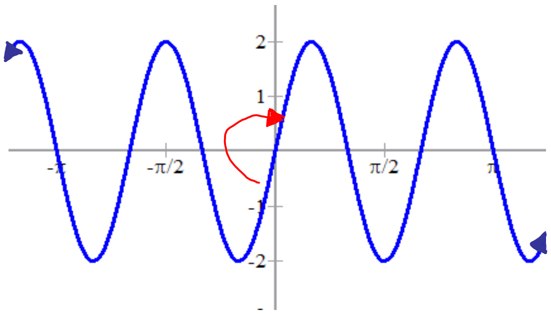
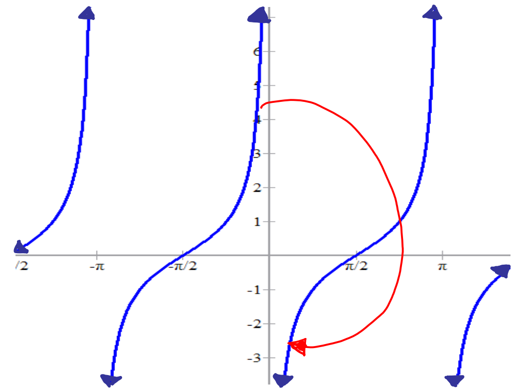
#1) Which of the following graphs does not have symmetry about the origin?

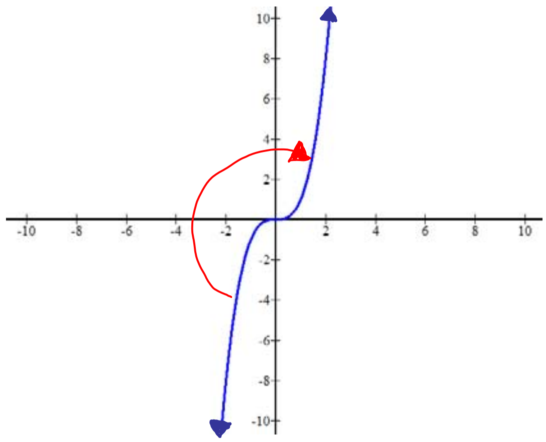
A)




B)




C)



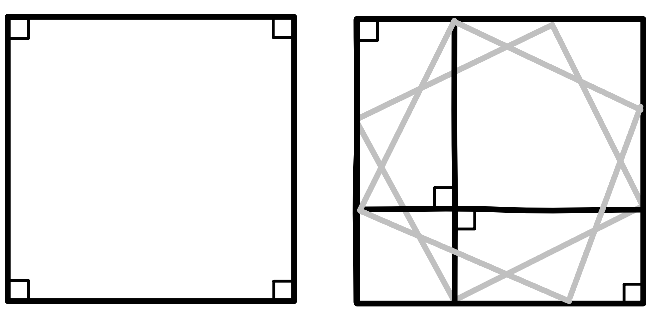

D) All of the above

E) None of the above

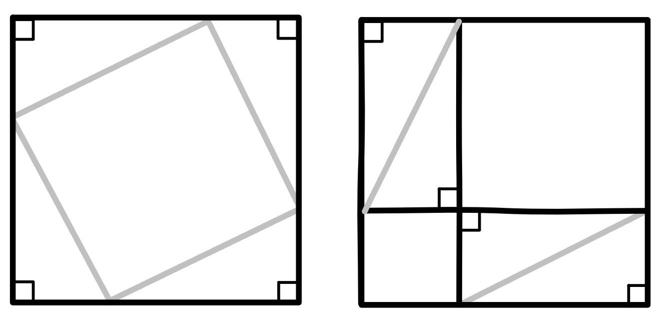
They all have symmetry about the origin.

#2) Which of the following provides a visual proof that $(a+b)^2 \neq a^2+b^2$ ?

