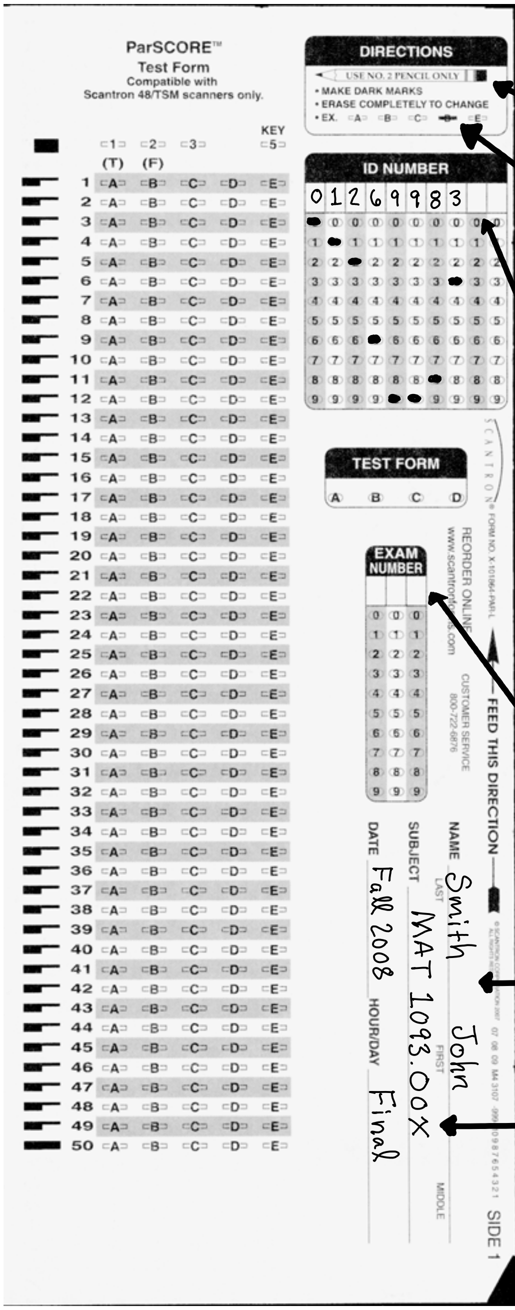
Precalculus **Spring 2009** **Final Exam** **Solutions**

Name____ Banner:_________________________

Do write your name in the blank.
Do write your Banner in the blank.



Do use a #2 pencil.
Do **NOT** use anything but a #2 pencil.

Do fill in the entire rectangle to mark your answer.
Do erase errors completely. If your eraser leaves smudges, consider carefully copying your answers to a new parscore without smudges.

Do ignore the @ symbol if it is in your banner I.D.
Do write the **last 8 digits** of your banner I.D. here.
Do start entering digits at the far left.
Do fill in the appropriate oval below each digit.

Do **NOT** fill in an exam number.

Do write your last name, first name and middle name here.

Do replace the x with the appropriate number for the section that you are enrolled. Look at the chart below if you don't know your section.

Class Days	Time	1093.section
MWF	10am	1093.002
MWF	2pm	1093.003

Do circle your answers on this exam. **Do** fill in the corresponding bubble on your ParScore.

Do **NOT** use a calculator. Do **NOT** use a formula sheet.

Do cover your scratch work. **Do** cover your answers on your exam. **Do** cover your Parscore.

Do **NOT** cheat. Do **NOT** even appear to be cheating.

Do notify me if something is illegible. **Do** ask me to clarify if a question is ambiguous.

Do use the back of the exam pages for scratch work. **Do** feel free to unstaple the pages of the exam.

Grades will be available in WebCT when the Parscore Office finishes grading your exams.
I don't know when this will be so do **NOT** ask me.

