PRECALCULUS **Spring 2008** **Final Exam**

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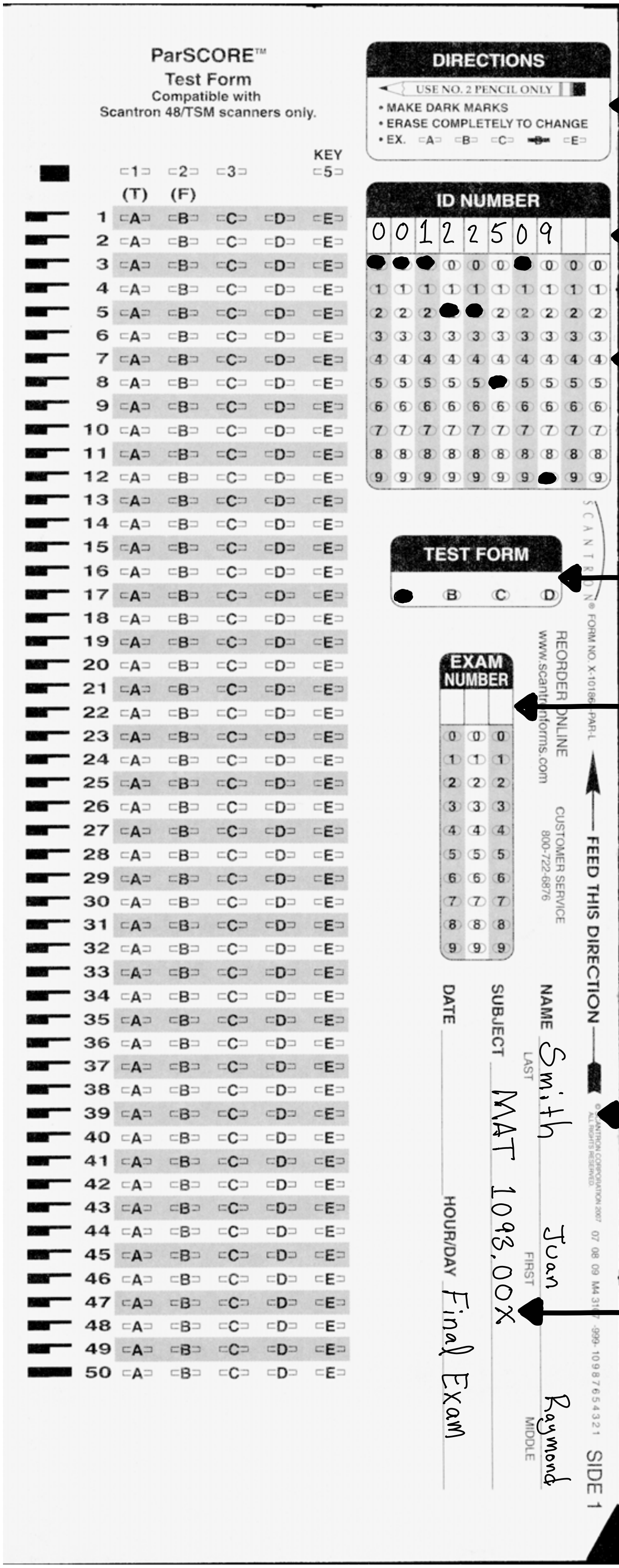
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Follow these directions.

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Write your last name, first name and middle name here.

Replace the x with the appropriate number for the section that you are enrolled. Look at the chart below if you don't know your section.

Class Days	Time	1093.section
MWF	8am	1093.001
MWF	10am	1093.002
MWF	2pm	1093.003
TTR	9:30am	1093.006

INSTRUCTIONS

Circle your answers on this exam and fill in the corresponding bubble on your ParScore.

You are NOT allowed to use calculators or formula sheets.

Separate the pages of the exam and use the back of the paper as scratch paper.
I'll have a stapler to staple your exam back together.

Cover your work and your Parscore. Don't cheat or appear to be cheating.

If something is illegible then please notify me.
If a question is ambiguous then please ask me to clarify.

DON'T GIVE UP! Do your best on every problem.
You are not supposed to already know the answer, you are to figure it out using what you know.

Use all of your available time. If you finish early, redo the problems to verify correctness. Don't "check your work" - redo it separately without looking at your previous work to avoid making the same mistakes twice.

When you are done, turn in this test and your Parscore.

OTHER INFORMATION

Your exam grades will be available in WebCT as soon as the Parscore Office uploads them to WebCT. When this will occur depends on how many other classes are ahead of us on the Parscore Office's list (They take care of all the Parscores at UTSA).

#1) While standing on the surface of the Earth you may feel like you are not moving, but indeed you are because the Earth is rotating and revolving around the Sun, which itself is orbiting the center of the Milky Way Galaxy. Considering only the rotation of the Earth, you have an angular speed of w = 1 rev/day. Considering that your linear speed depends upon where you are on the surface of the Earth. Which of the following statements is completely true?

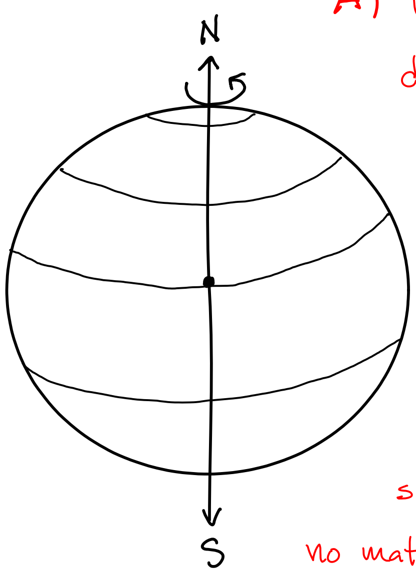
A) Your maximum linear speed caused by Earth's rotation occurs when you are on the Equator.

B) Your maximum angular speed occurs when you are on the North or South Pole.

C) If you drive North from UTSA keeping your speedometer at 70 mph, your distance from the Earth's axis of rotation becomes smaller as you approach the North Pole causing your total linear speed (speed of your car across the surface of the Earth plus the speed of the surface of the Earth's surface with respect to its center) to steadily increase.

D) All of the above.

E) None of the above



A) When standing on the equator, you are the furthest distance from the axis of rotation. You go around the biggest circle around the Earth as the Earth makes one revolution. So your maximum linear velocity occurs when on the equator.

B) Standing on the North Pole won't get you anywhere. You will simply spin in place. So, you'd have a linear speed $v = 0$. However, our angular speed stays the same no matter where you stand on the Earth. It is always 1 rev. per day.

C) Driving Northward at a constant speed in your car, you're speed with respect to the rest of the Universe is actually getting slower. Your car speedometer can't take into account the linear speed you get from being on the surface of the rotating Earth. As you approach the north pole, the added linear speed due to the rotating Earth becomes less and less until you reach the north pole where it doesn't contribute at all. Thus you are actually slowing down as you drive North.

#2) Which of the following pairs of equations have the same set of solutions?

