#1) $\sin(\pi/6) = 1/2$

A) $\sqrt{3}/2$ B) 0 C) $1/2$ D) 1 E) None of the above

#2) $\cos(2\pi/3) = -1/2$

A) $\sqrt{3}/2$ B) 1 C) 0 D) $-1/2$ E) None of the above

#3) $\sin(630^\circ) = \sin(630^\circ - 360^\circ) = \sin(270^\circ) = -1$

A) $-\sqrt{3}/2$ B) $\sqrt{2}/2$ C) 0 D) -1 E) None of the above

---

#4) $\tan(-11\pi/6) = \tan(-11\pi/6 + 2\pi) = \tan(-11\pi/6 + 12\pi/6) = \tan(\pi/6) = (1/2)/(\sqrt{3}/2) = 1/\sqrt{3}$

A) $\sqrt{3}/2$ B) $1/\sqrt{3}$ C) 0 D) 1 E) None of the above

---

#5) $\sec(1080^\circ) = \sec(1080^\circ - 3\cdot 360^\circ) = \sec(0^\circ) = 1/\cos(0^\circ) = 1/1 = 1$

A) $\sqrt{2}$ B) $-2\sqrt{3}/3$ C) 2 D) -1 E) None of the above

---

#6) $\csc(11\pi/6) = 1/\sin(11\pi/6) = 1/(-1/2) = -2$

A) $2/\sqrt{3}$ B) -2 C) $4/(\sqrt{6}-\sqrt{2})$ D) $\sqrt{3}$ E) None of the above

---

#7) $\cot(690^\circ) = \cot(690^\circ - 360^\circ) = \cot(330^\circ) = (\sqrt{3}/2)/(-1/2) = -\sqrt{3}$

A) $5\pi/2$ B) $1/\sqrt{3}$ C) 0 D) $-\sqrt{3}$ E) None of the above

---

#8) $\sin^2(7\pi/3) = \sin^2(7\pi/3 - 2\pi) = \sin^2(\pi/3) = (\sqrt{3}/2)^2 = 3/4$

A) $3\sqrt{3}/8$ B) $3/4$ C) $9/16$ D) 8 E) None of the above

---

#9) $\cot^2(9\pi/4) = \cot^2(9\pi/4 - 2\pi) = \cot^2(\pi/4) = 1^2 = 1$

A) $-\sqrt{3}$ B) -2 C) -1 D) 1 E) None of the above

#10) $\sin(27\pi/6) = \sin(27\pi/6 - 2\cdot 2\pi) = \sin(27\pi/6 - 24\pi/6) = \sin(\pi/2) = 1$

A) $1/2$ B) $-\sqrt{3}/2$ C) $\sqrt{3}/2$ D) 1 E) None of the above

---

#11) $\sin^{-1}(1/2) = \pi/6$

A) $\sqrt{3}/2$ B) 0 C) $1/2$ D) 1 E) None of the above

#12) $\cos^{-1}(1/2) = \pi/3$

A) $\pi/2$ B) $\pi/3$ C) 0 D) $\pi/6$ E) None of the above

#13) $\sin^{-1}(-1/2) = -\pi/6$

A) $\pi/2$ B) $-\pi/3$ C) 0 D) $-\pi/6$ E) None of the above

#14) $\cos^{-1}(-1/2) = 2\pi/3$

A) $\pi/2$ B) $-\pi/3$ C) 0 D) $-\pi/6$ E) None of the above

#15) $\arctan(-1) = -\pi/4$

A) $-\pi/4$ B) $3\pi/4$ C) $-3\pi/4$ D) -1 E) None of the above

---

#16) $\arcsin(1/\sqrt{2}) = \pi/4$

A) $\pi/6$ B) $3\pi/4$ C) $\pi/4$ D) $\sqrt{3}/2$ E) None of the above

---

#17) $\arccos(-\sqrt{3}/2) = 5\pi/6$

A) $5\pi/6$ B) $-\pi/6$ C) $-\pi/3$ D) $2\pi/3$ E) None of the above

---

#18) $\sin(\arccos(-\sqrt{2}/2)) = \sin(3\pi/4) = \sqrt{2}/2$

A) $-\sqrt{2}/2$ B) $-\pi/4$ C) $3\pi/4$ D) $\sqrt{2}/2$ E) None of the above

---

#19) $\cos(\arcsin(-\sqrt{2}/2)) = \cos(-\pi/4) = \sqrt{2}/2$

A) $-\sqrt{2}/2$ B) $-\pi/4$ C) $3\pi/4$ D) $\sqrt{2}/2$ E) None of the above

---

#20) $\sin(\arctan(-1)) = \sin(-\pi/4) = -\sqrt{2}/2$

A) $-\sqrt{2}/2$ B) $-\pi/4$ C) $3\pi/4$ D) $\sqrt{2}/2$ E) None of the above

#21) $\sin(\arccos(3/4)) = \sin(\theta)$ where $\theta = \arccos(3/4) \Rightarrow \cos(\theta) = 3/4 > 0 \Rightarrow \theta \in [0, \pi/2]$