#1) $\sin(-17\pi/6) = -\sin(17\pi/6) = -\sin(17\pi/6 - 2\pi) = -\sin(17\pi/6 - 12\pi/6) = -\sin(5\pi/6) = -1/2$

A) $\sqrt{3}/2$ B) $-1/2$ C) 0 D) 1 E) None of the above

#2) $\cos(-11\pi/3) = \cos(11\pi/3) = \cos(11\pi/3 - 2\cdot 2\pi) = \cos(11\pi/3 - 12\pi/3) = \cos(-\pi/3) = 1/2$

A) $\sqrt{3}/2$ B) $1/2$ C) 0 D) 1 E) None of the above

#3) $\sin(-570°) = -\sin(570°) = -\sin(570° - 360°) = -\sin(210°) = -(-1/2) = 1/2$

A) $-\sqrt{3}/2$ B) $1/2$ C) 0 D) -1 E) None of the above

#4) $\tan(-311\pi/4) = -\tan(311\pi/4 - 77\pi) = -\tan(311\pi/4 - 308\pi/4) = -\tan(3\pi/4) = 1$

A) $\sqrt{3}$ B) $-1/\sqrt{3}$ C) 0 D) 1 E) None of the above

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#5) $\sec(-930°) = \sec(930°) = \sec(930° - 2\cdot 360°) = \sec(210°) = 1/(-\sqrt{3}/2) = -2/\sqrt{3} = -2\sqrt{3}/3$

A) $\sqrt{2}$ B) $-2\sqrt{3}/3$ C) 2 D) -1 E) None of the above

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#6) $\csc(7\pi/12) = 4/(\sqrt{6}+\sqrt{2}) = 2/\sqrt{2+\sqrt{3}}$

A) $2/\sqrt{3}$ B) 2 C) $4/(\sqrt{6}+\sqrt{2})$ D) $\sqrt{3}$ E) None of the above

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#7) $\cot(600°) = \cot(600° - 360°) = \cot(240°) = (-1/2)/(-\sqrt{3}/2) = 1/\sqrt{3}$

A) $5\pi/2$ B) $1/\sqrt{3}$ C) 0 D) $\pi/2$ E) None of the above

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#8) $\sin^4(17\pi/3) = \sin^4(17\pi/3 - 3\cdot 2\pi) = \sin^4(17\pi/3 - 18\pi/3) = \sin^4(-\pi/3) = (-\sqrt{3}/2)^4 = 9/16$

A) $3/4$ B) $1/4$ C) $9/16$ D) $8+2\sqrt{2}$ E) None of the above

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#9) $\sec(11\pi/4)\csc(7\pi/4) = \sec(11\pi/4 - 2\pi)\csc(7\pi/4 - 2\pi) = \sec(3\pi/4)\csc(-\pi/4) = (-\sqrt{2})(-\sqrt{2}) = 2$

A) $-\sqrt{3}$ B) 2 C) -1 D) 1 E) None of the above

#10) $\cos^4(\pi/9) + \sin^4(19\pi/9) + 2\cos^2(\pi/9)\sin^2(19\pi/9) =$

A) $\sqrt{3}/2$ B) $1/2$ C) 0 D) 1 E) None of the above

$\cos^4(\pi/9) + \sin^4(19\pi/9) + 2\cos^2(\pi/9)\sin^2(19\pi/9) = [\cos^2(\pi/9) + \sin^2(19\pi/9)]^2$

$= [\cos^2(\pi/9) + \sin^2(19\pi/9 - 2\pi)]^2 = [\cos^2(\pi/9) + \sin^2(\pi/9)]^2 = 1^2 = 1$ since $\cos^2(\theta) + \sin^2(\theta) \equiv 1$

#11) $\cot(\arcsin(-\sqrt{3}/2)) = \cot(-\pi/3) = (1/2)/(-\sqrt{3}/2) = -1/\sqrt{3} = -\sqrt{3}/3$

A) $\sqrt{3}/3$ B) $1/2$ C) $-\pi/3$ D) $-\sqrt{3}/3$ E) None of the above

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#12) $\csc(\arccos(-1/\sqrt{2})) = \csc(3\pi/4) = \sqrt{2}$

