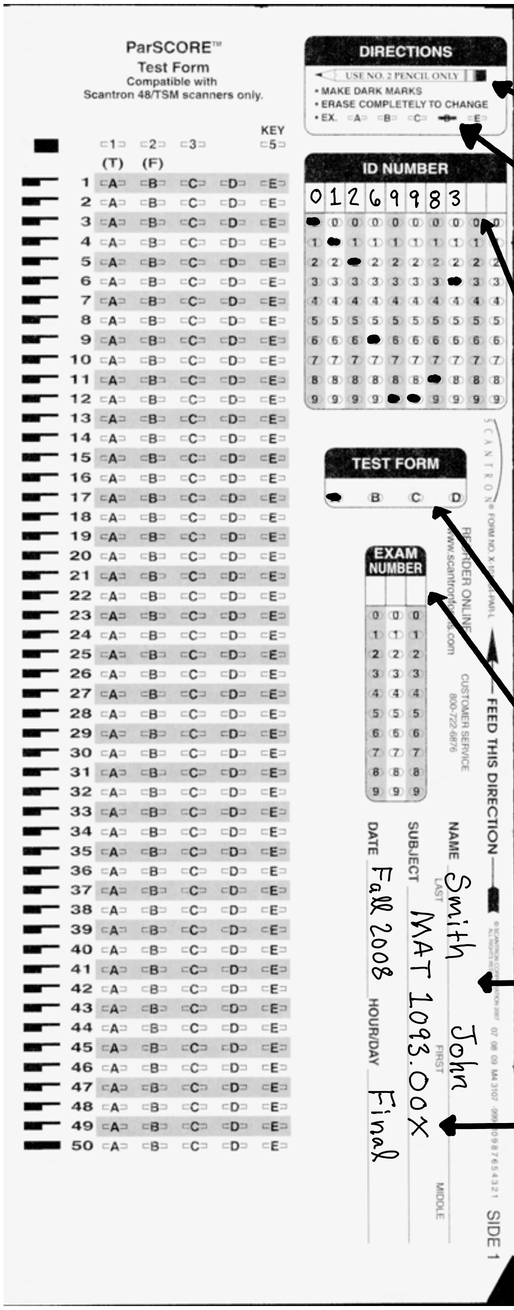
Precalculus **Spring 2009** **Exam 1** **Solutions**

Name__________________________________ Banner:______________________



Do write your name in the blank.
Do write your Banner in the blank.

Do use a #2 pencil.
Do **NOT** use anything but a #2 pencil.

Do fill in the entire rectangle to mark your answer.
Do erase errors completely. If your eraser leaves smudges, consider carefully copying your answers to a new parscore without smudges.

Do ignore the @ symbol if it is in your banner I.D.
Do write the **last 8 digits** of your banner I.D. here.
Do start entering digits at the far left.
Do fill in the appropriate oval below each digit.

Do fill in the oval for test form **A**.

Do **NOT** fill in an exam number.

Do write your last name, first name and middle name here.

Do replace the x with the appropriate number for the section that you are enrolled. Look at the chart below if you don't know your section.

Class Days	Time	1093.section
MWF	10am	1093.002
MWF	2pm	1093.003

Do circle your answers on this exam. **Do** fill in the corresponding bubble on your ParScore.

Do **NOT** use a calculator. Do **NOT** use a formula sheet.

Do cover your scratch work. **Do** cover your answers on your exam. **Do** cover your Parscore.

Do **NOT** cheat. Do **NOT** even appear to be cheating.

Do notify me if something is illegible. **Do** ask me to clarify if a question is ambiguous.

Do use the back of the exam pages for scratch work. **Do** feel free to unstaple the pages of the exam.

Grades will be available in WebCT when the Parscore Office finishes grading your exams.
I don't know when this will be so do **NOT** ask me.

#1) Which of the following is an identity?

A) $\cot(-x) = -1/\tan(x)$

B) $\tan(x + k\pi) = \tan(x)$; $k \in \mathbb{Z}$

C) $\sin(x) = \cos(x - \pi/2)$

D) All of the above

E) None of the above

A) True. $\cot(-x) = \dfrac{1}{\tan(-x)}$ Reciprocal Identity

$= \dfrac{1}{-\tan(x)}$ Symmetry Identity

B) True. $\tan(x + k\pi) = \tan(x)$ for any $x \in \mathbb{R}$ because tangent is π periodic

C) True. $\cos(x - \pi/2) = \cos[-(\pi/2 - x)]$

$= \cos(\pi/2 - x)$ Symmetry Identity

$= \sin(x)$ Cofunction Identity

#2) Which of the following is completely true?