$\cos^2(\theta) + \sin^2(\theta) \equiv 1 \Rightarrow \sin(\theta) = \sqrt{1-\cos^2(\theta)} = \sqrt{1-(3/4)^2} = \boxed{\sqrt{7}/4}$

A) $\sqrt{7}/4$ B) $2\sqrt{2}/3$ C) $3/2$ D) $\sqrt{3}/4$ E) None of the above

---

#22) $\cos(\arctan(3/4)) = \cos(\theta)$ where $\theta = \arctan(3/4) \Rightarrow \tan(\theta) = 3/4 > 0 \Rightarrow \theta \in [0, \pi/2)$

$1 + \tan^2(\theta) \equiv \sec^2(\theta) \equiv 1/\cos^2(\theta) \Rightarrow \cos(\theta) = \sqrt{1/[1+\tan^2(\theta)]} = \sqrt{1/[1+(3/4)^2]} = \boxed{4/5}$

A) $3/2$ B) $4/5$ C) $2/3$ D) $\sqrt{7}/2$ E) None of the above

---

#23) What is the period of $3\sin(5x-3)+2$? $T = 2\pi/\omega = \boxed{2\pi/5 = T}$

A) 3 B) $\pi/5$ C) $2\pi/5$ D) $\pi/3$ E) None of the above

#24) What is the period of $2\tan(3x-1)+1$? $T = \pi/\omega = \boxed{\pi/3 = T}$

A) 3 B) $\pi/5$ C) $2\pi/5$ D) $\pi/3$ E) None of the above

#25) If $\sin(\alpha) = 2/3$ and $\cos(\beta) = 1/3$ where $0 < \alpha < \pi/2$ and $0 < \beta < \pi/2$ then $\cos(\alpha+\beta) =$

A) $\sqrt{2}$ B) $\dfrac{\sqrt{5}-4\sqrt{2}}{9}$ C) $\dfrac{\sqrt{5}+4\sqrt{2}}{9}$ D) $\dfrac{\sqrt{5}}{3}$ E) None of the above

---

$\cos(\alpha) \equiv \sqrt{1-\sin^2(\alpha)} = \sqrt{1-(2/3)^2} = \sqrt{1-4/9} = \sqrt{5/9} = \sqrt{5}/3$

$\sin(\beta) \equiv \sqrt{1-\cos^2(\beta)} = \sqrt{1-(1/3)^2} = \sqrt{1-1/9} = \sqrt{8/9} = 2\sqrt{2}/3$

$\cos(\alpha+\beta) \equiv \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = \dfrac{\sqrt{5}}{3}\cdot\dfrac{1}{3} - \dfrac{2}{3}\cdot\dfrac{2\sqrt{2}}{3} = \boxed{\dfrac{\sqrt{5}-4\sqrt{2}}{9} = \cos(\alpha+\beta)}$

#26) If $\sin(\alpha) = 1/3$ and $\cos(\beta) = 3/4$ where $0 < \alpha < \pi/2$ and $0 < \beta < \pi/2$ then $\sin(\alpha+\beta) =$

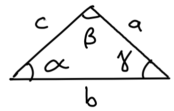
A) $\dfrac{\sqrt{2-\sqrt{3}}}{2}$ B) $\dfrac{\sqrt{2+\sqrt{3}}}{2}$ C) $\dfrac{4}{\sqrt{6}-\sqrt{2}}$ D) $\dfrac{3+2\sqrt{14}}{12}$ E) None of the above

---

$\cos(\alpha) \equiv \sqrt{1-\sin^2(\alpha)} = \sqrt{1-(1/3)^2} = \sqrt{1-1/9} = \sqrt{8/9} = 2\sqrt{2}/3$

$\sin(\beta) \equiv \sqrt{1-\cos^2(\beta)} = \sqrt{1-(3/4)^2} = \sqrt{1-9/16} = \sqrt{7}/4$

$\sin(\alpha+\beta) \equiv \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) = \dfrac{1}{3}\,\dfrac{3}{4} + \dfrac{2\sqrt{2}}{3}\,\dfrac{\sqrt{7}}{4} = \boxed{\dfrac{3+2\sqrt{14}}{12} = \sin(\alpha+\beta)}$

#27) If triangles are labeled in this way  and if $a=2$, $\beta=\gamma=15°$, solve for b, c and α.