A) $\sin\theta - \sqrt{3}\cos\theta = 2 \Leftrightarrow \sqrt{3}\sin\theta + \cos\theta = 2$

B) $2\sin^2\theta - 3\sin\theta = -1 \Leftrightarrow 2\cos^2\theta + \cos\theta = 1$

C) $\cos^2\theta - 2\sin^2\theta - 1 = 0 \Leftrightarrow \sin^2\theta - 2\cos^2\theta - 1 = 0$

D) All of the above.

E) None of the above.

A) $\sin\theta - \sqrt{3}\cos\theta = 2 \Leftrightarrow \frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta = \frac{2}{2} \Leftrightarrow \sin\left(\frac{\pi}{6}\right)\sin\theta - \cos\left(\frac{\pi}{6}\right)\cos\theta = 1$

$\Leftrightarrow \cos(\pi/6 + \theta) = 1 \Leftrightarrow \pi/6 + \theta = 2n\pi \Leftrightarrow \theta = -\pi/6 + 2n\pi \;;\; n \in \mathbb{Z}$

$\sqrt{3}\sin\theta + \cos\theta = 2 \Leftrightarrow \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{2}{2} \Leftrightarrow \sin\left(\frac{\pi}{3}\right)\sin\theta + \cos\left(\frac{\pi}{3}\right)\cos\theta = 1$

$\Leftrightarrow \cos(\pi/3 - \theta) = 1 \Leftrightarrow \pi/3 - \theta = 2n\pi \Leftrightarrow \theta = \pi/3 + 2n\pi \neq -\pi/6 + 2n\pi$

B) $2\sin^2\theta - 3\sin\theta = -1 \Leftrightarrow 2\sin^2\theta - 3\sin\theta + 1 = 0 \Leftrightarrow (2\sin\theta - 1)(\sin\theta - 1) = 0$

$$\Leftrightarrow \begin{cases} \sin\theta = 1/2 \Leftrightarrow \theta = \pi/6 + 2n\pi \text{ or } \theta = 5\pi/6 + 2n\pi \\ \sin\theta = 1 \Leftrightarrow \theta = \pi/2 + 2n\pi \end{cases}$$

$2\cos^2\theta + \cos\theta = 1 \Leftrightarrow 2\cos^2\theta + \cos\theta - 1 = 0 \Leftrightarrow (2\cos\theta - 1)(\cos\theta + 1) = 0$

$$\Leftrightarrow \begin{cases} \cos\theta = 1/2 \Leftrightarrow \theta = \pi/3 + 2n\pi \text{ or } \theta = 2\pi/3 + 2n\pi \\ \cos\theta = -1 \Leftrightarrow \theta = \pi + 2n\pi \end{cases}$$

The two equations have different solutions.

C) $\cos^2\theta - 2\sin^2\theta - 1 = 0 \Leftrightarrow \cos^2\theta - 2(1 - \cos^2\theta) - 1 = 0$

$\Leftrightarrow 3\cos^2\theta - 3 = 0 \Leftrightarrow \cos^2\theta = 1 \Leftrightarrow \cos\theta = \pm 1 \Leftrightarrow \theta = n\pi$

$\sin^2\theta - 2\cos^2\theta - 1 = 0 \Leftrightarrow \sin^2\theta - 2(1 - \sin^2\theta) - 1 = 0$

$\Leftrightarrow 3\sin^2\theta - 3 = 0 \Leftrightarrow \sin^2\theta = 1 \Leftrightarrow \sin\theta = \pm 1 \Leftrightarrow \theta = \pi/2 + n\pi \neq n\pi$

#3) Which of the following is an identity?

A) $(2a\sin\theta\cos\theta)^2 + a^2(\cos^2\theta - \sin^2\theta)^2 = a^2$

B) $(\tan\alpha + \tan\beta)(1 - \cot\alpha\cot\beta) + (\cot\alpha + \cot\beta)(1 - \tan\alpha\tan\beta) = 0$

C) $\dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} = \sec\theta + \tan\theta$

D) All of the above.

E) None of the above.

A) $(2a\sin\theta\cos\theta)^2 + a^2(\cos^2\theta - \sin^2\theta)^2 = a^2(2\sin\theta\cos\theta)^2 + a^2\cos^2(2\theta)$

$= a^2\sin^2(2\theta) + a^2\cos^2(2\theta) = a^2\left[\sin^2(2\theta) + \cos^2(2\theta)\right] = a^2$

B) $(\tan\alpha + \tan\beta)(1 - \cot\alpha\cot\beta) + (\cot\alpha + \cot\beta)(1 - \tan\alpha\tan\beta)$

$= (\tan\alpha + \tan\beta) - (\tan\alpha\cot\alpha\cot\beta + \tan\beta\cot\alpha\cot\beta)$

$+ (\cot\alpha + \cot\beta) - (\cot\alpha\tan\alpha\tan\beta + \cot\beta\tan\alpha\tan\beta)$

$= (\tan\alpha + \tan\beta) - (\cot\beta + \cot\alpha) + (\cot\beta + \cot\alpha) - (\tan\beta + \tan\alpha) = 0$

C) $\dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} = \dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} \cdot \dfrac{1/\cos\theta}{1/\cos\theta} = \dfrac{\sec\theta + 1 + \tan\theta}{\sec\theta + 1 - \tan\theta}$

$= \dfrac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)} \dfrac{\sec\theta + 1 + \tan\theta}{\sec\theta + 1 - \tan\theta} = \dfrac{(\sec\theta + \tan\theta)\left[(\sec\theta + \tan\theta) + 1\right]}{\underbrace{\sec^2\theta - \tan^2\theta} + \sec\theta + \tan\theta}$

$\tan^2\theta + 1 = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1$

$= \dfrac{(\sec\theta + \tan\theta)\left[(\sec\theta + \tan\theta) + 1\right]}{\left[1 + (\sec\theta + \tan\theta)\right]}$

$= (\sec\theta + \tan\theta)$

#4) Which of the following is an identity?

A) $\tan(\sin^{-1} v) = \dfrac{v}{\sqrt{1-v^2}}$

B) $\sin(\cos^{-1} v) = \sqrt{1-v^2}$

C) $\cos(\tan^{-1} v) = \dfrac{1}{\sqrt{1+v^2}}$

D) All of the above.

E) None of the above.

A) $\tan(\sin^{-1} v) = \tan\theta$ where $\sin^{-1} v = \theta \in [-\pi/2, \pi/2]$

$\Rightarrow \sin\theta = v = y/r$; let $y = v$ and $r = 1$

$x^2+y^2=r^2 \Rightarrow x = \pm\sqrt{r^2-y^2} = \pm\sqrt{1-v^2} \Rightarrow \tan(\sin^{-1} v) = \tan\theta = \dfrac{y}{x} = \dfrac{v}{\sqrt{1-v^2}}$

The only real complication here is why we take $+\sqrt{1-v^2}$ instead of $-\sqrt{1-v^2}$.

First off, $\theta \in [-\pi/2, \pi/2]$.