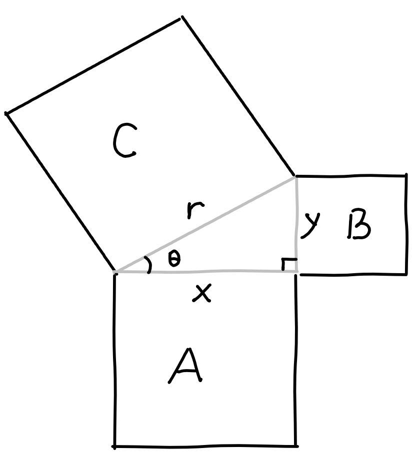
A)



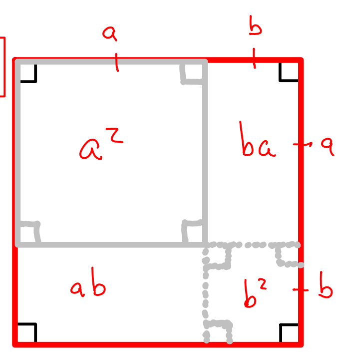
B)



C)




D)




$A = (a+b)^2$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$
$\neq a^2 + b^2$

E) None of the above

#3) Which of the following pairs of functions have the same graph?

A) $\sin(x-\pi)$ and $\cos(\pi-x)$

B) $\tan^2(x)$ and $\cot^2(x)+1$

C) $-\cos(x)$ and $\sin(x-\pi/2)$

D) $\sec(x)$ and $1/\sin(x)$

E) None of the above

---

A) $\sin(x-\pi)$ and $\cos(\pi-x)$ do not have the same graph. Verify this by showing they disagree at a single point. Consider $x=0$:

$\sin(0-\pi) = \sin(-\pi) = 0$ but

$\cos(\pi-0) = \cos(\pi) = -1 \neq 0$

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B) Using the Pythagorean Identity $\cot^2(x)+1 \equiv \csc^2 x$

We can see that $\tan^2(x) \neq \csc^2 x \equiv \cot^2(x)+1$

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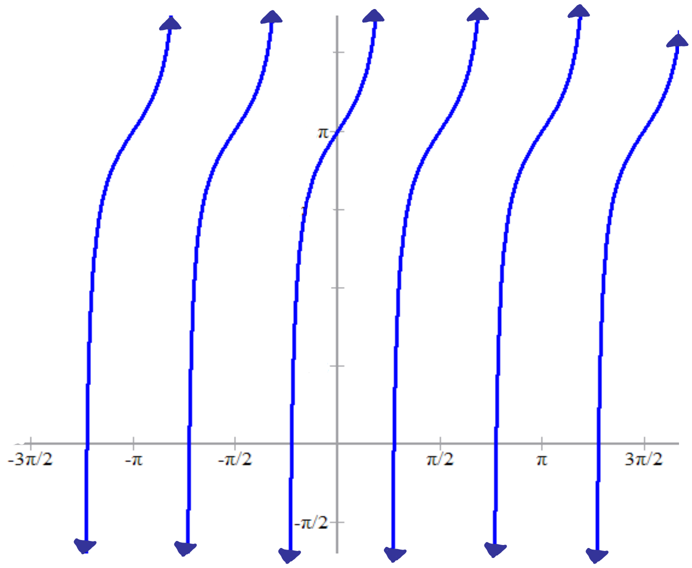
$\boxed{\text{C)}}$ $\sin(x-\pi/2) \equiv \sin[-(\pi/2-x)] \equiv -\sin(\pi/2-x) \equiv -\cos(x)$

---

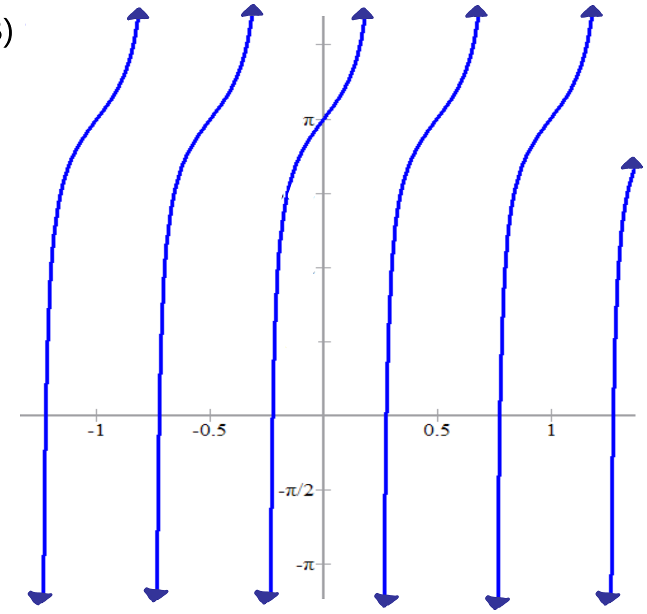
D) $1/\sin(x) \equiv \csc(x) \not\equiv \sec(x)$

