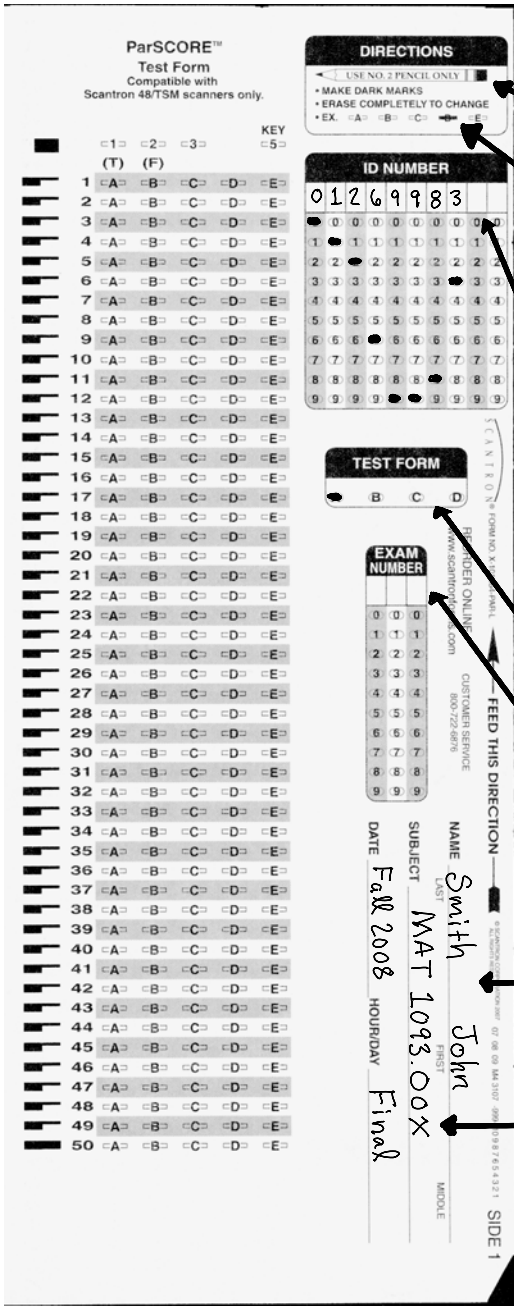
Precalculus **Spring 2009** **Exam 2 Solutions**

Name______________________________________ Banner:________________________



Do write your name in the blank.
Do write your Banner in the blank.

Do use a #2 pencil.
Do **NOT** use anything but a #2 pencil.

Do fill in the entire rectangle to mark your answer.
Do erase errors completely. If your eraser leaves smudges, consider carefully copying your answers to a new parscore without smudges.

Do ignore the @ symbol if it is in your banner I.D.
Do write the **last 8 digits** of your banner I.D. here.
Do start entering digits at the far left.
Do fill in the appropriate oval below each digit.

Do fill in the oval for test form **A**.

Do **NOT** fill in an exam number.

Do write your last name, first name and middle name here.

Do replace the x with the appropriate number for the section that you are enrolled. Look at the chart below if you don't know your section.

Class Days	Time	1093.section
MWF	10am	1093.002
MWF	2pm	1093.003

Do circle your answers on this exam. **Do** fill in the corresponding bubble on your ParScore.

Do **NOT** use a calculator. Do **NOT** use a formula sheet.

Do cover your scratch work. **Do** cover your answers on your exam. **Do** cover your Parscore.

Do **NOT** cheat. Do **NOT** even appear to be cheating.

Do notify me if something is illegible. **Do** ask me to clarify if a question is ambiguous.

Do use the back of the exam pages for scratch work. **Do** feel free to unstaple the pages of the exam.

Grades will be available in WebCT when the Parscore Office finishes grading your exams.
I don't know when this will be so do **NOT** ask me.

#1) $\sin(15\pi/6) = \sin(15\pi/6 - 2\pi) = \sin(15\pi/6 - 12\pi/6) = \sin(3\pi/6) = \sin(\pi/2) = 1$

A) $\sqrt{3}/2$ B) $1/2$ C) 0 D) 1 E) None of the above

#2) $\cos(13\pi/3) = \cos(13\pi/3 - 2 \cdot 2\pi) = \cos(13\pi/3 - 12\pi/3) = \cos(\pi/3) = 1/2$

A) $\sqrt{3}/2$ B) $1/2$ C) 0 D) 1 E) None of the above

#3) $\sin(585°) = \sin(585° - 360°) = \sin(225°) = -\sqrt{2}/2 = -1/\sqrt{2}$

A) $-\sqrt{3}/2$ B) $1/\sqrt{2}$ C) 0 D) -1 E) None of the above

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#4) $\tan(-33\pi/4) = \tan(-33\pi/4 + 8\pi) = \tan(-33\pi/4 + 32\pi/4) = \tan(-\pi/4) = -1$