A) $\sec(3/\pi) = \cos(\pi/3)$

B) $\boxed{\sin(13\pi/6) = \cos(7\pi/3)}$

C) $\tan(\theta) = 3/5 \Rightarrow \cos(\theta) = 3$

D) All of the above

E) None of the above

A) False. $\sec(3/\pi) = \sec\left[\left(\frac{3}{180}\right)^\circ\right] = \sec\left[\left(\frac{1}{60}\right)^\circ\right]$ since $\pi = 180^\circ$

$= \dfrac{1}{\cos[(1/60)^\circ]}$ Reciprocal Identity

$\neq \cos(\pi/3) = 1/2$

B) True. $\sin(13\pi/6) = \sin\left(\frac{12\pi}{6} + \frac{\pi}{6}\right) = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ since sine is 2π periodic

$\cos(7\pi/3) = \cos\left(\frac{6\pi}{3} + \frac{\pi}{3}\right) = \cos\left(2\pi + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ since cosine is 2π periodic

C) False. Given $\tan(\theta) = 3/5$

Consider using a Pythagorean and Reciprocal Identity:

$1 + \tan^2(\theta) = \sec^2(\theta) = 1/\cos^2(\theta)$

$\Rightarrow \cos^2(\theta) = \dfrac{1}{1+\tan^2(\theta)} = \dfrac{1}{1+(3/5)^2} = \dfrac{1}{34/25} = \dfrac{25}{34}$

$\Rightarrow \cos(\theta) = \pm\sqrt{\dfrac{25}{34}} = \pm\dfrac{5}{\sqrt{34}} \neq 3$

Of course $-1 \leq \cos(\theta) \leq 1$ so $\cos(\theta)$ could never be 3 anyways.

#3) Which of the following is completely true?

A) $\tan(\theta) = 10 \Rightarrow \sin(\theta) > 0$

B) $xa^2 + ya + z = 0 \Rightarrow a = \left(-y \pm \sqrt{y^2 - 4xz}\right)/2x$; $a, x, y, z \in \mathbb{R}$; $x \neq 0$

C) $360\pi^\circ = 2\pi$

D) All of the above

E) None of the above

A) False. Given $\tan(\theta) = 10 > 0$, θ could be in Q1 or Q4, but $\sin(\theta) > 0$ only when θ is in Q1 or Q2 and $\sin(\theta) < 0$ only when θ is in Q3 or Q4. So, if θ is in Q4, which it could be because $\tan(\theta) = 10 > 0$, then $\sin(\theta) < 0$. So, $\tan(\theta) = 10 \not\Rightarrow \sin(\theta) > 0$

B) True. Consider the Quadratic Formula: $ax^2+bx+c=0 \Leftrightarrow x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$
Just rename your values: $a \leftrightarrow x$, $b \to y$, $c \to z$ and you get
$xa^2 + ya + z = 0 \Leftrightarrow a = \left(-y \pm \sqrt{y^2 - 4xz}\right)/2x$
It is just the Quadratic Formula using different letters.

C) False. $360\pi^\circ = 360\pi^\circ\left(\dfrac{\pi}{180^\circ}\right) = 2\pi^2 \neq 2\pi$

#4) Which of the following is completely true?

A) $0 = \pi - \pi^3/3! + \pi^5/5! - \pi^7/7! + \cdots$

B) $2\cos(x) = 1 \Rightarrow x = \pm\pi/3$

C) $\cos^2(0.7684) + \sin^2(0.7684) = 1$

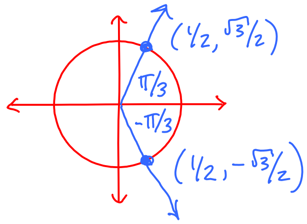
D) All of the above

E) None of the above

A) True. From the Power Series for sine: $\sin(x) = x - x^3/3! + x^5/5! - x^7/7! + \cdots$

We have, $0 = \sin(\pi) = \pi - \pi^3/3! + \pi^5/5! - \pi^7/7! + \cdots$

B) True. $2\cos(x) = 1 \Leftrightarrow \cos(x) = 1/2 \Rightarrow x = \pm\pi/3$



C) True. $\cos^2(x) + \sin^2(x) = 1$ is a Pythagorean Identity

$\Rightarrow \cos^2(0.7684) + \sin^2(0.7684) = 1$

#5) Which of the following is completely true?