A) $\sqrt{3}/2$ B) $\pi/4$ C) $1/\sqrt{2}$ D) $\sqrt{2}$ E) None of the above

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#13) $\cot(\arctan(\pi^\circ)) = 1/\tan(\arctan(\pi^\circ)) = 1/\pi^\circ$ since $\pi^\circ \neq \pi/2 + \pi K;\ K \in \mathbb{Z}$

A) $-1/\pi$ B) π° C) $1/\pi$ D) $1/\pi^\circ$ E) None of the above

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#14) $\sin(\tan^{-1}(-2/3)) =$

A) $-2/\sqrt{13}$ B) $\sqrt{5}/2$ C) $-2/\sqrt{5}$ D) $-2/\sqrt{11}$ E) None of the above

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$\sin(\tan^{-1}(-2/3)) = \sin(\theta)$ where $\theta = \tan^{-1}(-2/3) \in (-\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow \tan(\theta) = -2/3$ and $\sin(\theta) < 0$

$\tan(\theta) = -2/3 = y/x$; let $y = -2$ and $x = 3$, then $r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2+y^2} = \sqrt{3^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

$\Rightarrow \sin(\theta) = y/r = -2/\sqrt{13}$

#15) $\cot(\tan^{-1}(0.75)) = \cot(\tan^{-1}(3/4)) = \cot(\theta)$ where $\theta = \tan^{-1}(3/4) \Rightarrow \tan(\theta) = 3/4 \Rightarrow \cot(\theta) = 4/3$

A) $5/4$ B) $4/3$ C) $3/4$ D) $4/5$ E) None of the above

#16) $\arcsin(\cos(-17\pi/4)) = \arcsin(\cos(-17\pi/4 + 2\cdot 2\pi)) = \arcsin(\cos(-\pi/4)) = \arcsin(\sqrt{2}/2) = \pi/4$

A) $\sqrt{2}/2$ B) $3\pi/4$ C) 0 D) $\pi/4$ E) None of the above

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#17) $\arcsin(\sin(-13\pi/4)) = \arcsin(\sin(-13\pi/4 + 2\cdot 2\pi)) = \arcsin(\sin(3\pi/4)) = \arcsin(\sqrt{2}/2) = \pi/4$

A) $\sqrt{2}/2$ B) $3\pi/4$ C) 0 D) $\pi/4$ E) None of the above

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#18) $\tan^{-1}[\sec(420^\circ)\sin(60^\circ)] =$

A) 420° B) $8\pi/3$ C) 45° D) $\pi/3$ E) None of the above

$\tan^{-1}[\sec(420^\circ)\sin(60^\circ)] = \tan^{-1}[\sec(420^\circ - 360^\circ)\sin(60^\circ)] = \tan^{-1}[\sec(60^\circ)\sin(60^\circ)]$

$$= \tan^{-1}\left[\frac{\sin(60^\circ)}{\cos(60^\circ)}\right] = \tan^{-1}[\tan(60^\circ)] = 60^\circ = \pi/3$$

#19) $\sin(\arcsin(1-\pi)) = \sin(\arcsin(1-3.14...)) = \sin(\arcsin(-2.14...)) = \sin(\text{undefined}) = \text{undefined}$

A) $-\sqrt{3}/2$ B) $1-\pi$ C) $(1+\pi)/(1-\pi)$ D) 1 E) None of the above

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#20) $\arcsin(\sin(3-\pi)) = \arcsin(\sin(3-3.14...)) = \arcsin(\sin(-0.14...)) = -0.14... = 3-\pi$

since $-\pi/2 < 3-\pi < \pi/2$

A) 3 B) $\pi-3$ C) $\pi+3$ D) $3-\pi$ E) None of the above

#21) Solve for x: $\sin^2(x) + 1 = -\cos^2(x)$

A) $x \in \mathbb{R}$

B) $x = \pm 1$

C) $x = \pi/2 + 2\pi K;\ K \in \mathbb{Z}$

D) $x = 0$

E) None of the above

$\sin^2(x) + 1 = -\cos^2(x) \Rightarrow \sin^2(x) + \cos^2(x) = -1 \Rightarrow 1 = -1 \Rightarrow$ No solutions