A) $\sqrt{3}$ B) $-1/\sqrt{3}$ C) 0 D) 1 E) None of the above

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#5) $\sec(-225°) = \sec(-225° + 360°) = \sec(135°) = 1/(-1/\sqrt{2}) = -\sqrt{2}$

A) $\sqrt{2}$ B) $-1/\sqrt{2}$ C) 1 D) -1 E) None of the above

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#6) $\csc(2\pi/3) = 1/(\sqrt{3}/2) = 2/\sqrt{3}$

A) $2/\sqrt{3}$ B) -2 C) 0 D) $\sqrt{3}$ E) None of the above

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#7) $\cot(450°) = \cot(450° - 360°) = \cot(90°) = 0$

A) $5\pi/2$ B) $1/\sqrt{3}$ C) 0 D) $\pi/2$ E) None of the above

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#8) $\sin^2(7\pi) = \sin^2(7\pi - 3 \cdot 2\pi) = \sin^2(7\pi - 6\pi) = \sin^2(\pi) = (-1)^2 = 1$

A) $3/4$ B) $1/2$ C) -1 D) 1 E) None of the above

#9) $\tan(7\pi/4)\cot(7\pi/4) = (-1)(-1) = 1$

A) $-\sqrt{3}$ B) 2 C) -1 D) 1 E) None of the above

#10) $\cos^2(17\pi/9) + \sin^2(17\pi/9) = 1$ since $\cos^2(\theta) + \sin^2(\theta) \equiv 1$

A) $\sqrt{3}/2$ B) $1/2$ C) 0 D) 1 E) None of the above

#11) $\cos(\arcsin(\sqrt{2}/2)) = \cos(\pi/4) = \sqrt{2}/2 = 1/\sqrt{2}$

A) $\sqrt{2}/2$ B) $1/2$ C) $\pi/4$ D) $-\sqrt{2}/2$ E) None of the above

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#12) $\sin(\arccos(1/\sqrt{2})) = \sin(\pi/4) = \sqrt{2}/2 = 1/\sqrt{2}$

A) $\sqrt{3}/2$ B) $\pi/4$ C) $1/\sqrt{2}$ D) $-1/\sqrt{2}$ E) None of the above

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#13) $\cot(\arctan(500)) =$ $\cot(\theta)$ where $\theta = \arctan(500) \Rightarrow \tan(\theta) = 500 \Rightarrow \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{500}$

A) $-1/500$ B) 500 C) -500 D) $1/500$ E) None of the above

#14) $\tan(\sin^{-1}(0.7)) =$

A) 0.7 B) -0.51 C) -0.7 D) 7 E) None of the above

$\tan(\sin^{-1}(0.7)) = \tan(\theta)$ where $\theta = \sin^{-1}(0.7) \in (0, \pi/2) \Rightarrow \sin(\theta) = 0.7$ and $\tan(\theta) > 0$

$\sin(\theta) = 0.7 = y/r$; let $y = 0.7$ and $r = 1$, then $x^2 + y^2 = r^2 \Rightarrow x = \pm\sqrt{r^2 - y^2} = \pm\sqrt{1 - 0.7^2} = \pm\sqrt{1 - 0.49} = \pm\sqrt{0.51}$

$\tan(\theta) = y/x = 0.7/\sqrt{0.51}$

#15) $\cos(\tan^{-1}(0.5)) =$

A) $\sqrt{2}/5$ B) $-5/\sqrt{2}$ C) $\pi/4$ D) $-1/2$ E) None of the above

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$\cos(\tan^{-1}(0.5)) = \cos(\theta)$ where $\theta = \tan^{-1}(0.5) \in (0, \pi/2) \Rightarrow \tan(\theta) = 0.5$ and $\cos(\theta) > 0$

$\tan(\theta) = 0.5 = y/x$; let $y = 0.5$ and $x = 1$, then $r^2 = x^2 + y^2 \Rightarrow r = \pm\sqrt{x^2 + y^2} = \pm\sqrt{1 + 0.5^2} = \pm\sqrt{1 + 0.25} = \pm\sqrt{1.25} = \pm\sqrt{5/4} = \pm\sqrt{5}/2$

$\Rightarrow \cos(\theta) = x/r = 1/(\sqrt{5}/2) = 2/\sqrt{5}$

#16) $\arcsin(\cos(3\pi/4)) = \arcsin(-\sqrt{2}/2) = -\pi/4$

A) $\sqrt{2}/2$ B) $3\pi/4$ C) 0 D) $\pi/4$ E) None of the above

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#17) $\arcsin(\sin(3\pi/4)) = \arcsin(\sqrt{2}/2) = \pi/4$