If $\theta \in [-\pi/2, 0)$ then $\sin\theta = v < 0 \Rightarrow \tan\theta < 0$ so we want

$\tan\theta = \dfrac{v}{\sqrt{1-v^2}} < 0$ since $v < 0$ as a consequence of $\theta \in [-\pi/2, 0)$

If $\theta \in [0, \pi/2]$ then $\sin\theta = v \geq 0 \Rightarrow \tan\theta \geq 0$ so again we want

$\tan\theta = \dfrac{v}{\sqrt{1-v^2}} \geq 0$ since $v \geq 0$ as a consequence of $\theta \in [0, \pi/2]$

Basically, the sign is contained in v. When $v < 0$, $\tan\theta < 0$ and when $v > 0$, $\tan\theta > 0$, so we don't need a minus sign. If we wanted them to always have opposite signs, we'd add the minus sign. If this is too abstract, you can always just plug in a few numbers to see if you need to put the minus sign or not. Try it.

B) $\sin(\cos^{-1} v) = \sin\theta$ where $\cos^{-1} v = \theta \in [0, \pi]$

$\Rightarrow \cos\theta = v = x/r$; let $x = v$ and $r = 1$

$\Rightarrow y = \pm\sqrt{r^2 - x^2} = \pm\sqrt{1 - v^2} \Rightarrow \sin(\cos^{-1} v) = \sin\theta = \frac{y}{r} = \sqrt{1 - v^2}$

We want the positive one because $\theta \in [0, \pi]$.

C) $\cos(\tan^{-1} v) = \cos\theta$ where $\tan^{-1} v = \theta \in (-\pi/2, \pi/2)$

$\Rightarrow \tan\theta = v = y/x$; let $y = v$ and $x = 1$

$\Rightarrow r = \pm\sqrt{x^2 + y^2} = \pm\sqrt{1 + v^2} \Rightarrow \cos(\tan^{-1} v) = \cos\theta = \frac{x}{r} = \frac{1}{\sqrt{1 + v^2}}$

We want the positive one because $\theta \in (-\pi/2, \pi/2)$.

Go back and review these problems. Look at the answers and the questions together. Try to see through the steps and reason your way to the answer. You can actually do these in your head in a few seconds if you actually understand the problem. $\cos(\tan^{-1} v)$ is just cosine of an angle such that tangent of that same angle is v.

Do this kind of reflection for all of the problems you solve. Could you have done it an easier way? Can you do it in your head?

#5) Solve for θ: $\sqrt{2\sin(\theta^2)} = \sqrt{\sqrt{3}}$

A) $\theta \in \left\{ \pm\sqrt{\pi/3} + 2k\pi, \pm\sqrt{2\pi/3} + 2k\pi \;;\; k \in \mathbb{Z} \right\}$

B) $\theta \in \left\{ \pm\sqrt{\pi/3 + 2k\pi}, \pm\sqrt{2\pi/3 + 2k\pi} \;;\; k \in \mathbb{Z} \right\}$

C) $\theta \in \left\{ \sqrt{\pi/3 + 2k\pi}, \sqrt{2\pi/3 + 2k\pi} \;;\; k \in \mathbb{Z} \right\}$

D) $\theta \in \left\{ \pi/3 + 2k\pi, 2\pi/3 + 2k\pi \;;\; k \in \mathbb{Z} \right\}$

E) None of the above.

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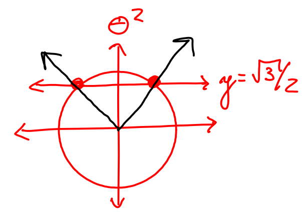
both sides of original equation are positive.

$\sqrt{x^2} = |x|$

$$\sqrt{2\sin(\theta^2)} = \sqrt{\sqrt{3}} \iff \left(\sqrt{2\sin(\theta^2)}\right)^2 = \left(\sqrt{\sqrt{3}}\right)^2 \iff |2\sin(\theta^2)| = |\sqrt{3}|$$

$$\iff \sin(\theta^2) = \sqrt{3}/2$$



$$\iff \theta^2 = \begin{cases} \pi/3 + 2k\pi \\ 2\pi/3 + 2k\pi \end{cases} \quad \text{where } k \in \mathbb{Z}$$

$$\iff \theta = \begin{cases} \pm\sqrt{\pi/3 + 2k\pi} \\ \pm\sqrt{2\pi/3 + 2k\pi} \end{cases} \quad \text{where } k \in \mathbb{Z}$$

$$\iff \boxed{\theta \in \left\{ \pm\sqrt{\pi/3 + 2k\pi}, \pm\sqrt{2\pi/3 + 2k\pi} \;;\; k \in \mathbb{Z} \right\}}$$

#6) Solve for θ: $\sin(3\theta) = -\sin\theta$

A) $\theta \in \{2k\pi,\ \pi + 2k\pi;\ k \in \mathbb{Z}\}$

B) $\theta \in \{k\pi;\ k \in \mathbb{Z}\}$

C) $\theta \in \{k\pi/2;\ k \in \mathbb{Z}\}$

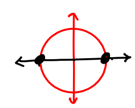
D) $\theta \in \{k\pi/2 + k\pi;\ k \in \mathbb{Z}\}$

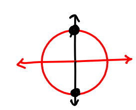
E) None of the above.

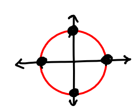
$$\sin(3\theta) = \sin(2\theta)\cos\theta + \cos(2\theta)\sin\theta$$
$$= 2\sin\theta\cos^2\theta + (2\cos^2\theta - 1)\sin\theta$$
$$= 4\sin\theta\cos^2\theta - \sin\theta$$

Therefore,

$$\sin(3\theta) = -\sin\theta \iff 4\sin\theta\cos^2\theta - \sin\theta = -\sin\theta$$
$$\iff 4\sin\theta\cos^2\theta = 0$$
$$\iff \sin\theta\cos^2\theta = 0$$
$$\iff \begin{cases} \sin\theta = 0 \iff \theta \in \{k\pi;\ k \in \mathbb{Z}\} \\ \cos\theta = 0 \iff \theta \in \{\pi/2 + k\pi;\ k \in \mathbb{Z}\} \end{cases}$$
$$\iff \theta \in \{k\pi/2;\ k \in \mathbb{Z}\}$$







#7) Solve for θ: $\cos(3\theta) = -3\cos^3\theta$

A) $\theta \in \{ \pm\sqrt[3]{\pi/6 + 2k\pi}\,;\ k \in \mathbb{Z} \}$

B) $\theta \in \{ k\pi,\ (3k+1)\pi/2\,;\ k \in \mathbb{Z} \}$

C) $\theta \in \{ k\pi/2\,;\ k \in \mathbb{Z} \}$

D) $\boxed{\theta \in \{ (2k+1)\pi/2\,;\ k \in \mathbb{Z} \}}$

E) None of the above.

$$\cos(3\theta) \equiv \cos(2\theta)\cos\theta - \sin\theta\sin(2\theta)$$

$$\equiv (1-2\sin^2\theta)\cos\theta - 2\sin^2\theta\cos\theta$$

$$= \cos\theta\,(1-4\sin^2\theta) = \cos\theta\,(\underbrace{1-\sin^2\theta}_{\cos^2\theta} - 3\sin^2\theta)$$

$$= -3\cos^3\theta\sin^2\theta$$

Thus,

$$\cos(3\theta) = -3\cos^3\theta \iff -3\cos^3\theta\sin^2\theta = -3\cos^3\theta$$

$$\iff -3\cos^3\theta\sin^2\theta + 3\cos^3\theta = 0$$

$$\iff -\cos^3\theta\,(1-\sin^2\theta) = 0$$

$$\iff \cos^5\theta = 0$$

$$\iff \cos\theta = 0$$

$$\iff \theta \in \{ \pi/2 + k\pi\,;\ k \in \mathbb{Z} \}$$

$$\iff \theta \in \{ (2k+1)\pi/2\,;\ k \in \mathbb{Z} \}$$

#8) Solve for θ: $\sin^2(4\theta) - \cos^2(4\theta) = 0$

A) $\theta \in \{(2k+1)\pi/4, (2k+1)\pi/2 \,;\, k \in \mathbb{Z}\}$

B) $\theta = k\pi/8 \,;\, k \in \mathbb{Z}$

C) $\theta = k\pi/4 \,;\, k \in \mathbb{Z}$

D) No real valued solutions.