A) The domain of $\tan(2x)$ is all $x \in \mathbb{R} \setminus \{(2K+1)\pi/2 ; K \in \mathbb{Z}\}$.

B) The domain of $1/\sin(x)$ is the same as the domain of $\cos(x)$.

C) The domain of $\sin(x)$ is the same as the range of $\cot(x)$.

D) All of the above

E) None of the above

A) False. You can try graphing $\tan(2x)$ and finding the values where it is undefined by looking for vertical asymptotes.

$\tan(2x)$ has period $T = \pi/\omega = \pi/2$ so vertical asymptotes will be a distance $\pi/2$ apart and,

$\tan(\pi/2) = \tan[2(\pi/4)]$ is undefined. So,

The domain of $\tan(2x)$ is all $x \in \mathbb{R} \setminus \{\pi/4 + (\pi/2)K ; K \in \mathbb{Z}\}$

$= x \in \mathbb{R} \setminus \{(2K+1)\pi/4 ; K \in \mathbb{Z}\} \neq x \in \mathbb{R} \setminus \{(2K+1)\pi/2 ; K \in \mathbb{Z}\}$.

In fact, $x \in \mathbb{R} \setminus \{(2K+1)\pi/2 ; K \in \mathbb{Z}\}$ is the domain of $\tan(x)$, not $\tan(2x)$.

B) False. The domain of $1/\sin(x)$ contains any real number except for when $\sin(x) = 0$ which happens when $x = \pi K ; K \in \mathbb{Z}$. But, the domain of $\cos(x)$ has no exceptions. The domain of $\cos(x)$ is $\mathbb{R}$.

C) True. The <u>domain</u> of $\sin(x)$ is $\mathbb{R}$ and the <u>range</u> of $\cot(x)$ is also $\mathbb{R}$.

#6) Which of the following is completely true?

A) $\sin(x) = \sqrt{3}/2 \Rightarrow \cos(x) = 1/2$

B) $\cos(x) = 1/2 \Rightarrow \sin(x) = \sqrt{3}/2$

C) $\tan(x) = 1 \Rightarrow \sin(x) = \sqrt{2}/2$

D) All of the above

E) None of the above

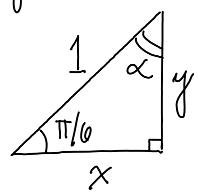
A) False. $\sin(2\pi/3) = \sqrt{3}/2$ but $\cos(2\pi/3) = -1/2 \neq 1/2$

B) False. $\cos(-\pi/3) = 1/2$ but $\sin(-\pi/3) = -\sqrt{3}/2 \neq \sqrt{3}/2$

C) False. $\tan(5\pi/4) = 1$ but $\sin(5\pi/4) = -\sqrt{2}/2 \neq \sqrt{2}/2$

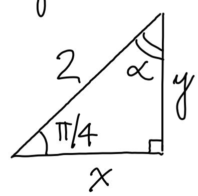
#7) Which of the following is completely true?

A) According to the following diagram (which is not necessarily drawn to scale),



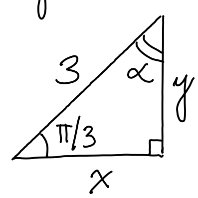
$\alpha = 30^\circ$, $x = 1/2$, and $y = \sqrt{3}/2$.

B) According to the following diagram (which is not necessarily drawn to scale),



$\alpha = 45^\circ$, $x = \sqrt{2}$, and $y = \sqrt{2}$.

C) According to the following diagram (which is not necessarily drawn to scale),

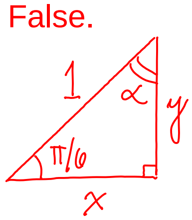


$\alpha = \pi/6$, $x = 3\sqrt{3}/2$, and $y = 3/2$.

D) All of the above

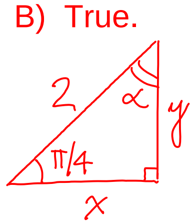
E) None of the above

A) False.



$y = y/1 = \sin(\pi/6) = 1/2 \neq \sqrt{3}/2$

B) True.