A) $\sqrt{2}/2$ B) $3\pi/4$ C) 0 D) $\pi/4$ E) None of the above

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#18) $\tan^{-1}(\tan(405^\circ)) = \tan^{-1}(\tan(405^\circ - 360^\circ)) = \tan^{-1}(\tan(45^\circ)) = 45^\circ = \pi/4$

A) 405° B) $9\pi/4$ C) -1 D) 1 E) None of the above

#19) $\sin(\arcsin(2)) = \sin(\text{undefined}) = \text{undefined}$; The domain of arcsine is $[-1,1]$.

A) $-\sqrt{3}/2$ B) 2 C) -1 D) 1 E) None of the above

#20) $\arcsin(\sin(2)) = 2$ since 2 is within the range of arcsine which is $[-\pi/2, \pi/2]$; $-\pi/2 < 2 < \pi/2$

A) $\sqrt{3}/2$ B) $1/2$ C) 2 D) 1 E) None of the above

#21) Which of the following is an identity?

A) $\tan(p)\cot(p) = 1$

B) $\tan^2(q) - 1 = \sec^2(q)$

C) $\cos(2a+2b) = 2\cos(a+b)$

D) All of the above

E) None of the above

A) True. $\tan(p)\cot(p) = \tan(p) \cdot \frac{1}{\tan(p)} = \frac{\tan(p)}{\tan(p)} = 1$

B) False. There is a Pythagorean Identity that looks similar, $\tan^2(q) + 1 \equiv \sec^2(q)$, but $\tan^2(q) - 1 = \sec^2(q)$ has a minus instead of a plus.

$\sec^2(q)$ cannot equal both $\tan^2(q) - 1$ and $\tan^2(q) + 1$ but we know $\sec^2(q) \equiv \tan^2(q) + 1$ is true so $\sec^2(q) \not\equiv \tan^2(q) - 1$.

Or, you just plug in a value for q to show this isn't an identity. Try $q = 0$:

$\sec^2(0) = 1/\cos^2(0) = 1/1 = 1$ but $\tan^2(0) - 1 = 0 - 1 = -1$

C) False. Consider $a = 0$ and $b = \pi$. Then

$\cos(2a+2b) \equiv 2\cos(a+b) \Rightarrow \cos(2\pi) = 2\cos(\pi) \Rightarrow 1 = 2(-1) \Rightarrow 1 = -2$

which is <u>Not</u> true so $\cos(2a+2b) \not\equiv 2\cos(a+b)$.

#22) Which of the following is an identity?

A) $\tan^2(t)+1 = \csc^2(t)$

B) $\sin(2a+2b) = 2\cos(a+b)\sin(a+b)$

C) $\cot^2(t)-1 = \sec^2(t)$

D) All of the above

E) None of the above

A) False. $\tan^2(t)+1=\csc^2(t)$ cannot be an identity because when $t=\pi/3$ we have

$\tan^2(\pi/3)+1 = (\sqrt{3})^2+1 = 4 \neq \csc^2(\pi/3) = [1/(\sqrt{3}/2)]^2 = 4/3$

B) True. Using the Double Angle Identity for $\sin\theta$: $\sin(2\theta) \equiv 2\cos(\theta)\sin(\theta)$ we have,

$\sin(2a+2b) \equiv \sin[2(a+b)] \equiv 2\cos(a+b)\sin(a+b)$

C) False. $\cot^2(t)-1 = \sec^2(t)$ cannot be an identity because when $t=\pi/3$ we have

$\cot^2(\pi/3)-1 = (1/\sqrt{3})^2-1 = 1/3-1 = -2/3 \neq \sec^2(\pi/3) = [1/(1/2)]^2 = 2^2 = 4$

#23) Which of the following is an identity?

A) $\sin(p+q) = \sin(p)\sin(q) + \cos(p)\cos(q)$

B) $p\sin(q) = \sin(2pq)$

C) $\tan(a+b)/\cot(a+b) = \tan^2(a+b)$

D) All of the above

E) None of the above

A) False. $\sin(p+q) = \sin(p)\sin(q) + \cos(p)\cos(q)$ cannot be an identity because when $p=q=0$ we have $\sin(p+q) = \sin(0+0) = \sin(0) = 0$ but $\sin(p)\sin(q) + \cos(p)\cos(q) = \sin(0)\sin(0) + \cos(0)\cos(0) = 0\cdot 0 + 1\cdot 1 = 1$

B) False. $p\sin(q) = \sin(2pq)$ cannot be an identity because when $p=2$ and $q=\pi/4$ we have $p\sin(q) = 2\sin(\pi/4) = 2(\sqrt{2}/2) = \sqrt{2} \neq \sin(2pq) = \sin(2\cdot 2\cdot \pi/4) = \sin(\pi) = -1$

C) True. $\tan(a+b)/\cot(a+b) \equiv \dfrac{\tan(a+b)}{1/\tan(a+b)} \equiv \tan(a+b)\cdot\tan(a+b) \equiv \tan^2(a+b)$

#24) Which of the following is an identity?

