $2 - 2\cos^2(x) \not\equiv 1 + \cos(2x)$

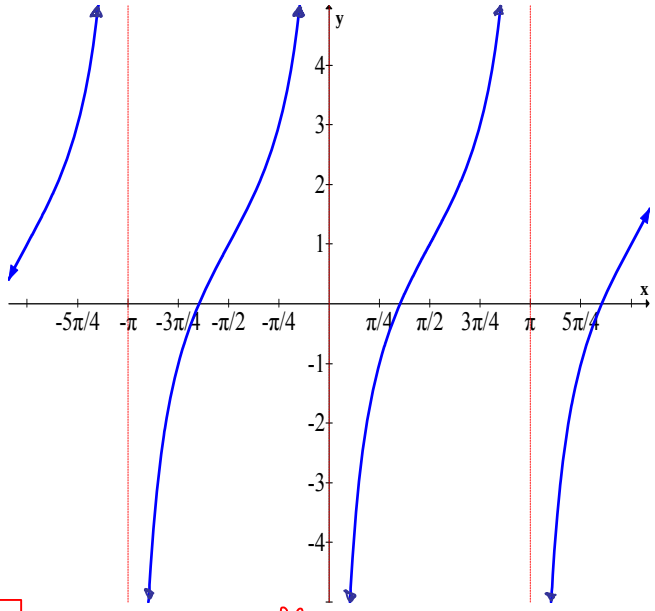
since $2 - 2\cos^2(0) \neq 1 + \cos(2 \cdot 0)$

since $2 - 2(1) \neq 1 + (1)$

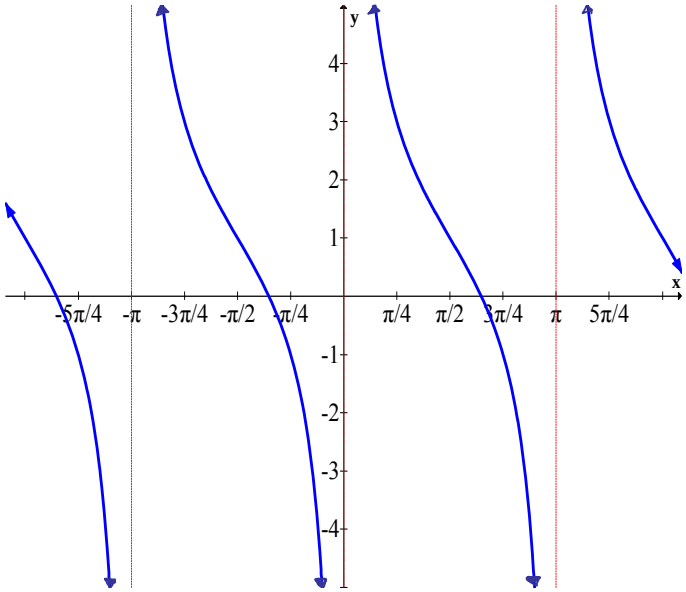
since $0 \neq 2$

#11) Which of the following is the graph of the function f(x) = -2tan(x) + 1?

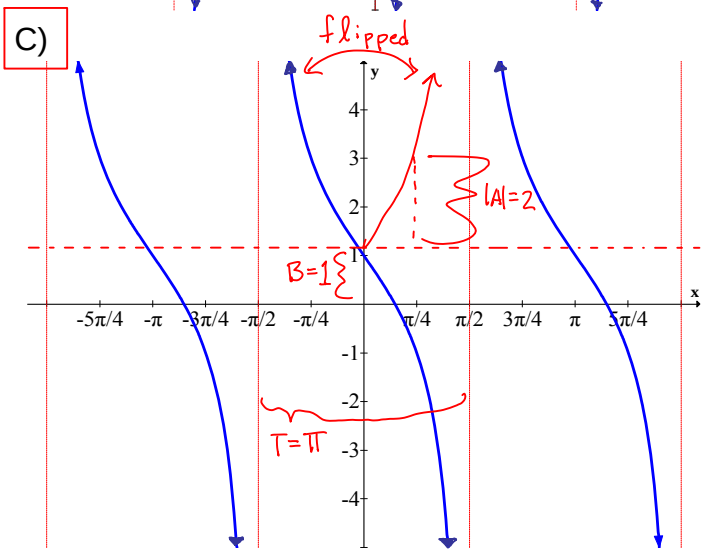
A)