A) $\alpha = 150°$, $b = c = \sqrt{3} - \sqrt{2}$

B) $\alpha = 150°$, $b = \sqrt{6} - \sqrt{2}$, $c = \sqrt{6} + \sqrt{2}$

C) $\alpha = 150°$, $b = c = \sqrt{6} - \sqrt{2}$

D) $\alpha = 150°$, $b = c = \sqrt{6} + \sqrt{2}$

E) None of the above

$\alpha+\beta+\gamma = 180° \Rightarrow \alpha = 180° - \beta - \gamma = 180° - 15° - 15° = \boxed{150° = \alpha}$

$\sin(15°) = \sin(45° - 30°) = \sin(45°)\cos(30°) - \cos(45°)\sin(30°) = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{b}{\sin(\beta)} = \frac{a}{\sin(\alpha)} \Rightarrow b = \frac{a\sin(\beta)}{\sin(\alpha)} = \frac{2\sin(15°)}{\sin(150°)} = 2\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\frac{1}{1/2} = \boxed{\sqrt{6} - \sqrt{2} = b}$

$\frac{c}{\sin(\gamma)} = \frac{a}{\sin(\alpha)} \Rightarrow c = \frac{a\sin(\gamma)}{\sin(\alpha)} = \frac{2\sin(15°)}{\sin(150°)} = 2\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\frac{1}{1/2} = \boxed{\sqrt{6} - \sqrt{2} = c}$

#28) Convert 212 degrees to radians: $212° = 212°(\pi/180°) = \boxed{53\pi/45 = 212°}$

A) 212π B) $\pi/212$ C) 212 D) $53\pi/45$ E) None of the above

---

#29) Linear speed is given by $v = s/t$ and angular speed is $\omega = \theta/t$ where s is arclength and t is time. Assume the Earth's orbit to be circular and the distance between the Earth and Sun to be 1 AU (AU stands for Astronomical Unit). What is the Earth's linear speed relative to the sun?

$\omega = 2\pi/1yr$ $\qquad s = r\theta$

$v = s/t = r\theta/t = r\omega = 1AU \cdot 2\pi/1yr = \boxed{2\pi\, AU/yr = v}$

A) $1AU/yr$ B) $2\pi\, AU/yr$ C) $\pi\, AU/yr$ D) $\pi^2 AU/yr$ E) None of the above

#30) $2\sin(\pi/12)\cos(\pi/12) = \sin(2\cdot\pi/12) = \sin(\pi/6) = \boxed{1/2 = 2\sin(\pi/12)\cos(\pi/12)}$

A) $(\sqrt{6}+\sqrt{2})/4$ B) $(\sqrt{6}-\sqrt{2})/4$ C) $\sqrt{3}/2$ D) $1/2$ E) None of the above

---

#31) What is the domain of $\sin(x)$?

A) $[-1, 1]$ B) $\mathbb{Z}$ C) $\mathbb{N}$ D) $\mathbb{R}$ E) None of the above

---

#32) Complete the identity: $\sin(\pi/2 - \theta) = \cos(\theta)$

A) $\cos(\theta)$ B) $-\sin(\theta)$ C) $\sin(\theta/2)$ D) $-\cos(\theta)$ E) None of the above

---

#33) If $\tan(\theta) = 3/2$ and $\cos(\theta) > 0$ then $\sin(\theta) =$

A) 3 B) 2 C) $3/\sqrt{13}$ D) $3/\sqrt{7}$ E) None of the above

---

Let $y=3$ and $x=2$, then $r = \sqrt{x^2+y^2} = \sqrt{2^2+3^2} = \sqrt{13} \Rightarrow \sin(\theta) = y/r = \boxed{3/\sqrt{13} = \sin(\theta)}$

#34) If $\cot^2(\theta) = 4$ then $\csc^2(\theta) = 1 + \cot^2(\theta) = 1 + 4 = \boxed{5 = \csc^2(\theta)}$

A) $1/2$ B) 2 C) 17 D) 13 E) None of the above

#35) Complete the identity: $\cos(-x) = \cos(x)$

A) $\cos(x)$ B) $\sin(x)$ C) $-\cos(x)$ D) $\sin(x - \pi/2)$ E) None of the above

---

#36) What is the amplitude of $4\sin(3x-2)+1$? $A = |4| = 4$

A) $2/3$ B) 1 C) 4 D) $2\pi/3$ E) None of the above

#37) The following graph is a graph of what function?

A) $-4\sin(x)$

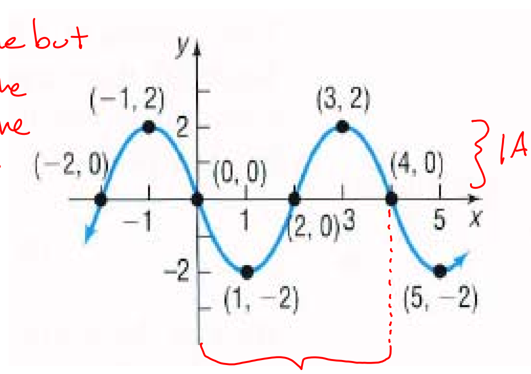
B) $(1/2)\sin(\pi x)$

C) $2\sin(-4x)$

D) $2\sin(-\frac{\pi}{2}x)$

E) None of the above

Looks like sine but flipped over the x axis so the x has a minus with it.