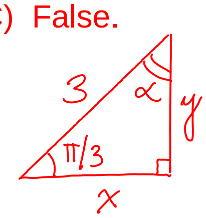


$y/2 = \sin(\pi/4) = \sqrt{2}/2 \Rightarrow y = \sqrt{2}$

$x/2 = \cos(\pi/4) = \sqrt{2}/2 \Rightarrow x = \sqrt{2}$

$\pi/4 + \alpha = \pi/2 \Rightarrow \alpha = \pi/2 - \pi/4 = \pi/4 = 45^\circ$

C) False.



$y/3 = \sin(\pi/3) = \sqrt{3}/2 \Rightarrow y = 3\sqrt{3}/2 \neq 3/2$

#8) Which of the following is completely true?

A) $\tan(510^\circ) = \cot(-\pi/6)$

B) $-\cos(570^\circ) = \sin(10\pi/3)$

C) $\boxed{\sin(405^\circ) = \cos(-\pi/4)}$

D) All of the above

E) None of the above

A) False. $\tan(510^\circ) = \tan(360^\circ + 150^\circ) = \tan(150^\circ) = \dfrac{1/2}{-\sqrt{3}/2} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

but $\cot(-\pi/6) = \dfrac{\sqrt{3}/2}{-1/2} = -\sqrt{3} \neq -\dfrac{\sqrt{3}}{3} = \tan(510^\circ)$

B) False. $-\cos(570^\circ) = -\cos(360^\circ + 210^\circ) = -\cos(210^\circ) = -(-\sqrt{3}/2) = \sqrt{3}/2$ but

$\sin(10\pi/3) = \sin(6\pi/3 + 4\pi/3) = \sin(2\pi + 4\pi/3) = \sin(4\pi/3) = -\sqrt{3}/2$

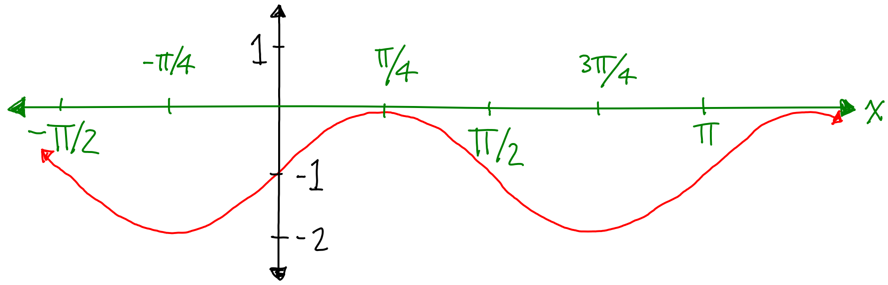
C) True. $\sin(405^\circ) = \sin(360^\circ + 45^\circ) = \sin(45^\circ) = \sqrt{2}/2 = \cos(-\pi/4)$

#9) Which of the following is completely true?

A) $\boxed{\sin(\pi/360)+\sin(2\pi/360)+\sin(3\pi/360)+\cdots+\sin(718\pi/360)+\sin(719\pi/360)+\sin(720\pi/360)=0}$

B) If $(6,3)$ is on the terminal side of an angle θ, then $\cos(\theta)=3/2$

C) The following is a graph of the function $y=2\sin(2x)-1$



D) All of the above

E) None of the above

A) True. Using the unit circle as a guide, we can tell by inspection that

$\sin(\pi/360)=-\sin(719\pi/360) \Rightarrow \sin(\pi/360)+\sin(719\pi/360)=0$

$\sin(2\pi/360)=-\sin(718\pi/360) \Rightarrow \sin(2\pi/360)+\sin(718\pi/360)=0$

$\vdots$

$\sin(359\pi/360)=-\sin(361\pi/360) \Rightarrow \sin(359\pi/360)+\sin(361\pi/360)=0$ and $\sin(720\pi/360)=0$

So, $\sin(\pi/360)+\sin(2\pi/360)+\sin(3\pi/360)+\cdots+\sin(718\pi/360)+\sin(719\pi/360)+\sin(720\pi/360)$

$$=\underbrace{\sin(\pi/360)+\sin(719\pi/360)}_{=0}+\underbrace{\sin(2\pi/360)+\sin(718\pi/360)}_{=0}+\cdots+\underbrace{\sin(359\pi/360)+\sin(361\pi/360)}_{=0}+\underbrace{\sin(720\pi/360)}_{=0}=0$$