E) None of the above.

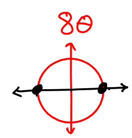
$$\sin^2(4\theta) - \cos^2(4\theta) = 0 \iff 1 - 2\cos^2(4\theta) = 0$$

$$\iff -(2\cos^2(4\theta) - 1) = 0$$

$$\iff \cos(8\theta) = 0$$

$$\iff 8\theta = k\pi \,;\, k \in \mathbb{Z}$$

$$\iff \theta = k\pi/8 \,;\, k \in \mathbb{Z}$$



#9) Solve for θ: $\sqrt{3}\cos(\sqrt{3}\theta) + \sin(\sqrt{3}\theta) = \sqrt{3}$

A) $\theta \in \{ \pm\pi/\sqrt{3} + 2k\pi/\sqrt{3} \; ; \; k \in \mathbb{Z} \}$

B) $\theta \in \{ \pm 2\sqrt{3}k\pi/3 \; ; \; k \in \mathbb{Z} \}$

C) $\boxed{\theta \in \{ \sqrt{3}\pi/9 + 2\sqrt{3}k\pi/3, \; 2\sqrt{3}k\pi/3 \; ; \; k \in \mathbb{Z} \}}$

D) No real valued solutions.

E) None of the above.

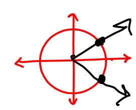
$$\sqrt{3}\cos(\sqrt{3}\theta) + \sin(\sqrt{3}\theta) = \sqrt{3} \iff \frac{\sqrt{3}}{2}\cos(\sqrt{3}\theta) + \frac{1}{2}\sin(\sqrt{3}\theta) = \frac{\sqrt{3}}{2}$$

$$\iff \cos(\pi/6)\cos(\sqrt{3}\theta) + \sin(\pi/6)\sin(\sqrt{3}\theta) = \sqrt{3}/2$$

$$\iff \cos(\sqrt{3}\theta - \pi/6) = \sqrt{3}/2$$

$$\iff \sqrt{3}\theta - \pi/6 = \pm\pi/6 + 2k\pi \; ; \; k \in \mathbb{Z}$$

$\sqrt{3}\theta - \pi/6$

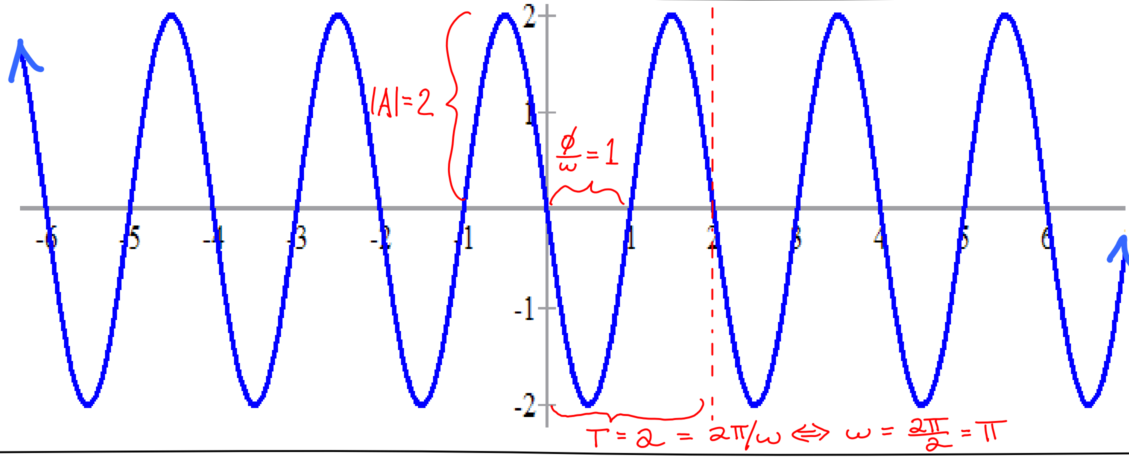


$$\iff \sqrt{3}\theta = \begin{cases} \pi/3 + 2k\pi \\ 2k\pi \end{cases} \; ; \; k \in \mathbb{Z}$$

$$\iff \theta = \begin{cases} \sqrt{3}\pi/9 + 2\sqrt{3}k\pi/3 \\ 2\sqrt{3}k\pi/3 \end{cases} \; ; \; k \in \mathbb{Z}$$

$$\iff \boxed{\theta \in \{ \sqrt{3}\pi/9 + 2\sqrt{3}k\pi/3, \; 2\sqrt{3}k\pi/3 \; ; \; k \in \mathbb{Z} \}}$$

#10) This is a graph of which of the following functions?



A) $y=2\cos(\pi x)$

B) $\boxed{y=2\sin(\pi x-\pi)}$

C) $y=2\cos(2x-\pi)$

D) $y=2\sin(\pi x/2+\pi/2)$

E) None of the above.

$\frac{\phi}{\omega}=1$ and $\omega=\pi \Rightarrow \phi=\pi+\pi KT=\pi+2\pi K=(2K+1)\pi$ where $K\in\mathbb{Z}$

for $K=0$ we have $\phi=\pi$

#11) If $f(z) = \sin(2/z)$ then

A) $f(-z) = f(z)$ $\boxed{\textbf{B)}\quad f(2/z) = \sin z}$ **C)** $f(2) = \pi/2$

D) All of the above. **E)** None of the above.

$$f(z) = \sin(2/z) \Rightarrow f(2/z) = \sin\left(\frac{2}{2/z}\right) = \sin(z)$$

#12) Suppose $f(y) = \sin\left(\frac{1}{y}\right)$. Which of the following is true for $f(y)$:

A) f is defined for all $y = \theta$,

B) f is defined for all y such that $-\infty < y \leq 0$ and $0 < y < \infty$,

C) f is undefined for $y = \cos\left[n\pi + \frac{\pi}{2}\right]$ if $n \in \mathbb{Z}$,

D) $f(y)$ is undefined for all y such that $\theta = 0$,

E) None of the above.

A) False. $f(0)$ is undefined.

B) False. $f(0)$ is undefined.

C) True. $\cos\left[n\pi + \frac{\pi}{2}\right] = 0$ for any $n \in \mathbb{Z}$ and $f(0)$ is undefined.