#4) Which of the following is the graph of $y = \pi + \tan(-2x - \pi)$

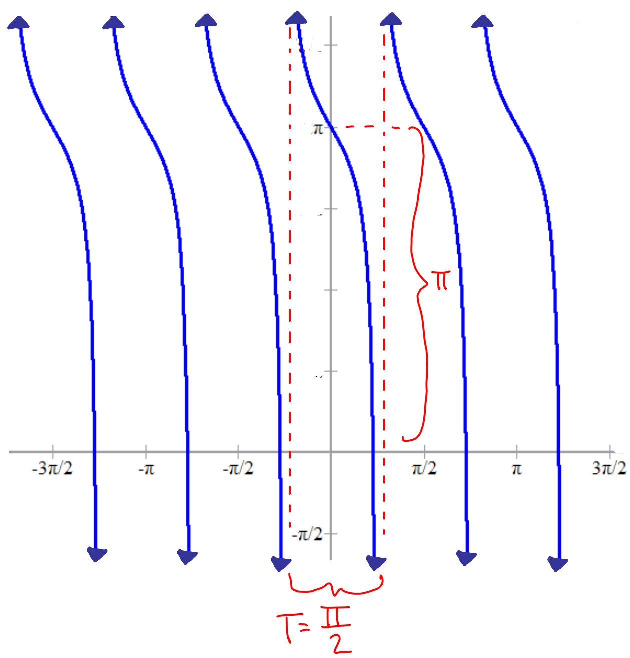
A)



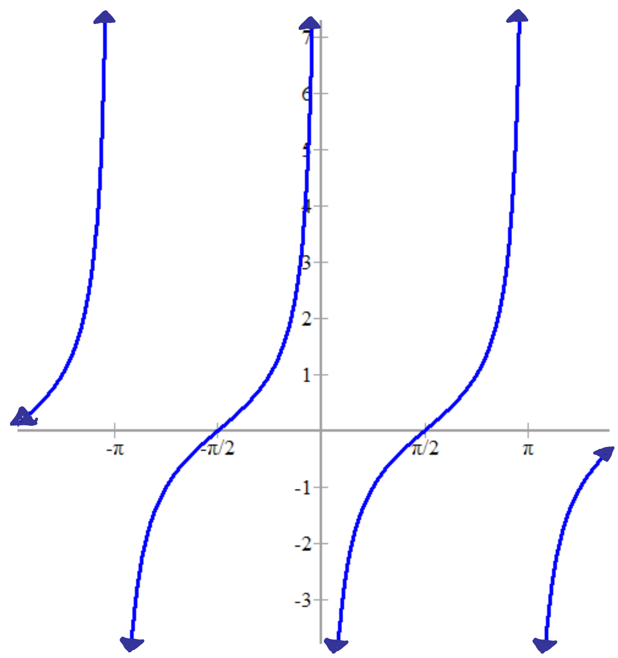

B)




$\boxed{\text{C)}}$




D)