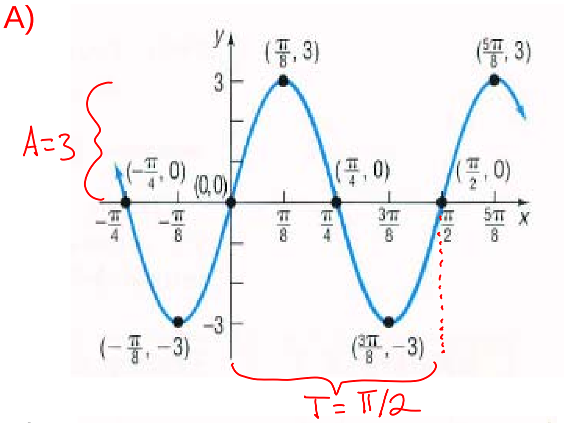


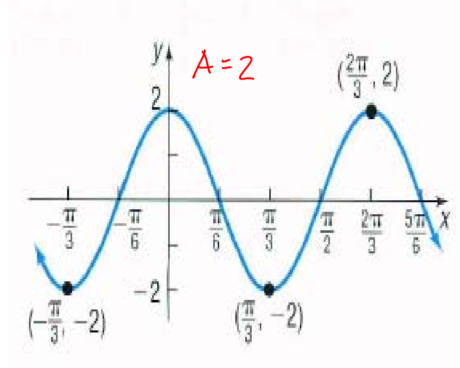
$\}\ |A| = 2$

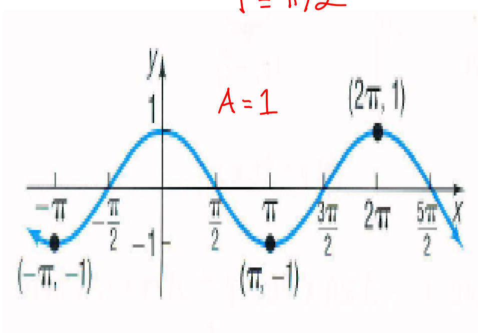
$T = 4 = 2\pi/\omega$

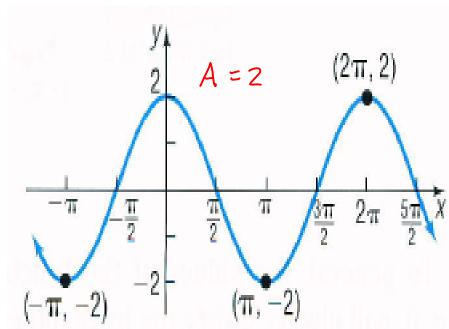
$\Rightarrow \omega = 2\pi/4 = \pi/2$

#38) Which is the graph of $f(x) = 3\sin(4x)$? $T = 2\pi/4 = \pi/2$, $A = 3$

A) 

B) 

C) 

D) 

E) None of the above

#39) The following graph is a graph of what function? $T = \pi = \pi/\omega \Rightarrow \omega = 1$

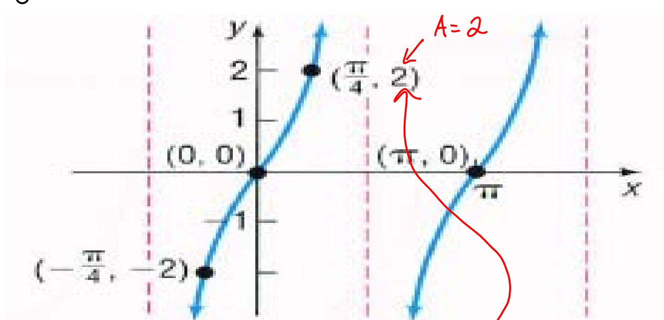
A) $\tan(2x)$

B) $2\cot(x)$

C) $2\tan(x)$

D) $2\tan(-\frac{\pi}{2}x)$

E) None of the above



$\tan(\pi/4) = 1 \Rightarrow 2\tan(\pi/4) = 2$

#40) Solve for x: $3\sin^{-1}(x) = \pi/2 \Rightarrow \sin^{-1}(x) = \pi/6 \Rightarrow x = \sin(\pi/6) = \boxed{1/2 = x}$

A) 0 B) 1 C) $\sqrt{3}/2$ D) $1/2$ E) None of the above

#41) $\sin(75°) =$

A) $(\sqrt{6} + \sqrt{2})/4$ B) $(\sqrt{6} - \sqrt{2})/4$ C) $\sqrt{2 - \sqrt{3}}/2$ D) $5\pi/12$ E) None of the above

---

$\sin(75°) = \sin(45° + 30°) = \sin(45°)\cos(30°) + \cos(45°)\sin(30°) = (\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2) = \boxed{(\sqrt{6} + \sqrt{2})/4 = \sin(75°)}$

#42) Solve for x: $2\sin(3x) = 1 \Leftrightarrow \sin(3x) = 1/2$

A) $x = \frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k; k \in \mathbb{Z}$

B) $x = \frac{\pi}{18} + \frac{2}{3}\pi k, \frac{5\pi}{18} + \frac{2}{3}\pi k; k \in \mathbb{Z}$