D) False. $f(y)$ is a function of the variable y only. The variable θ is irrelevant.

#13) Which of the following is completely true?

A) $\cos(1^\circ) - \cos(2^\circ) + \cos(3^\circ) - \cos(4^\circ) + \cdots + \cos(359^\circ)$

$= -\sin(1^\circ) + \sin(2^\circ) - \sin(3^\circ) + \sin(4^\circ) + \cdots - \sin(359^\circ)$

B) $\cos^2(1^\circ) + \cos^2(2^\circ) + \cdots + \cos^2(359^\circ) = 1$

C) $\boxed{\tan(1^\circ) + \tan(2^\circ) + \cdots + \tan(89^\circ) + \tan(91^\circ) + \cdots + \tan(179^\circ) = \tan(181^\circ) + \cdots + \tan(269^\circ) + \tan(271^\circ) + \cdots + \tan(359^\circ)}$

D) All of the above.

E) None of the above.

A) $\cos(1^\circ) - \cos(2^\circ) + \cos(3^\circ) - \cos(4^\circ) + \cdots + \cos(359^\circ)$

$= \cos(1^\circ) + \cos(179^\circ) + \cos(3^\circ) + \cos(177^\circ) + \cdots + \cos(79^\circ) + \cos(91^\circ)$

$- [\cos(2^\circ) + \cos(178^\circ) + \cos(4^\circ) + \cos(176^\circ) + \cdots \cos(88^\circ) + \cos(92^\circ)] + \overset{0}{\cancel{\cos}(90^\circ)} + \overset{-1}{\cancel{\cos}(180^\circ)}$

$= \cos(1^\circ) - \cos(1^\circ) + \cos(3^\circ) - \cos(3^\circ) + \cdots + \cos(79^\circ) - \cos(79^\circ)$

$- [\cos(2^\circ) - \cos(2^\circ) + \cos(4^\circ) - \cos(4^\circ) + \cdots + \cos(88^\circ) - \cos(88^\circ)] - 1$

$= 0 - 0 - 1 = -1$ but

$-\sin(1^\circ) + \sin(2^\circ) - \sin(3^\circ) + \sin(4^\circ) + \cdots - \sin(359^\circ)$

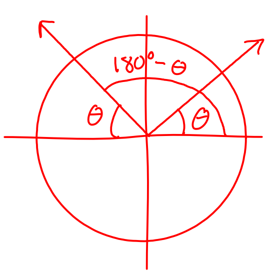
$= -[\sin(1^\circ) + \sin(359^\circ) + \sin(3^\circ) + \sin(357^\circ) + \cdots + \sin(179^\circ) + \sin(181^\circ)]$

$+ \sin(2^\circ) + \sin(358^\circ) + \sin(4^\circ) + \sin(356^\circ) + \cdots + \sin(178^\circ) + \sin(182^\circ) + \overset{0}{\cancel{\sin}(180^\circ)}$

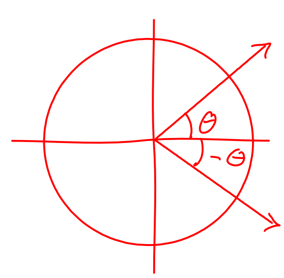
$= -[\sin(1^\circ) - \sin(1^\circ) + \sin(3^\circ) - \sin(3^\circ) + \cdots + \sin(179^\circ) - \sin(179^\circ)]$

$+ \sin(2^\circ) - \sin(2^\circ) + \sin(4^\circ) - \sin(4^\circ) + \cdots + \sin(178^\circ) - \sin(178^\circ) = 0 \neq -1$

This can be visualized simply by looking at the unit circle:



$\cos\theta = -\cos(180^\circ - \theta)$



$\sin\theta = -\sin(-\theta)$

B) $\cos^2(1^\circ) + \cos^2(2^\circ) + \cdots + \cos^2(359^\circ) = \overbrace{\cos^2(1^\circ) + \cos^2(2^\circ) + \cdots}^{>0} + \overset{1}{\cancel{\cos(180^\circ)}}^2 + \overbrace{\cdots + \cos^2(359^\circ)}^{>0} > 1 \Rightarrow \neq 1$

C) $\tan(1^\circ) + \tan(2^\circ) + \cdots + \tan(89^\circ) + \tan(91^\circ) + \cdots + \tan(179^\circ)$

$= \tan(1^\circ) + \tan(179^\circ) + \tan(2^\circ) + \tan(178^\circ) + \cdots + \tan(89^\circ) + \tan(91^\circ)$

$= \tan(1^\circ) - \tan(1^\circ) + \tan(2^\circ) - \tan(2^\circ) + \cdots + \tan(89^\circ) - \tan(89^\circ) = 0$ and

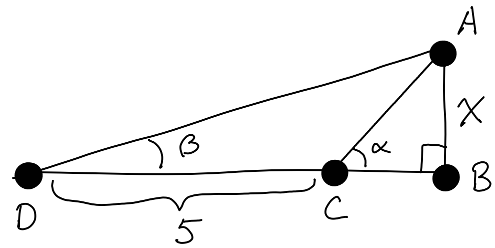
$\tan(181^\circ) + \cdots + \tan(269^\circ) + \tan(271^\circ) + \cdots + \tan(359^\circ)$

$= \tan(181^\circ) + \tan(359^\circ) + \tan(182^\circ) + \tan(358^\circ) + \cdots + \tan(269^\circ) + \tan(271^\circ)$

$= \tan(181^\circ) - \tan(181^\circ) + \tan(182^\circ) - \tan(182^\circ) + \cdots + \tan(269^\circ) - \tan(269^\circ) = 0$ ✓

#14) For the following diagram, given

$\alpha = 50°$ and $\beta = 20°$, solve for x.



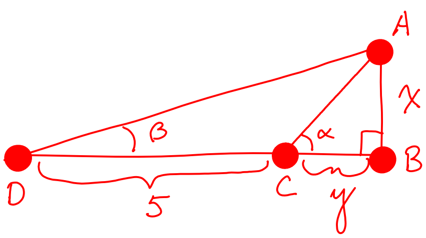
A) $\dfrac{5\tan(50°)\tan(20°)}{\tan(50°)+\tan(20°)}$

B) $\dfrac{5\tan(50°)\tan(20°)}{\tan(20°)-\tan(50°)}$

C) $\boxed{\dfrac{5\tan(50°)\tan(20°)}{\tan(50°)-\tan(20°)}}$

D) $5[\tan(70°)-\tan(50°)+\tan(20°)]$

E) None of the above.