E) None of the above

---

$y = \pi + \tan(-2x - \pi) = \tan[-(2x+\pi)] = -\tan(2x+\pi) + \pi$

flip over x axis

Vertical shift $= \pi$

$\omega = 2$

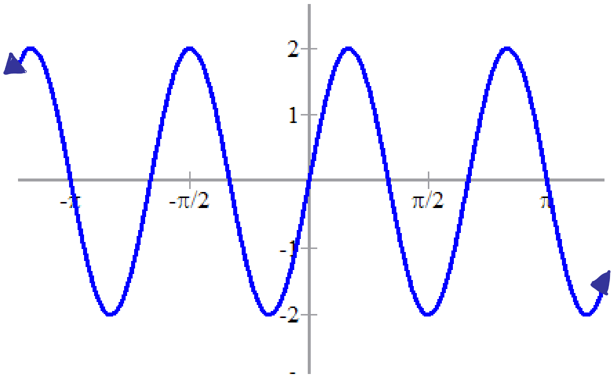
$T = \pi/\omega = \pi/2$

$\phi = -\pi$

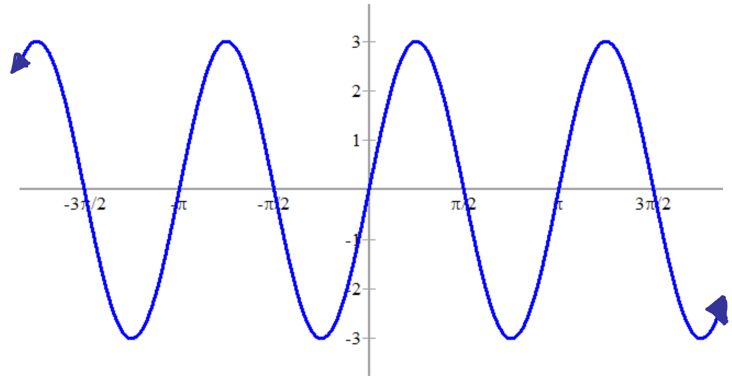
Phase shift $= \phi/\omega = -\pi/2$ which is a multiple of the period so it will not make a difference in the graph.

#5) Which of the following is the graph of $y = 1 - 3\cos(2x) - 1$

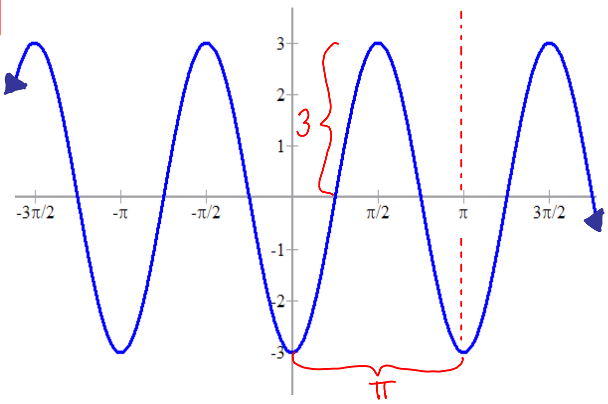
A)



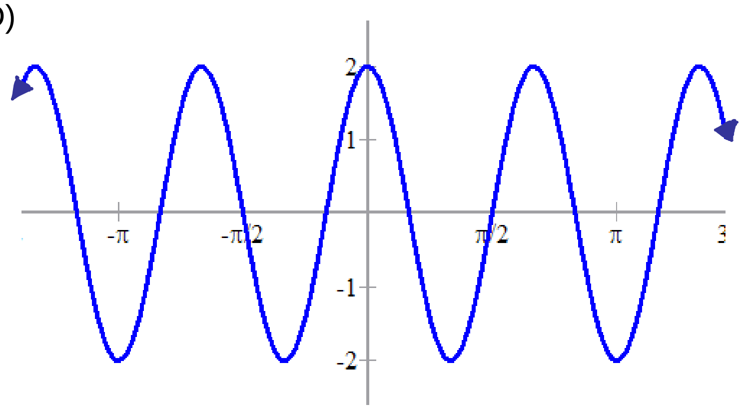
B)



$\boxed{\text{C)}}$



D)



E) None of the above

$y = 1 - 3\cos(2x) - 1 = -3\cos(2x)$

flip over x axis

Amplitude $= 3$

$\omega = 2$

$T = 2\pi/\omega = 2\pi/2 = \pi$

#6) Which of the following statements is completely true?

A) $\sin(-76\pi/3) = -\sin(73\pi/6)$

B) $-\cos(17\pi/8) = \cos(-15\pi/8)$

C) $\boxed{-\tan(32\pi/7) = \tan(-39\pi/7)}$

D) All of the above

E) None of the above

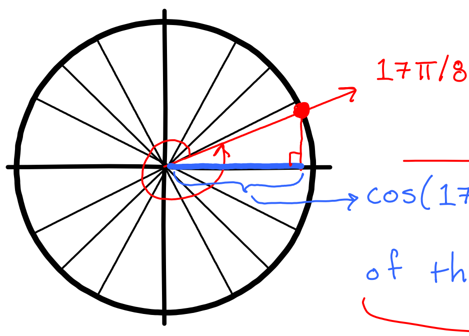
---

A) $\sin(-76\pi/3) = -\sin(76\pi/3) = -\sin(24\pi + 4\pi/3) = -\sin(4\pi/3) = \sqrt{3}/2$