C) $x = \frac{\pi}{18} + 2\pi k, \frac{5\pi}{18} + 2\pi k; k \in \mathbb{Z}$

D) $x = \frac{\pi}{6} + \frac{2}{3}\pi k, \frac{5\pi}{6} + \frac{2}{3}\pi k; k \in \mathbb{Z}$

E) None of the above

$$\Leftrightarrow 3x = \begin{cases} \frac{\pi}{6} + 2\pi k \Rightarrow \boxed{x = \frac{\pi}{18} + \frac{2}{3}\pi k} \\ \frac{5\pi}{6} + 2\pi k \Rightarrow \boxed{x = \frac{5\pi}{18} + \frac{2}{3}\pi k} \end{cases}$$

#43) Solve for x: $2\sin(3x)\cos(3x) = 1 \Leftrightarrow \sin(6x) = 1$

A) $x = \frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k; k \in \mathbb{Z}$

B) $x = \frac{\pi}{12} + \frac{\pi}{3}k; k \in \mathbb{Z}$

C) $x = \pm\frac{\pi}{2} + 2\pi k; k \in \mathbb{Z}$

D) $x = \frac{\pi}{6} + \frac{\pi}{3}k, \frac{5\pi}{6} + \frac{\pi}{3}k; k \in \mathbb{Z}$

E) None of the above

$$\Leftrightarrow 6x = \frac{\pi}{2} + 2\pi k \Leftrightarrow \boxed{x = \frac{\pi}{12} + \frac{\pi}{3}k}$$

#44) Solve for x: $\sin^2(x) - \cos^2(x) - 1 = \cos(x) \Leftrightarrow -[1 - \sin^2(x)] - \cos^2(x) - \cos(x) = 0$

A) $x = \pm\frac{\pi}{2} + 2\pi k; k \in \mathbb{Z}$

B) $x = \frac{\pi}{2} + \frac{\pi}{3}k, \frac{5\pi}{6} + \frac{\pi}{3}k; k \in \mathbb{Z}$

C) $x = \frac{\pi}{2} + \pi k, \pm\frac{\pi}{3} + 2\pi k; k \in \mathbb{Z}$

D) $x = \frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k; k \in \mathbb{Z}$

E) None of the above

$$\Leftrightarrow -\cos^2(x) - \cos^2(x) - \cos(x) = 0$$

$$\Leftrightarrow -2\cos^2(x) - \cos(x) = 0$$

$$\Leftrightarrow \cos(x)[2\cos(x) + 1] = 0$$

$$\Leftrightarrow \begin{cases} \cos(x) = 0 \Leftrightarrow \boxed{x = \frac{\pi}{2} + \pi k} \\ 2\cos(x) + 1 = 0 \Leftrightarrow \cos(x) = -1/2 \Leftrightarrow \boxed{x = \pm\frac{2\pi}{3} + 2\pi k} \end{cases}$$

#45) Solve for x: $\tan(x) - \cot(x) = 0 \Leftrightarrow \tan(x) = \cot(x) \Leftrightarrow \frac{\tan(x)}{\cot(x)} = 1$

A) $x = \frac{\pi}{6} + 2\pi k, \frac{5\pi}{6} + 2\pi k; k \in \mathbb{Z}$

B) $x = \frac{\pi}{4} + \frac{\pi}{2}k; k \in \mathbb{Z}$

C) $x = \pm\frac{\pi}{4} + 2\pi k; k \in \mathbb{Z}$

D) $x = \pm\frac{\pi}{2} + 2\pi k; k \in \mathbb{Z}$

E) None of the above

$$\Leftrightarrow \tan^2(x) = 1 \Leftrightarrow \tan(x) = \pm 1$$

$$\Leftrightarrow \boxed{x = \frac{\pi}{4} + \frac{\pi}{2}k}$$

#46) Solve for x: $\sin(2x)\sin(x) - \cos(x) = 0 \Leftrightarrow 2\sin(x)\cos(x)\sin(x) - \cos(x) = 0$

$\Leftrightarrow \cos(x)[2\sin^2(x) - 1] = 0$

$\Leftrightarrow -\cos(x)[1 - 2\sin^2(x)] = 0$

$\Leftrightarrow -\cos(x)\cos(2x) = 0$

$\Leftrightarrow \begin{cases} \cos(x) = 0 \Leftrightarrow \boxed{x = \frac{\pi}{2} + \pi K} \\ \cos(2x) = 0 \Leftrightarrow 2x = \frac{\pi}{2} + 2\pi K \Leftrightarrow \boxed{x = \frac{\pi}{4} + \frac{\pi}{2}K} \end{cases}$

A) $x = \frac{\pi}{2} + 2\pi K,\ \frac{\pi}{4} + \frac{\pi}{2}K;\ K \in \mathbb{Z}$