$\tan(50°) = \dfrac{x}{y} \Rightarrow y = \dfrac{x}{\tan(50°)}$

$\tan(20°) = \dfrac{x}{5+y} = \dfrac{x}{5+\dfrac{x}{\tan(50°)}} = \dfrac{x\tan(50°)}{x+5\tan(50°)}$

$\Rightarrow \tan(20°)[x+5\tan(50°)] = x\tan(50°)$

$\Rightarrow x[\tan(20°)-\tan(50°)] = -5\tan(50°)\tan(20°)$

$\Rightarrow \boxed{x = \dfrac{5\tan(50°)\tan(20°)}{\tan(50°)-\tan(20°)}}$

#15) Solve for θ: $2\sin(\theta^2+2\theta+1) = 1$

A) $\boxed{\theta \in \left\{ \pm\sqrt{\pi/6 + 2\pi k} - 1,\ \pm\sqrt{5\pi/6 + 2\pi k} - 1\ ;\ k \in \mathbb{Z} \right\}}$

B) $\sin(\theta^2) = 1/2,\ \sin(2\theta) = 1/2$

C) $\theta \in \left\{ \pi/6 + 2\pi k,\ 5\pi/6 + 2\pi k\ ;\ k \in \mathbb{Z} \right\}$

D) $\theta \in \left\{ (\pi/6 + 2\pi k)^2 + 2(\pi/6 + 2\pi k) + 1,\ (5\pi/6 + 2\pi k)^2 + 2(5\pi/6 + 2\pi k) + 1\ ;\ k \in \mathbb{Z} \right\}$

E) None of the above.

$$2\sin(\theta^2+2\theta+1) = 1 \Leftrightarrow \sin\left[(\theta+1)^2\right] = 1/2 \Leftrightarrow (\theta+1)^2 = \begin{cases} \pi/6 + 2n\pi \\ 5\pi/6 + 2n\pi \end{cases}$$

$$\Leftrightarrow \theta + 1 = \begin{cases} \pm\sqrt{\pi/6 + 2n\pi} \\ \pm\sqrt{5\pi/6 + 2n\pi} \end{cases} \Leftrightarrow \boxed{\theta = \begin{cases} \pm\sqrt{\pi/6 + 2n\pi} - 1 \\ \pm\sqrt{5\pi/6 + 2n\pi} - 1 \end{cases}}$$

#16 - 30 are EXTRA CREDIT

#16) $r[\cos\theta \pm i\sin\theta] = re^{\pm i\theta}$ in complex exponential notation. What is $\tan\theta$?

A) $\dfrac{e^{i\theta}+e^{-i\theta}}{2}$ **B)** $\dfrac{e^{i\theta}+e^{-i\theta}}{2i}$ **C)** $\dfrac{e^{i\theta}-e^{-i\theta}}{2}$

D) $\dfrac{e^{i\theta}-e^{-i\theta}}{2i}$ **E)** None of the above.

$\cos\theta + i\sin\theta + \cos\theta - i\sin\theta = e^{i\theta} + e^{-i\theta}$

$\Leftrightarrow 2\cos\theta = e^{i\theta} + e^{-i\theta}$

$\Leftrightarrow \cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$

$\cos\theta + i\sin\theta - [\cos\theta - i\sin\theta] = e^{i\theta} - e^{-i\theta}$

$\Leftrightarrow 2i\sin\theta = e^{i\theta} - e^{-i\theta}$

$\Leftrightarrow \sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

$$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{e^{i\theta}-e^{-i\theta}}{2i}}{\frac{e^{i\theta}+e^{-i\theta}}{2}} = \frac{e^{i\theta}-e^{-i\theta}}{i[e^{i\theta}+e^{-i\theta}]}$$

#17) $\log_{10}\left(x^2\sqrt{x^3+1}\right) = \log_{10}(x^2) + \log_{10}(\sqrt{x^3+1}) = 2\log_{10}(x) + (1/2)\log_{10}(x^3+1)$

A) $2\log_{10}(x) + \dfrac{\log_{10}(x^3+1)}{2}$

B) $2(1/2)(3)\left[\log_{10}(x) + \log_{10}(\sqrt{x^3+1})\right]$

C) $\log_{10}(x^2)\cdot\log_{10}(\sqrt{x^3+1})$

D) All of the above.

E) None of the above.

#18) Solve for x: $2^{x+1}\cdot 16^{-x} = 1/2$

A) $x = 3/2$ **B)** $x = 2/3$ **C)** $x = 0$ **D)** $x = \log_2$ **E)** None of the above.

$2^{x+1}\cdot 16^{-x} = 1/2 \Leftrightarrow 2^{x+1}\cdot 2^{-4x} = 2^{-3x+1} = 2^{-1} \Leftrightarrow -3x+1 = -1$

$\Leftrightarrow \boxed{x = 2/3}$

#19) Which of the following is completely true? Note: $\cos\theta + i\sin\theta = e^{i\theta}$

A) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

B) $0 = 1 + e^{i\pi}$ and $1 = e^{i2\pi}$

C) If $z = x + iy$ then e^z has radius $r = e^x$ and angle $\theta = y$

D) All of the above.

E) None of the above.

A) $$\left.\begin{aligned} \cos\theta + i\sin\theta &= e^{i\theta} \\ \Leftrightarrow \cos\theta - i\sin\theta &= e^{-i\theta} \end{aligned}\right\} \Leftrightarrow \cos\theta + i\sin\theta + \cos\theta - i\sin\theta = e^{i\theta} + e^{-i\theta}$$

$$\Leftrightarrow 2\cos\theta = e^{i\theta} + e^{-i\theta} \Leftrightarrow \boxed{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$

$$\left.\begin{aligned} \cos\theta + i\sin\theta &= e^{i\theta} \\ \Leftrightarrow \cos\theta - i\sin\theta &= e^{-i\theta} \end{aligned}\right\} \Leftrightarrow \cos\theta + i\sin\theta - \cos\theta - i\sin\theta = e^{i\theta} - e^{-i\theta}$$

$$\Leftrightarrow 2i\sin\theta = e^{i\theta} - e^{-i\theta} \Leftrightarrow \boxed{\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}}$$

B) $$\cos\theta + i\sin\theta = e^{i\theta} \Leftrightarrow \cancel{\cos}(\pi)^{-1} + i\,\cancel{\sin}(\pi)^{0} = e^{i\pi} \Leftrightarrow -1 = e^{i\pi} \Leftrightarrow \boxed{0 = e^{i\pi} + 1}$$

$$\cos\theta + i\sin\theta = e^{i\theta} \Leftrightarrow \cancel{\cos}(2\pi)^{1} + i\,\cancel{\sin}(2\pi)^{0} = e^{i2\pi} \Leftrightarrow \boxed{1 = e^{i2\pi}}$$

C) $$e^z = e^{x+iy} = e^x e^{iy} = e^x(\cos y + i\sin y) = r(\cos\theta + i\sin\theta) \Leftrightarrow r = e^x \text{ and } \theta = y$$

#20) Completely factor the polynomial: $z^6 + 64 = 0$

A) $(z-2)(z+2)(z-2i)(z+2i)[z-(\sqrt{2}+\sqrt{2}i)][z+(\sqrt{2}+\sqrt{2}i)]=0$

B) $(z-2)(z+2)[z-(\sqrt{2}+\sqrt{2}i)][z+(\sqrt{2}+\sqrt{2}i)][z-(\sqrt{3}-i)][z+(\sqrt{3}-i)]=0$

C) $(z-2i)(z+2i)[z-(1+\sqrt{3}i)][z+(1+\sqrt{3}i)][z-(1-\sqrt{3}i)][z+(1-\sqrt{3}i)]=0$

D) $\boxed{[z-(\sqrt{3}+i)][z-2i][z-(-\sqrt{3}+i)][z-(-\sqrt{3}-i)][z-(-2i)][z-(\sqrt{3}-i)]=0}$

E) None of the above.