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#22) Solve for x: $2\cos(2\pi x) = 1$

A) $x = \pm\pi/3$

B) $x = \pm\pi/6 + \pi K/2;\ K \in \mathbb{Z}$

C) $x = \pm 1/6 + K;\ K \in \mathbb{Z}$

D) $x = \pm\pi/3 + \pi K;\ K \in \mathbb{Z}$

E) None of the above

$2\cos(2\pi x) = 1 \Leftrightarrow \cos(2\pi x) = 1/2 \Leftrightarrow 2\pi x = \pm\pi/3 + 2\pi K \Leftrightarrow x = \pm 1/6 + K;\ K \in \mathbb{Z}$

#23) Solve for θ: $\cos(8\theta) - 8\sin^2(2\theta) + 1 = 0$

A) $\theta = \pm\pi/3$

B) $\theta = \pi K/2 ;\ K \in \mathbb{Z}$

C) $\theta = \pm\pi/6 + K ;\ K \in \mathbb{Z}$

D) $\theta = \pm\pi/3 + \pi K ;\ K \in \mathbb{Z}$

E) None of the above

$$\cos(8\theta) - 8\sin^2(2\theta) + 1 = 0$$

$$\Leftrightarrow -8\sin^2(2\theta) + \cos(8\theta) + 1 = 0$$

$$\Leftrightarrow -8\sin^2(2\theta) + [1 - 2\sin^2(4\theta)] + 1 = 0 \qquad \text{since } \cos(2x) \equiv 1 - 2\sin^2(x)$$

$$\Leftrightarrow -8\sin^2(2\theta) + 1 - 2\sin^2(4\theta) + 1 = 0$$

$$\Leftrightarrow -8\sin^2(2\theta) + 1 - 2[1 - 2\sin^2(2\theta)]^2 + 1 = 0 \qquad \text{since } \cos(2x) \equiv 1 - 2\sin^2(x)$$

$$\Leftrightarrow -8\sin^2(2\theta) + 2 - 2[1 - 4\sin^2(2\theta) + 4\sin^4(2\theta)] = 0$$

$$\Leftrightarrow \cancel{-8\sin^2(2\theta)} + 2 - 2 + \cancel{8\sin^2(2\theta)} - 8\sin^4(2\theta) = 0$$

$$\Leftrightarrow -8\sin^4(2\theta) = 0$$

$$\Leftrightarrow \sin^4(2\theta) = 0$$

$$\Leftrightarrow \sin(2\theta) = 0 \Leftrightarrow 2\theta = \pi K \Leftrightarrow \theta = \pi K/2 ;\ K \in \mathbb{Z}$$

#24) Solve for x: $\sin(x+\pi/8)\cos(x+15\pi/8) = -\sin(x+15\pi/8)\cos(-x-\pi/8)$

A) $x \in \{\pi/4 + 2\pi K, 3\pi/4 + 2\pi K;\ K \in \mathbb{Z}\}$

B) $x = \pi K;\ K \in \mathbb{Z}$

C) $x = \pm 7\pi/8 + \pi K/2;\ K \in \mathbb{Z}$

D) $x = \pi K/2 \quad ;\ K \in \mathbb{Z}$

E) None of the above

$\sin(x+\pi/8)\cos(x+15\pi/8) = -\sin(x+15\pi/8)\cos(-x-\pi/8)$

$\Leftrightarrow \sin(x+\pi/8)\cos(x+15\pi/8) + \sin(x+15\pi/8)\cos(-x-\pi/8) = 0$

$\Leftrightarrow \sin(x+\pi/8)\cos(x+15\pi/8) + \cos[-(x+\pi/8)]\sin(x+15\pi/8) = 0$