B) $x = \pm\frac{\pi}{4} + 2\pi K,\ \frac{\pi}{2} + 2\pi K;\ K \in \mathbb{Z}$

C) $x = \frac{\pi}{4} + 2\pi K,\ \frac{\pi}{2} + 2\pi K;\ K \in \mathbb{Z}$

D) $x = \pm\frac{\pi}{4} + 2\pi K;\ K \in \mathbb{Z}$

E) None of the above

#47) Solve for x: $\cos^2(x) + \sin^2(x) = 1 \Leftrightarrow x \in \mathbb{R}$

A) $x = \frac{\pi}{2} + 2\pi K;\ K \in \mathbb{Z}$

B) $x = \pm 1$

C) $x \in \mathbb{R}$

D) $x = 2\pi K;\ K \in \mathbb{Z}$

E) None of the above

#48) Complete the identity: $\left(\frac{1 + \tan(\theta)}{1 + \cot(\theta)}\right)^2 = \left(\frac{\tan(\theta)[\cot(\theta) + 1]}{1 + \cot(\theta)}\right)^2$

$= \tan^2(\theta)\left(\frac{1 + \cot(\theta)}{1 + \cot(\theta)}\right)^2$

$= \boxed{\tan^2(\theta)}$

A) $\cot^2(\theta)$

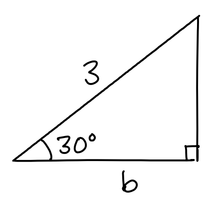
B) $\tan^2(\theta)$

C) $\sin^2(\theta)/\cos(\theta)$

D) $\tan(\theta)$

E) None of the above

#49) Solve for b in the given triangle:



A) $b = 3/2$

B) $b = 3\sqrt{3}/2$

C) $b = \sqrt{3}/6$

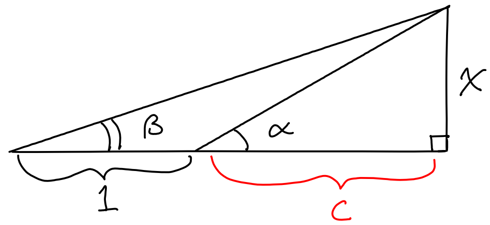
D) $b = \sqrt{3}/3$

E) None of the above

$\cos(30°) = \text{adj}/\text{hyp} = b/3$

$\Rightarrow b = 3\cos(30°) = \boxed{3\sqrt{3}/2 = b}$

#50) Solve for x in terms of α and β.



A) $x = 1/[\cot(\beta)+\cot(\alpha)]$

B) $x = 1/[\cot(\beta)-\cot(\alpha)]$

C) $x = 1/[\tan(\beta)-\tan(\alpha)]$

D) $x = 1/[\tan(\beta)+\tan(\alpha)]$

E) None of the above

$\tan(\beta) = \text{opp}/\text{adj} = x/(1+c) \Rightarrow x = (1+c)\tan(\beta) \Rightarrow c = x/\tan(\beta) - 1 = x\cot(\beta) - 1 = c \quad (1)$

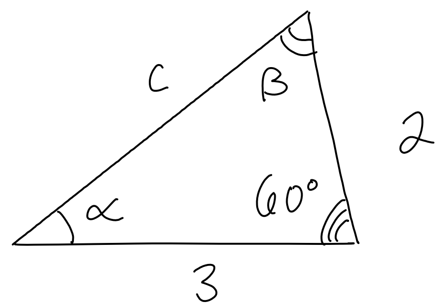
$\tan(\alpha) = \text{opp}/\text{adj} = x/c \Rightarrow c = x/\tan(\alpha) = x\cot(\alpha) = c \quad (2)$

$(2)-(1) \Rightarrow 0 = c - c = x\cot(\alpha) - [x\cot(\beta)-1] = x\cot(\alpha) - x\cot(\beta) + 1 = 0$

$\Rightarrow x[\cot(\alpha) - \cot(\beta)] = -1$

$\Rightarrow \boxed{x = 1/[\cot(\beta)-\cot(\alpha)]}$

#51) Solve the triangle:



A) $c = \sqrt{7}$, $\alpha = \arccos(\sqrt{7})$, $\beta = 130° - \arccos(\sqrt{7})$

B) $c = 7$, $\alpha = \arcsin(7)$, $\beta = 130° - \arcsin(7)$

C) $c = 1/\sqrt{7}$, $\alpha = \arcsin(\sqrt{7})$, $\beta = 130° - \arcsin(\sqrt{7})$

D) $c = \sqrt{7}$, $\alpha = \arcsin(1/\sqrt{7})$, $\beta = 130° - \arcsin(1/\sqrt{7})$

E) None of the above

$a = 2,\ b = 3,\ \gamma = 60°$

$c^2 = a^2 + b^2 - 2ab\cos(\gamma) = 2^2 + 3^2 - 2\cdot 2\cdot 3\cos(60°) = 4 + 9 - 12(1/2) = 7 \Rightarrow \boxed{c = \sqrt{7}}$

$\frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c} \Rightarrow \sin(\alpha) = \frac{a}{c}\sin(\gamma) = \frac{2}{\sqrt{7}}\sin(60°) = \frac{2}{\sqrt{7}}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{7}} \Rightarrow \boxed{\alpha = \arcsin(\sqrt{3/7})}$

$\alpha + \beta + \gamma = 180° \Rightarrow \beta = 180° - \alpha - \gamma = 180° - \arcsin(\sqrt{3/7}) - 60° = \boxed{120° - \arcsin(\sqrt{3/7}) = \beta}$

#52) Find the area of a triangle with sides of lengths 4, 5 and 7 using Heron's Area Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = (a+b+c)/2$.