$z^6+64=0 \Leftrightarrow z^6=-64=2^6[\cos(\pi)+i\sin(\pi)]$

The solutions must all have radius $\sqrt[6]{2^6}=2$ and have angles θ_K such that

$6\theta_K=\pi+2\pi K \Rightarrow \theta_K=\frac{\pi}{6}+\frac{2\pi K}{6} \Rightarrow \theta_0=\frac{\pi}{6},\ \theta_1=\frac{3\pi}{6}=\frac{\pi}{2},\ \theta_2=\frac{5\pi}{6},\ \theta_3=\frac{7\pi}{6},\ \theta_4=\frac{9\pi}{6}=\frac{3\pi}{2},\ \theta_5=\frac{11\pi}{6}$

$z_0=2[\cos(\frac{\pi}{6})+i\sin(\frac{\pi}{6})]=2[\frac{\sqrt{3}}{2}+\frac{i}{2}]=\sqrt{3}+i$

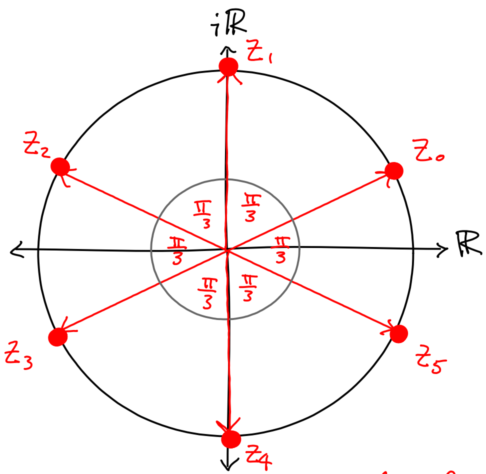
$z_1=2[\cos(\frac{\pi}{2})+i\sin(\frac{\pi}{2})]=2[0+i]=2i$

$z_2=2[\cos(\frac{5\pi}{6})+i\sin(\frac{5\pi}{6})]=2[-\frac{\sqrt{3}}{2}+\frac{i}{2}]=-\sqrt{3}+i$

$z_3=2[\cos(\frac{7\pi}{6})+i\sin(\frac{7\pi}{6})]=2[-\frac{\sqrt{3}}{2}-\frac{i}{2}]=-\sqrt{3}-i$

$z_4=2[\cos(\frac{3\pi}{2})+i\sin(\frac{3\pi}{2})]=2[0-i]=-2i$

$z_5=2[\cos(\frac{11\pi}{6})+i\sin(\frac{11\pi}{6})]=2[\frac{\sqrt{3}}{2}-\frac{i}{2}]=\sqrt{3}-i$



Notice the six solutions of our 6th root are all equally spaced around the circle of radius 2. Also, since the polynomial has all real coefficients, the solutions come in complex conjugate pairs. Since we know the solutions we can write down the factors.

$z^6+64=0 \Leftrightarrow (z-z_0)(z-z_1)(z-z_2)(z-z_3)(z-z_4)(z-z_5)=0$

$\Leftrightarrow \boxed{[z-(\sqrt{3}+i)][z-2i][z-(-\sqrt{3}+i)][z-(-\sqrt{3}-i)][z-(-2i)][z-(\sqrt{3}-i)]=0}$

$\Leftrightarrow (z-\sqrt{3}-i)(z-2i)(z+\sqrt{3}-i)(z+\sqrt{3}+i)(z+2i)(z-\sqrt{3}+i)=0$

#21) Which of the following is completely true?

A) $2^{3x} = 3^{2x+1} \Leftrightarrow 3^{2x} = 2^{3x+1}$

B) $\boxed{\log_6(x+3) + \log_6(x+4) = 1 \Leftrightarrow \log_6\left[\frac{1}{x+3}\right] + \log_6\left[\frac{1}{x+4}\right] = -1}$

C) $16^{x+2} = 8^{x-3} \Leftrightarrow 27^{x+3} = 81^{x-2}$

D) All of the above.

E) None of the above.

A) $2^{3x} = 3^{2x+1} \Leftrightarrow \log_2(2^{3x}) = \log_2(3^{2x+1}) \Leftrightarrow 3x = (2x+1)\log_2(3)$

$\Leftrightarrow 3x - 2\log_2(3)x = \log_2(3) \Leftrightarrow x(3 - 2\log_2(3)) = \log_2(3)$

$\Leftrightarrow x = \dfrac{\log_2(3)}{3 - 2\log_2(3)}$

$3^{2x} = 2^{3x+1} \Leftrightarrow \log_2(3^{2x}) = \log_2(2^{3x+1}) \Leftrightarrow 2x\log_2(3) = 3x+1$

$\Leftrightarrow 2x\log_2(3) - 3x = 1 \Leftrightarrow x(2\log_2(3) - 3) = 1$

$\Leftrightarrow x = \dfrac{1}{2\log_2(3) - 3} \neq \dfrac{\log_2(3)}{3 - 2\log_2(3)}$

$\boxed{\text{B)}}$ $\log_6(x+3) + \log_6(x+4) = 1 \Rightarrow \log_6[(x+3)(x+4)] = 1 \Leftrightarrow (x+3)(x+4) = 6^1$

$\Leftrightarrow x^2 + 7x + 12 - 6 = x^2 + 7x + 6 = 0 \Leftrightarrow (x+1)(x+6) = 0 \Leftrightarrow x = -1, -6$

But $x = -6$ cannot be a solution since $\log_6(x+3)$ is undefined for $x = -6$.

Although it isn't necessary to solve since...

$\log_6\left[\frac{1}{x+3}\right] + \log_6\left[\frac{1}{x+4}\right] = -1 \Leftrightarrow \log_6[(x+3)^{-1}] + \log_6[(x+4)^{-1}] = -1$

$\Leftrightarrow -\log_6(x+3) - \log_6(x+4) = -1 \Leftrightarrow \log_6(x+3) + \log_6(x+4) = 1$

Thus the two equations are equivalent and have the same solutions.

C) $16^{x+2} = 8^{x-3} \Leftrightarrow (2^4)^{x+2} = (2^3)^{x-3} \Leftrightarrow 2^{4(x+2)} = 2^{3(x-3)}$

$\Leftrightarrow 4(x+2) = 3(x-3) \Leftrightarrow x = -17$

$27^{x+3} = 81^{x-2} \Leftrightarrow (3^3)^{x+3} = (3^4)^{x-2} \Leftrightarrow 3^{3(x+3)} = 3^{4(x-2)}$

$\Leftrightarrow 3(x+3) = 4(x-2) \Leftrightarrow x = 17 \neq -17$

#22)

Find the product of the eight 8th roots of unity (1).
(In other words, find the roots and multiply them all together.)