$-\sin(73\pi/6) = -\sin(12\pi + \pi/6) = -\sin(\pi/6) = -1/2 \neq \sqrt{3}/2$

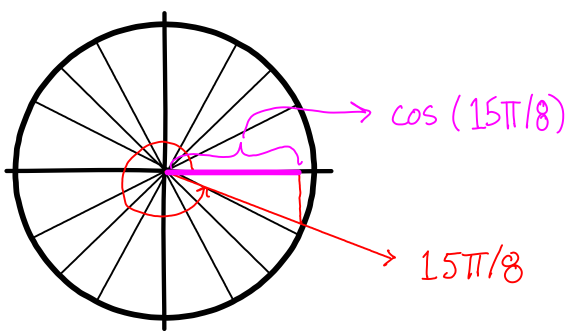
---

B) $-\cos(17\pi/8)$




cos(17π/8) Take the negative of this length to get −cos(17π/8)

$\cos(-15\pi/8) = \cos(15\pi/8)$




$\cos(15\pi/8) = \cos(17\pi/8) \neq -\cos(17\pi/8)$

---

C) $-\tan(32\pi/7) = \tan(-32\pi/7)$

$= \tan(-32\pi/7 - \pi)$ since tangent is $\pi$ periodic

$= \tan(-39\pi/7)$

#7) Assuming the Earth's orbit is circular, what is it's approximate angular speed assuming the Earth is 8 lightminutes away from the sun? (Note that 1 lightminute is a length equal to the distance light travels in 1 minute and also note that there are 365 days in 1 year).

---

A) $365^\circ / 1\text{ orbit}$

B) $365\text{ days} / 2\pi\text{ lightyears}$

C) $365\text{ days} / \text{orbit}$

D) All of the above

E) None of the above

---

Angular speed, $\omega := \theta / t$

The Earth performs $\underbrace{1\text{ orbit}}$ per year, → This is an angle equal to 360° or $2\pi$ radians.

thus $\omega = \dfrac{1\text{ orbit}}{1\text{ year}} = \dfrac{1\text{ orbit}}{365\text{ days}}$

---

A) $365^\circ / 1\text{ orbit} = 2\pi / 2\pi = 1 \neq \dfrac{1\text{ orbit}}{1\text{ year}}$

---

B) $365\text{ days} / 2\pi\text{ lightyears}$ has units of days per lightyear which is not an angle per time so therefore couldn't be a angular speed.

---

C) $365\text{ days}/\text{orbit}$ has units of days per orbit which is time per angle, not angle per time so therefore couldn't be a angular speed.

#8) Which of the following statements is completely true if tan(x) = 3/2 and sin(x) < 0?

A) $\cos(x) = -3/\sqrt{13}$ and $\sin(x) = -2/\sqrt{13}$

B) $\csc(x) = -\sqrt{13}/2$ and $\sec(x) = \sqrt{13}/3$

C) $\cos(x) = \pm 2/\sqrt{13}$ and $\sin(x) = \pm 3/\sqrt{13}$

D) $\cos(x) = 2/\sqrt{13}$ and $\sin(x) = -3/\sqrt{13}$

$\boxed{\text{E) None of the above}}$

If $\sin(x) < 0$ and $\tan(x) = 3/2 > 0$, then $\cos(x) < 0$ since

$$\frac{3}{2} = \tan(x) \equiv \frac{\sin(x)}{\cos(x)} \qquad \frac{\text{negative}}{\text{negative}} = \text{positive}$$

With this alone we can discard B, C, and D since they don't have $\cos(x) < 0$.

A quick check to see if option A is not valid can be performed using the Pythagorean Identity: $\cos^2 x + \sin^2 x \equiv 1$

Using $\cos(x) = -3/\sqrt{13}$ and $\sin(x) = -2/\sqrt{13}$, we have

$$\cos^2(x) + \sin^2(x) = (-3/\sqrt{13})^2 + (-2/\sqrt{13})^2 = 9/13 + 4/13 = 1$$

So we still can't dismiss option A as a possible answer.