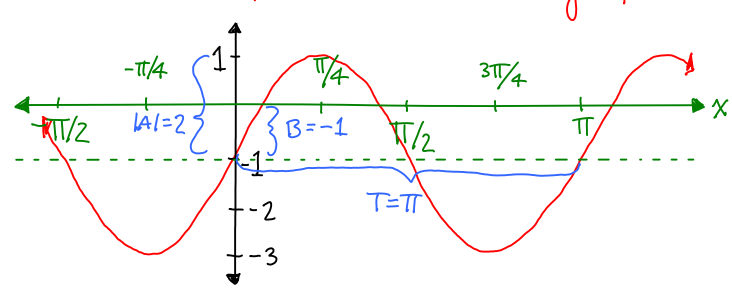
Draw a picture using the unit circle to convince yourself.

B) False. If $(6,3)$ is on the terminal side of an angle θ,

then $r=\sqrt{x^2+y^2}=\sqrt{6^2+3^2}=\sqrt{36+9}=\sqrt{45}=\sqrt{3\cdot3\cdot5}=3\sqrt{5}$

So, $\cos(\theta)=\dfrac{x}{r}=\dfrac{6}{3\sqrt{5}}=\dfrac{2}{\sqrt{5}}\neq\dfrac{3}{2}$

C) False. The graph of the function $y=2\sin(2x)-1$ is provided below. The graph has a period, $T=2\pi/\omega=2\pi/2=\pi$, has an amplitude $|A|=|2|=2$, has a vertical shift of -1, and has no horizontal shift.



#10) Which of the following is completely true?

A) The graph of y = sin(x) + 1 displays symmetry about the origin.

B) $\boxed{\text{The function } f(x) = \cos(x) + 1 \text{ has even symmetry.}}$

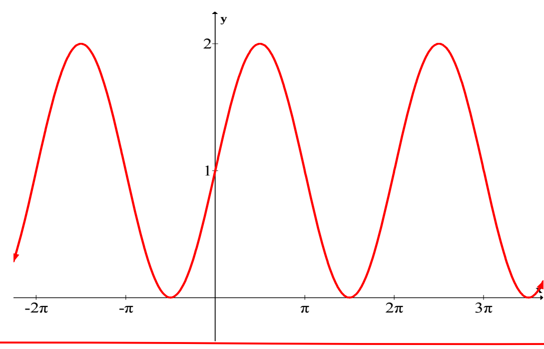
C) The function g(x) = 2cos(x) is antisymmetric.

D) All of the above

E) None of the above

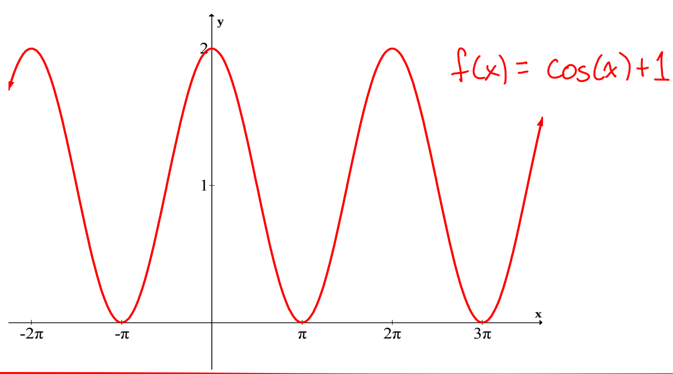
A) False. Below is the graph of $f(x) = \sin(x) + 1$. If you rotate it by 180° about the origin you will have a different graph. Another way to approach it is to notice that

$$f(-x) = \sin(-x) + 1 = -\sin(x) + 1 \neq -f(x) = -\sin(x) - 1$$



B) True. $f(x) = \cos(x) + 1$ has even symmetry. It's graph is symmetric about the y-axis, because if you reflect the graph over y-axis it will look the same. You can also tell that $f(x) = \cos(x) + 1$ has even symmetry because

$$f(-x) = \cos(-x) + 1 = \cos(x) + 1 = f(x)$$



C) False. $g(x) = 2\cos(x)$ is a symmetric function because

$$g(-x) = 2\cos(-x) = 2\cos(x) = g(x)$$

Or, you can tell by looking at it's graph which is symmetric about the y-axis.