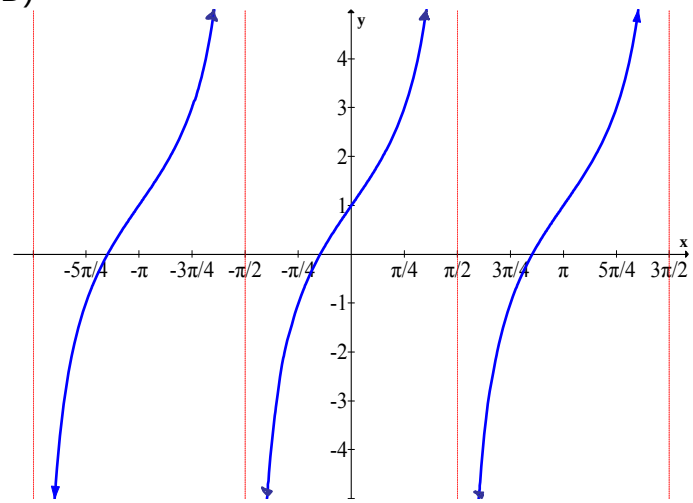
B)



C)



D)



E) None of the above

#12) Which of the graphs in problem 11 are antisymmetric (have odd symmetry)?

A) The graphs from choices A and B.

B) The graphs from choices C and D.

C) The graphs from choices A and D.

D) All of the graphs are antisymmetric.

E) None of the graphs are antisymmetric.

Antisymmetric $\Leftrightarrow$ $f(-x) \equiv -f(x)$

$\Leftrightarrow$ Symmetric about the origin.

$\Leftrightarrow$ Rotating 180° about the origin leaves the graph unchanged.

$\Leftrightarrow$ The function has odd symmetry.

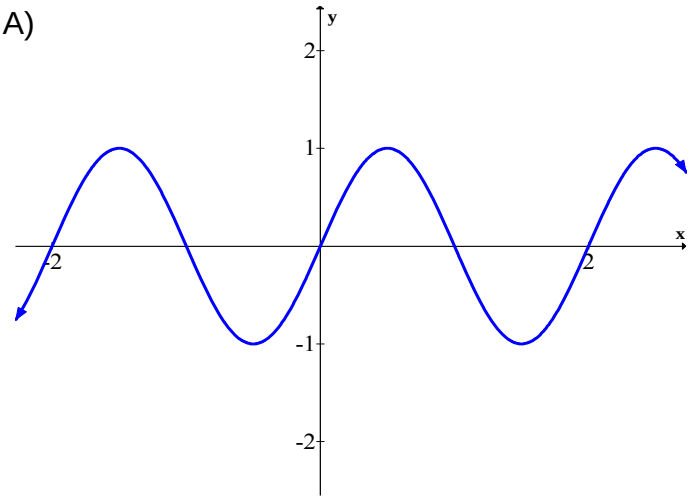
$\Leftrightarrow$ Flipping over the y-axis gives the same result as flipping over the x-axis.

#13) Which of the following is the graph of the function $f(x) = \cos(-\pi x - \pi)$?

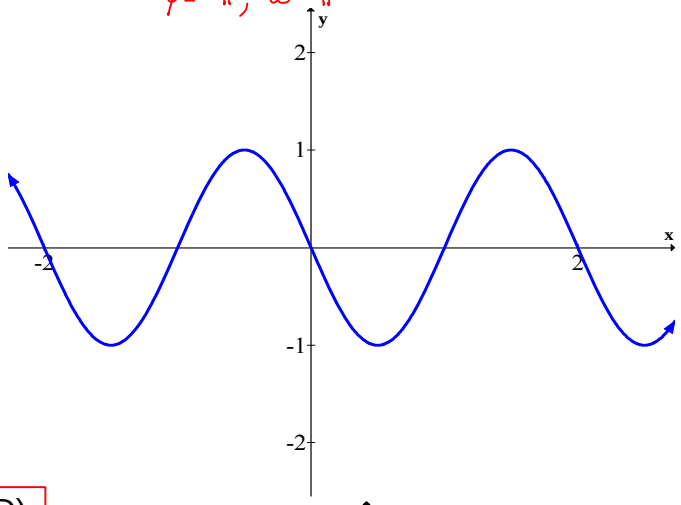
$= \cos(\pi x + \pi)$ since $\cos(-\theta) \equiv \cos(\theta)$