$\Leftrightarrow \sin(x+\pi/8)\cos(x+15\pi/8) + \cos(x+\pi/8)\sin(x+15\pi/8) = 0$ since $\cos(-\theta) \equiv \cos(\theta)$

$\Leftrightarrow \sin[(x+\pi/8) + (x+15\pi/8)] = 0$ since $\sin(\alpha+\beta) \equiv \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\Leftrightarrow \sin(2x + 16\pi/8) = 0$

$\Leftrightarrow \sin(2x + 2\pi) = 0$

$\Leftrightarrow \sin(2x) = 0$ since $\sin(\theta) \equiv \sin(\theta + 2\pi K)$ $K \in \mathbb{Z}$

$\Leftrightarrow 2x = \pi K;\ K \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{2}K;\ K \in \mathbb{Z}$

#25) Solve for x: $3\sin^2(x) - \cos^2(x) = 1$

A) $x \in \{\pi/4 + 2\pi K, 3\pi/4 + 2\pi K;\ K \in \mathbb{Z}\}$

B) $x = 2\pi K;\ K \in \mathbb{Z}$

C) $x = \pi/4 + \pi K/2;\ K \in \mathbb{Z}$

D) $x = \pi/8 + \pi K/2;\ K \in \mathbb{Z}$

E) None of the above

$$3\sin^2(x) - \cos^2(x) = 1$$

$$\Leftrightarrow 2\sin^2(x) - 1 + \sin^2(x) - \cos^2(x) = 0$$

$$\Leftrightarrow 2\sin^2(x) - 1 - [\cos^2(x) - \sin^2(x)] = 0$$

$$\Leftrightarrow -\cos(2x) - \cos(2x) = 0$$

$$\Leftrightarrow -2\cos(2x) = 0$$

$$\Leftrightarrow \cos(2x) = 0$$

$$\Leftrightarrow 2x = \pi/2 + \pi K$$

$$\Leftrightarrow x = \pi/4 + \pi K/2;\ K \in \mathbb{Z}$$

#26) Solve for x: $4\cos^2(4x) - 1 = 0$

A) $x \in \{\pi/4 + 2\pi K, 3\pi/4 + 2\pi K; K \in \mathbb{Z}\}$

B) $x = 2\pi K; K \in \mathbb{Z}$

C) $x \in \{\pm\pi/12 + \pi K/2, \pm\pi/6 + \pi K/2; K \in \mathbb{Z}\}$

D) $x = \pi/8 + \pi K/2; K \in \mathbb{Z}$

E) None of the above

$$4\cos^2(4x) - 1 = 0$$

$$\Leftrightarrow [2\cos(4x) - 1][2\cos(4x) + 1] = 0$$

$$\Leftrightarrow \left\{\begin{matrix} 2\cos(4x) - 1 = 0 \\ 2\cos(4x) + 1 = 0 \end{matrix}\right\} \Leftrightarrow \cos(4x) = \pm 1/2 \Leftrightarrow 4x \in \{\pm\pi/3 + 2\pi K, \pm 2\pi/3 + 2\pi K; K \in \mathbb{Z}\}$$

$$\Leftrightarrow x \in \{\pm\pi/12 + \pi K/2, \pm\pi/6 + \pi K/2; K \in \mathbb{Z}\}$$

#27) Solve for x: $\cos^4(4x) - \sin^4(4x) = \cos^4(2x) - \sin^4(2x)$

A) $x \in \{\pi k/4, \pm \pi/6 + \pi k/2;\ k \in \mathbb{Z}\}$

B) $x \in \{\pi/4 + 2\pi k, 3\pi/4 + 2\pi k;\ k \in \mathbb{Z}\}$

C) $x = \pi/4 + \pi k/2;\ k \in \mathbb{Z}$

D) $x = \pi/8 + \pi k/2;\ k \in \mathbb{Z}$

E) None of the above

$$\cos^4(4x) - \sin^4(4x) = \cos^4(2x) - \sin^4(2x)$$

$$\Leftrightarrow [\cos^2(4x) - \sin^2(4x)][\cos^2(4x) + \sin^2(4x)] = [\cos^2(2x) - \sin^2(2x)][\cos^2(2x) + \sin^2(2x)]$$