Let's go ahead and just solve for sinx and cosx using the given information

$$0 < \tan(x) = 3/2 \quad \text{and} \quad \sin(x) < 0 \quad \Rightarrow \quad \cos(x) < 0$$

<u>Solution 1</u>: using the trig definitions.

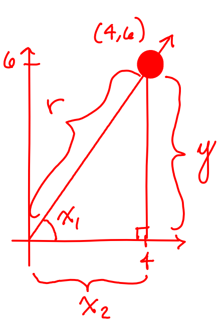
not the same x's, so I gave them subscripts to differentiate them.

$\tan(x_1) := y/x_2 = 3/2$ Choose $y = 6$ and $x = 4$ then

$$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2+y^2} = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

Thus, $\sin(x_1) := y/r = 6/2\sqrt{13} = 3/\sqrt{13}$, but $\sin(x) < 0$ so we know

$$\boxed{\sin(x) = -3/\sqrt{13}}$$




We worked the problem in quadrant 1 but our angle was actually in quadrant 3 so we included the minus sign at the end.

Knowing $\sin(x) = -3/\sqrt{13}$ , we can easily find $\cos(x)$ since

$$\cos^2(x) + \sin^2(x) \equiv 1 \Leftrightarrow \cos(x) = \pm\sqrt{1-\sin^2(x)} = \pm\sqrt{1-(-3/\sqrt{13})^2}$$

$$= \pm\sqrt{1-9/13} = \pm\sqrt{4/13} = \pm 2/\sqrt{13}$$

But we know $\cos(x) < 0$ so $\boxed{\cos(x) = -2\sqrt{13}}$

Option A is <u>not</u> a correct answer.

<u>Solution 2</u>: using the trig identities.

Since we know $\sin(x) < 0$ and $\tan(x) = 3/2 > 0 \Rightarrow \cos(x) < 0$

We just need an identity relating $\tan(x)$ and $\sin(x)$ or $\tan(x)$ and $\cos(x)$.

Our Pythagorean identities will do.

$$1 + \tan^2(x) \equiv \sec^2(x) \equiv 1/\cos^2(x)$$

$$\Leftrightarrow \cos(x) = \frac{\pm 1}{\sqrt{1+\tan^2(x)}} = \frac{\pm 1}{\sqrt{1+(3/2)^2}} = \frac{\pm 1}{\sqrt{1+9/4}} = \pm\frac{1}{\sqrt{13/4}} = \pm\frac{1}{\sqrt{13}/2}$$

$$= \pm 2/\sqrt{13}$$ But we know $\cos(x) < 0$

So, $\boxed{\cos(x) = -2/\sqrt{13}}$

Also, $\cot^2(x) + 1 \equiv \csc^2(x) \Leftrightarrow 1/\tan^2(x) + 1 \equiv 1/\sin^2(x)$

$$\Leftrightarrow \sin(x) = \frac{\pm 1}{\sqrt{1+1/\tan^2(x)}} = \frac{\pm 1}{\sqrt{1+1/(3/2)^2}} = \frac{\pm 1}{\sqrt{1+1/\frac{9}{4}}} = \frac{\pm 1}{\sqrt{1+4/9}} = \frac{\pm 1}{\sqrt{13/9}}$$

$$= \frac{\pm 1}{\sqrt{13/9}} = \frac{\pm 1}{\sqrt{13}/3} = \pm 3/\sqrt{13}$$ But we know $\sin(x) < 0$

So, $\boxed{\sin(x) = -3/\sqrt{13}}$

Option A is <u>not</u> a correct answer.

Of course we could have disqualified option A by simply applying the Quotient identity $\tan(x) \equiv \dfrac{\sin(x)}{\cos(x)}$

Option A says $\cos(x) = -3/\sqrt{13}$ and $\sin(x) = -2/\sqrt{13}$, but if that were true then $\tan(x) = \dfrac{-2/\sqrt{13}}{-3/\sqrt{13}} = \dfrac{2}{3}$ but the problem stated that $\tan(x) = 3/2$.

#9) Which of the following statements is completely true?