A) $\sin(\arccos(x)) = \sqrt{1-x^2}$

B) $\cos(\arcsin(x)) = \sqrt{1-x^2}$

C) $\sec^2(\arctan(x)) = 1+x^2$

D) All of the above

E) None of the above

A) True. $\sin(\arccos(x)) = \sin(\theta)$ where $\theta = \arccos(x) \in [0,\pi] \Rightarrow \cos(\theta) = x$ and $\sin(\theta) \geq 0$.

$\cos^2(\theta) + \sin^2(\theta) \equiv 1 \Rightarrow \sin(\theta) = \sqrt{1-\cos^2(\theta)} = \boxed{\sqrt{1-x^2} = \sin(\arccos(x))}$

B) True. $\cos(\arcsin(x)) = \cos(\theta)$ where $\theta = \arcsin(x) \in [-\pi/2, \pi/2] \Rightarrow \sin(\theta) = x$ and $\cos(\theta) \geq 0$.

$\cos^2(\theta) + \sin^2(\theta) \equiv 1 \Rightarrow \cos(\theta) = \sqrt{1-\sin^2(\theta)} = \boxed{\sqrt{1-x^2} = \cos(\arcsin(x))}$

C) True. $\sec^2(\arctan(x)) = \sec^2(\theta)$ where $\theta = \arctan(x) \in (-\pi/2, \pi/2) \Rightarrow \tan(\theta) = x$

Using the Pythagorean Identity $1+\tan^2(\theta) \equiv \sec^2(\theta)$, we have

$\sec^2(\theta) \equiv 1+\tan^2(\theta) = \boxed{1+x^2 = \sec^2(\arctan(x))}$

#25) Which of the following is completely true?

A) $\cos(5\pi/12) = (\sqrt{6} - \sqrt{2})/4$

B) $\sin(75°) = (\sqrt{6} + \sqrt{2})/4$

C) $\cos(75°) = \sqrt{2 - \sqrt{3}}/2$

D) All of the above

E) None of the above

A) True. $\cos(5\pi/12) = \cos(\pi/4 + \pi/6)$

$= \cos(\pi/4)\cos(\pi/6) - \sin(\pi/4)\sin(\pi/6)$ (Sum Identity)

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

B) True. $\sin(75°) = \sin(45° + 30°)$

$= \sin(45°)\cos(30°) + \cos(45°)\sin(30°)$ (Sum Identity)

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

C) True. $\cos(75°) = \cos(5\pi/12) = \frac{\sqrt{6} - \sqrt{2}}{4}$ from part A, but $\frac{\sqrt{6} - \sqrt{2}}{4} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

which can be seen by squaring both sides of the equality:

$\frac{\sqrt{6} - \sqrt{2}}{4} = \frac{\sqrt{2 - \sqrt{3}}}{2} \Leftrightarrow \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2 = \left(\frac{\sqrt{2 - \sqrt{3}}}{2}\right)^2 \Leftrightarrow \frac{(\sqrt{6} - \sqrt{2})(\sqrt{6} - \sqrt{2})}{16} = \frac{2 - \sqrt{3}}{4}$

$\Leftrightarrow \frac{(\sqrt{6})^2 - 2\sqrt{6}\sqrt{2} + (\sqrt{2})^2}{16} = \frac{2 - \sqrt{3}}{4} \Leftrightarrow \frac{6 - 2\sqrt{12} + 2}{16} = \frac{2 - \sqrt{3}}{4}$

$\Leftrightarrow \frac{8 - 4\sqrt{3}}{16} = \frac{2 - \sqrt{3}}{4} \Leftrightarrow \frac{2 - \sqrt{3}}{4} = \frac{2 - \sqrt{3}}{4}$

Or, instead of using the Sum Identity for Cosine as in Part A, we could have used the Half Angle Identity for Cosine: $\cos(\theta/2) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$

$\Rightarrow \cos(75°) = \sqrt{\frac{1 + \cos(150°)}{2}} = \sqrt{\frac{1 + (-\sqrt{3}/2)}{2}} = \sqrt{\frac{1}{2} - \frac{\sqrt{3}/2}{2}} = \sqrt{\frac{2}{4} - \frac{\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2} = \cos(75°)}$