A) 1

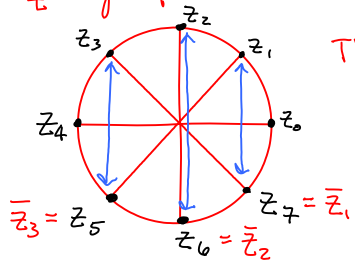
B) $\boxed{-1}$

C) i

D) $-i$

E) None of the above

The easy way: Roots obviously have radius $r=1$, so roots are equally spaced around the unit circle. 1 is an obvious root.



The roots come in complex conjugate pairs, and $z\bar{z} = (x+yi)(x-yi) = x^2+y^2 = r^2$

$\bar{z}_0 = z_0$ and $\bar{z}_4 = z_4$ since they are real.

$\Rightarrow z_0 z_1 z_2 z_3 z_4 z_5 z_6 z_7 = r^6 (1)(-1)$

A longer solution:

$1 = 1[\cos(2k\pi) + i\sin(2k\pi)] \Rightarrow$ roots have radius $\sqrt[8]{1} = 1$ and angles θ_k such that

$$8\theta_k = 2\pi k \Rightarrow \theta_k = \frac{2\pi k}{8} = \frac{\pi k}{4} = \left\{0, \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}, \frac{7\pi}{4}\right\}$$

If we were to multiply all the roots together, their angles would add giving an angle of

$$\theta = 0 + \frac{\pi}{4} + \frac{2\pi}{4} + \frac{3\pi}{4} + \frac{4\pi}{4} + \frac{5\pi}{4} + \frac{6\pi}{4} + \frac{7\pi}{4} = \frac{28\pi}{4} = 7\pi$$

and radii multiply when multiplying complex numbers so our product would have a radius of $r = 1^8 = 1$

So our product is $1[\cos(7\pi) + i\sin(7\pi)] = \boxed{-1}$

#23) Write the given complex number, z, in x + iy standard form.

$$z = \sqrt{8}\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$$

A) $\sqrt{6} - \sqrt{2}\,i$

B) $\cos\left(-\frac{\pi}{3}\right) + \sqrt{2}\,i$

C) $\boxed{\sqrt{2} - \sqrt{6}\,i}$

D) All of the above.

E) None of the above.

$$z = \sqrt{8}\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right] = \sqrt{8}\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = \boxed{\sqrt{2} - \sqrt{6}\,i = z}$$

#24) Find all solutions, real or not, to the following equation: $x^3 + 2x^2 + 2x = 0$

A) $\boxed{x \in \{0, -1 \pm i\}}$

B) $x \in \{0, 1 \pm i\}$

C) $x \in \{0, \pm 2i\}$

D) All of the above.

E) None of the above.

$$x^3 + 2x^2 + 2x = 0 \iff x(x^2 + 2x + 2) = 0$$

$$\iff \begin{cases} \boxed{x = 0} \\ x^2 + 2x + 2 = 0 \end{cases} \iff x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{-4}}{2} = \boxed{-1 \pm i = x}$$

#25) Expand: $\log_4(5x^3y)$

A) $3\log_4(5xy)$

B) $5\log_4(5x^3y)$

C) $\boxed{\log_4(5) + 3\log_4(x) + \log_4(y)}$

D) All of the above.

E) None of the above.

$$\log_4(5x^3y) = \log_4(5) + \log_4(x^3) + \log_4(y)$$
$$= \log_4(5) + 3\log_4(x) + \log_4(y)$$

#26) Simplify: $\frac{1}{3}\left[\log_2(x) + \log_2(x+1)\right]$

A) $\log_2(x^2+x)^3$

B) $\log_2\left[x^{1/3} + (x+1)^{1/3}\right]$

C) $\boxed{\log_2\left[\sqrt[3]{x(x+1)}\right]}$

D) All of the above.

E) None of the above.

$$\frac{1}{3}\left[\log_2(x) + \log_2(x+1)\right] = \frac{1}{3}\left[\log_2\left(x(x+1)\right)\right] = \log_2\left[\left(x(x+1)\right)^{1/3}\right] = \log_2\left[\sqrt[3]{x(x+1)}\right]$$

#27) Solve for x: $e^{2x} - 3e^x + 2 = 0$

A) $x = \ln(2)$

B) $x \in \{0, \sqrt{e}\}$

C) $\boxed{x \in \{0, \ln(2)\}}$

D) All of the above.

E) None of the above.

$$e^{2x} - 3e^x + 2 = 0 \iff (e^x)^2 - 3e^x + 2 = 0$$

$$\iff (e^x - 1)(e^x - 2) = 0 \iff \begin{cases} e^x = 1 \iff x = \ln(1) = 0 \\ e^x = 2 \iff x = \ln(2) \end{cases}$$

#28) Solve for x: $\log_3(5x-1) = \log_3(x+7)$

A) $x = \sqrt{2}$

B) $\boxed{x = 2}$

C) $x = \log_3(2)$

D) All of the above.

E) None of the above.

$$\log_3(5x-1) = \log_3(x+7) \iff 5x - 1 = x + 7$$

$$\iff 4x = 8 \iff \boxed{x = 2}$$

#29) Which of the following is a cube root of z = -2 + 2i

A) $\sqrt{2}\left[\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right]$

B) $1 + i$

C) $\sqrt{2}\left[\cos\left(\frac{19\pi}{12}\right) + i\sin\left(\frac{19\pi}{12}\right)\right]$

D) All of the above.

E) None of the above.

$$z = -2 + 2i = 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2\sqrt{2}\left[\cos(3\pi/4) + i\sin(3\pi/4)\right]$$

So, the three cube roots, z_K with $K=0, 1$, and 2, are given by

$$z_K = (2\sqrt{2})^{1/3}\left[\cos\left(\frac{3\pi/4}{3} + \frac{2\pi K}{3}\right) + i\sin\left(\frac{3\pi/4}{3} + \frac{2\pi K}{3}\right)\right]$$

$$= \sqrt{2}\left[\cos\left(\frac{\pi}{4} + \frac{2\pi K}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{2\pi K}{3}\right)\right]$$

Thus,

$$z_0 = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right] = \sqrt{2}\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right] = \boxed{1 + i = z_0}$$

$$z_1 = \sqrt{2}\left[\cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)\right] = \boxed{\sqrt{2}\left[\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right] = z_1}$$

$$z_2 = \sqrt{2}\left[\cos\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)\right] = \boxed{\sqrt{2}\left[\cos\left(\frac{19\pi}{12}\right) + i\sin\left(\frac{19\pi}{12}\right)\right] = z_2}$$

#30) If $z_1 = 24[\cos(300°) + i\sin(300°)]$ and $z_2 = 8[\cos(75°) + i\sin(75°)]$ then what is $\frac{z_1}{z_2}$?

A) $-\frac{3\sqrt{2}}{2}(1+i)$

B) $-3/2 - 3i/2$

C) $3[\cos(75°) + i\sin(75°)]$

D) All of the above.

E) None of the above.

$$\frac{z_1}{z_2} = \frac{24}{8}\left[\cos(300° - 75°) + i\sin(300° - 75°)\right] = 3\left[\cos(225°) + i\sin(225°)\right] = 3\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right]$$

$$= \boxed{-\frac{3\sqrt{2}}{2}(1+i) = \frac{z_1}{z_2}}$$