A) $23\sqrt{7}$ B) 140 C) 70 D) $4\sqrt{6}$ E) None of the above

---

$s = (a+b+c)/2 = (4+5+7)/2 = 16/2 = 8$

$\Rightarrow A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{8(8-4)(8-5)(8-7)} = \sqrt{8(4)(3)(1)}$

$= \sqrt{2^5 \cdot 3} = \sqrt{2^4}\sqrt{2 \cdot 3} = \boxed{4\sqrt{6} = A}$

#53) Convert the polar coordinate $(3, \pi/6)$ to rectangular.

A) $(\sqrt{3}/2, 1/2)$ B) $(1/2, 3/2)$ C) $(\sqrt{3}/2, 3/2)$ D) $(3\sqrt{3}/2, 3/2)$ E) None of the above

---

$\cos(\theta) := x/r \Rightarrow x = r\cos(\theta) = 3\cos(\pi/6) = \boxed{3\sqrt{3}/2 = x}$

$\sin(\theta) := y/r \Rightarrow y = r\sin(\theta) = 3\sin(\pi/6) = 3 \cdot 1/2 = \boxed{3/2 = y}$

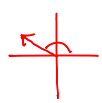
#54) Convert the rectangular coordinate $(-\sqrt{3}, 1)$ to polar.

A) $(\sqrt{3}, -\pi/6)$ B) $(2, -\pi/6)$ C) $(2, 5\pi/6)$ D) $(3, \pi/3)$ E) None of the above

---

$r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{4} = \boxed{2 = r}$

$(-\sqrt{3}, 1) = 2(-\sqrt{3}/2, 1/2)$



$\theta = 5\pi/6$ by inspection

or use $\theta = \arctan(y/x) + \pi = \arctan(-1/\sqrt{3}) + \pi = -\pi/6 + \pi = \boxed{5\pi/6 = \theta}$

#55) If $z = 2[\cos(33°) + i\sin(33°)]$ and $w = 3[\cos(27°) + i\sin(27°)]$ then $z \cdot w =$

A) $6i$ B) $2 + 2\sqrt{2}i$ C) $3 + 3\sqrt{3}i$ D) $33 + 27i$ E) None of the above

---

$z \cdot w = 2[\cos(33°) + i\sin(33°)] \cdot 3[\cos(27°) + i\sin(27°)] = 2 \cdot 3[\cos(33°+27°) + i\sin(33°+27°)]$

$= 6[\cos(60°) + i\sin(60°)] = 6[1/2 + (\sqrt{3}/2)i] = \boxed{3 + 3\sqrt{3}i = z \cdot w}$

#56) $(1-\sqrt{3}i)^8 =$

A) $-2^7-2^7\sqrt{3}i$ B) $1-3^4 i$ C) $8-8\sqrt{3}i$ D) $2^8+2^8\sqrt{3}i$ E) None of the above

---

$r=\sqrt{x^2+y^2}=\sqrt{1^2+(-\sqrt{3})^2}=\sqrt{4}=2 \Rightarrow (1-\sqrt{3}i)^8=[2(1/2-i\sqrt{3}/2)]^8$

$\Rightarrow \theta=-\pi/3$ by inspection.

$\Rightarrow (1-\sqrt{3}i)^8=[2(1/2-i\sqrt{3}/2)]^8=2^8[\cos(-8\pi/3)+i\sin(-8\pi/3)]$

$=2^8(-1/2-i\sqrt{3}/2)=\boxed{-2^7-2^7\sqrt{3}i=(1-\sqrt{3}i)^8}$

#57) $(2+3i)/(1-2i)=$

A) $2-\frac{3}{2}i$ B) $-\frac{4}{3}+\frac{7}{3}i$ C) $-\frac{2}{3}+3i$ D) $1+i$ E) None of the above

---

$$\frac{2+3i}{1-2i}=\left(\frac{2+3i}{1-2i}\right)\left(\frac{1+2i}{1+2i}\right)=\frac{2+6i^2+3i+4i}{1-2i^2}=\frac{-4+7i}{3}=\boxed{-\frac{4}{3}+\frac{7}{3}i=\frac{2+3i}{1-2i}}$$

#58) Which of the following is a fourth root of i?