$$\Leftrightarrow [\cos^2(4x) - \sin^2(4x)](1) = [\cos^2(2x) - \sin^2(2x)](1)$$

$$\Leftrightarrow \cos^2(4x) - \sin^2(4x) - [\cos^2(2x) - \sin^2(2x)] = 0$$

$$\Leftrightarrow \cos^2(4x) - (1 - \cos^2(4x)) - \cos(4x) = 0$$

$$\Leftrightarrow 2\cos^2(4x) - \cos(4x) - 1 = 0$$

$$\Leftrightarrow \cos(4x) = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} = \frac{1 \pm 3}{4}$$

$$\Leftrightarrow \begin{cases} \cos(4x) = 1/4 + 3/4 = 1 \Leftrightarrow 4x = \pi k \Leftrightarrow x = \pi k/4 \\ \cos(4x) = 1/4 - 3/4 = -1/2 \Leftrightarrow 4x = \pm 2\pi/3 + 2\pi k \Leftrightarrow x = \pm \pi/6 + \pi k/2 \end{cases}$$

$$\Leftrightarrow x \in \{\pi k/4, \pm \pi/6 + \pi k/2;\ k \in \mathbb{Z}\}$$

#28) Solve for x: $\sqrt{\cos(x^2+2x+1)} = -1$

A) $x \in \{\pi/4 + 2\pi K, 3\pi/4 + 2\pi K; K \in \mathbb{Z}\}$

B) $x = 2\pi K; K \in \mathbb{Z}$

C) $x = \pi/4 + \pi K/2; K \in \mathbb{Z}$

D) $x = \pi/8 + \pi K/2; K \in \mathbb{Z}$

E) None of the above

$$\sqrt{\cos(x^2+2x+1)} = -1 \quad \Leftarrow \quad \cos(x^2+2x+1) = 1 \Rightarrow \quad \sqrt{\cos(x^2+2x+1)} = 1$$

$$\Updownarrow$$

$$x^2+2x+1 = 2\pi K$$

$$\Leftrightarrow (x+1)^2 = 2\pi K$$

$$\Leftrightarrow x+1 = \pm\sqrt{2\pi K}$$

$$\Leftrightarrow x = \pm\sqrt{2\pi K} - 1; K \in \mathbb{Z}, K \geq 0$$

$$\Leftrightarrow x \in \{-1, \sqrt{2\pi}-1, \sqrt{4\pi}-1, \ldots, -\sqrt{2\pi}-1, -\sqrt{4\pi}-1, \ldots; K \in \mathbb{Z}\}$$

Check solutions and pick the ones that satisfy $\sqrt{\cos(x^2+2x+1)} = -1$

$x = -1$: $\sqrt{\cos(x^2+2x+1)} = \sqrt{\cos(0)} = \sqrt{1} = 1 \neq -1 \Rightarrow$ not a solution

$x = \sqrt{2\pi K} - 1$: $\sqrt{\cos(x^2+2x+1)} = \sqrt{\cos[(x+1)^2]} = \sqrt{\cos[(\sqrt{2\pi K}-1+1)^2]}$

$= \sqrt{\cos(2\pi K)} = \sqrt{1} = 1 \neq -1 \Rightarrow$ not a solution

$x = -\sqrt{2\pi K} - 1$: $\sqrt{\cos(x^2+2x+1)} = \sqrt{\cos[(x+1)^2]} = \sqrt{\cos[(-\sqrt{2\pi K}-1+1)^2]}$

$= \sqrt{\cos(2\pi K)} = \sqrt{1} = 1 \neq -1 \Rightarrow$ not a solution

Thus, there are no real solutions. This result makes sense because $\sqrt{\cos(x^2+2x+1)}$ is positive but -1 is negative. A positive value can't also be negative so $\sqrt{\cos(x^2+2x+1)} = -1$ can't have any solutions.