$\Rightarrow \phi = -\pi,\ \omega = \pi$

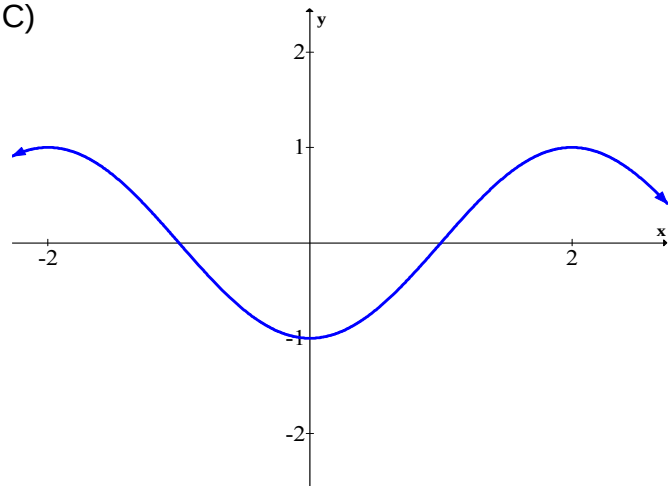
A)



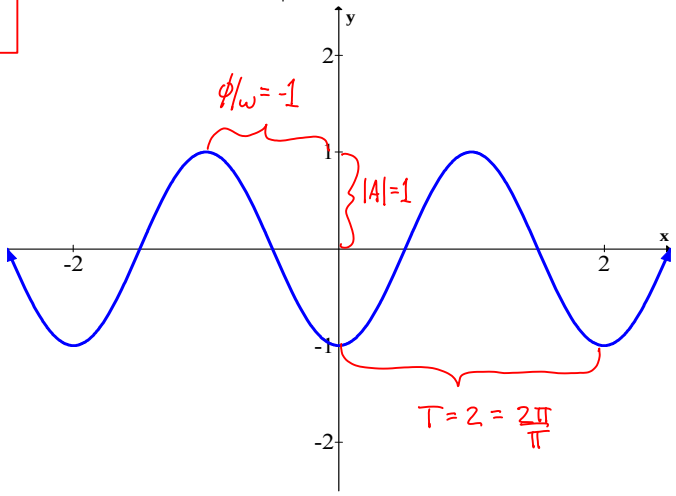
B)



C)



D)



E) None of the above

#14) Which of the graphs in problem 13 are symmetric (have even symmetry)?

A) The graphs from choices A and B.

B) The graphs from choices C and D.

C) The graphs from choices A and D.

D) All of the graphs are symmetric.

E) None of the graphs are symmetric.

Symmetric $\Leftrightarrow$ $f(-x) = f(x)$

$\Leftrightarrow$ The function has even symmetry.

$\Leftrightarrow$ Symmetric about the y-axis.

$\Leftrightarrow$ Flipping over the y-axis leaves the graph unchanged.

#15) Which of the graphs in problem 13 are symmetric about the origin?

A) The graphs from choices A and B.

B) The graphs from choices C and D.

C) The graphs from choices A and D.

D) All of the graphs are symmetric about the origin.

E) None of the graphs are symmetric about the origin.

Antisymmetric $\Leftrightarrow$ $f(-x) \equiv -f(x)$

$\Leftrightarrow$ Symmetric about the origin.

$\Leftrightarrow$ Rotating 180° about the origin leaves the graph unchanged.

$\Leftrightarrow$ The function has odd symmetry.

$\Leftrightarrow$ Flipping over the y-axis gives the same result as flipping over the x-axis.