A) $\cos(17\pi/8) = \sin(19\pi/8)$

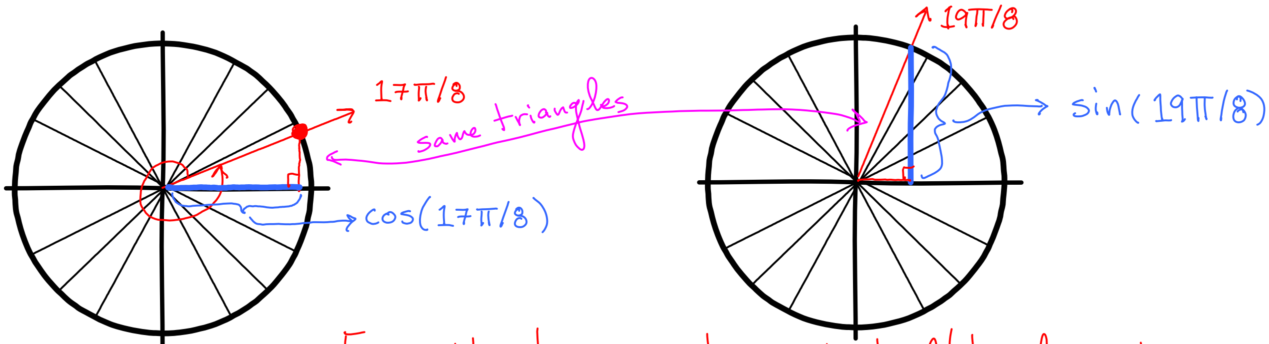
B) $\tan(x - \pi/2) \equiv \cot(-x)$

C) $-\sec(\frac{3\pi}{2} + x) \equiv -\csc(x)$

D) All of the above

E) None of the above

A) $\cos(17\pi/8) = \cos(2\pi + \pi/8)$ $\quad$ $\sin(19\pi/8) = \sin(2\pi + 3\pi/8)$




From the diagrams above, it should be clear that

$$\cos(17\pi/8) = \sin(19\pi/8)$$

B) $\tan(x - \pi/2) = \tan[-(\pi/2 - x)]$

$= -\tan(\pi/2 - x)$ because tangent is odd

$= -\cot(x)$ cofunction identity

$= \cot(-x)$ because cotangent is odd

C) $-\sec(\frac{3\pi}{2} + x) = -\sec(\frac{3\pi}{2} + x - 2\pi)$ because secant is $2\pi$ periodic

$= -\sec(x - \pi/2) = -\sec[-(\pi/2 - x)]$

$= -\sec(\pi/2 - x)$ because secant is even

$= -\csc(x)$ cofunction identity

Options A, B, and C are all true.

#10) Which of the following statements is completely true?

A) $\cos(2) > \sin(1)$

B) $\cos(\theta+\pi/2) \equiv \cos(\theta-\pi/2)$

C) $\dfrac{\pi/4}{\tan(1)} = \dfrac{1}{\cot(\pi/4)}$

D) All of the above

E) $\boxed{\text{None of the above}}$

A) Consider $\pi = 3.14... = 3 + 0.14...$

So, $\pi/3 = 3.14.../3 \approx 1$ but $\pi/3 > 1$ by $0.14.../3$

Therefore,

$$\left.\begin{array}{l}\sin(1) \approx \sin(\pi/3) > 0 \\ \cos(2) \approx \cos(2\pi/3) < 0\end{array}\right\} \Rightarrow \cos(2) < \sin(1) \Rightarrow \text{Option A is false.}$$

B) Consider $\theta = \pi/2$, then

$\cos(\theta+\pi/2) = \cos(\pi/2+\pi/2) = \cos(\pi) = -1$ but

$\cos(\theta-\pi/2) = \cos(\pi/2-\pi/2) = \cos(0) = 1$

So, $\cos(\theta+\pi/2) \not\equiv \cos(\theta-\pi/2) \Rightarrow$ Option B is false.

C) $\dfrac{1}{\cot(\pi/4)} = \dfrac{1}{1} = 1$ but $\tan(1) \neq \pi/4$ so $\dfrac{\pi/4}{\tan(1)} \neq 1 = \dfrac{1}{\cot(\pi/4)}$

$\Rightarrow$ Option C is false.