A) $\cos(3\pi/7)+i\sin(3\pi/7)$

B) -1

C) $\sqrt{2}/2+i\sqrt{2}/2$

D) $\cos(\pi/8)+i\sin(\pi/8)$

E) None of the above

$i=\cos(\pi/2)+i\sin(\pi/2) \Rightarrow \sqrt[4]{i}=\cos\left(\frac{\pi/2+2\pi k}{4}\right)+i\sin\left(\frac{\pi/2+2\pi k}{4}\right)$

$k=0 \Rightarrow \cos\left(\frac{\pi/2+2\pi\cdot 0}{4}\right)+i\sin\left(\frac{\pi/2+2\pi\cdot 0}{4}\right)=\cos(\pi/8)+i\sin(\pi/8)$

$k=1 \Rightarrow \cos\left(\frac{\pi/2+2\pi}{4}\right)+i\sin\left(\frac{\pi/2+2\pi}{4}\right)=\cos(5\pi/8)+i\sin(5\pi/8)$

$k=2 \Rightarrow \cos\left(\frac{\pi/2+2\pi\cdot 2}{4}\right)+i\sin\left(\frac{\pi/2+2\pi\cdot 2}{4}\right)=\cos(9\pi/8)+i\sin(9\pi/8)$

$k=3 \Rightarrow \cos\left(\frac{\pi/2+2\pi\cdot 3}{4}\right)+i\sin\left(\frac{\pi/2+2\pi\cdot 3}{4}\right)=\cos(13\pi/8)+i\sin(13\pi/8)$

#59) If $\vec{u} = (3,4)$ and $\vec{v} = (-1,2)$ then $3\vec{u} - \vec{v} = 3(3,4) - (-1,2) = \boxed{(10,10) = 3\vec{u} - \vec{v}}$

A) $(10,10)$ B) $(0,2)$ C) $(8,14)$ D) $(3,-1)$ E) None of the above

---

#60) Solve for x: $e^{-x^2} = (e^x)^2 \cdot 1/e^4$

A) $-1 \pm \sqrt{5}$ B) $e^{\pm\sqrt{5}}$ C) $1 \pm \sqrt{5}$ D) $\pm\sqrt{5}$ E) None of the above

---

$$e^{-x^2} = (e^x)^2 \cdot 1/e^4 \Leftrightarrow e^{-x^2} = e^{2x} e^{-4} = e^{2x-4} \Leftrightarrow -x^2 = 2x - 4$$

$$\Leftrightarrow x^2 + 2x - 4 = 0 \Leftrightarrow x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{4+16}}{2} = \boxed{-1 \pm \sqrt{5} = x}$$

#61) Solve for x: $\log_x(\sqrt{10}) = 1/2 \Leftrightarrow x^{1/2} = \sqrt{10} \Leftrightarrow \boxed{x = 10}$

A) 10^2 B) 10 C) 2 D) $\sqrt{2}$ E) None of the above

#62) What is the domain of the function $f(x) = \ln(x - \pi)$?

A) $\mathbb{R}$ B) $x > \pi$ C) $0 < x < \pi$ D) $x > 0$ E) None of the above

---

$x - \pi > 0 \Leftrightarrow \boxed{x > \pi}$

#63) Solve for x: $\log_2(8^x) + 3 = 0$

A) $x = 0$ B) $x = \pm 1$ C) $x = 1$ D) $x = -1$ E) None of the above

---

$$\log_2(8^x) + 3 = 0 \Leftrightarrow \log_2(8^x) = -3 \Leftrightarrow 8^x = 2^{-3} \Leftrightarrow (2^3)^x = 2^{-3}$$

$$\Leftrightarrow 2^{3x} = 2^{-3} \Leftrightarrow 3x = -3 \Leftrightarrow \boxed{x = -1}$$

#64) Solve for x: $4e^{x+1} + 1 = 6$

A) $\ln(5/4)$ B) $\ln(4/5) + 1$ C) $\ln(5/4) - 1$ D) $\ln(5/4) + 1$ E) None of the above

---

$$4e^{x+1} + 1 = 6 \Leftrightarrow 4e^{x+1} = 5 \Leftrightarrow e^{x+1} = 5/4 \Leftrightarrow x + 1 = \ln(5/4)$$

$$\Leftrightarrow \boxed{x = \ln(5/4) - 1}$$

#65) Solve for x: $\log_4(4^x) = -1 \Leftrightarrow 4^x = 4^{-1} \Leftrightarrow \boxed{x = -1}$

A) $x = e$ B) $x = 0$ C) $x = -1$ D) $x = \pm 1$ E) None of the above

---

#66) $\log\left[\sqrt{x^2+1}\cdot\sqrt[3]{x^3-1}\right] = \log\left(\sqrt{x^2+1}\right) + \log\left(\sqrt[3]{x^3-1}\right) = \frac{1}{2}\log(x^2+1) + \frac{1}{3}\log(x^3-1)$

A) $\frac{1}{2}\log(x^2+1) + \frac{1}{3}\log(x^3-1)$

B) $2\log(x)$

C) $\frac{1}{6}\log(x^2+1)\cdot\log(x^3-1)$

D) $\log^2(x^2+1) + \log^3(x^3-1)$

E) None of the above

#67) $3\log(x) - 4\log(y^2) + 5\log(z^3) = \log(x^3) + \log(y^{-8}) + \log(z^{15}) = \log(x^3 z^{15}/y^8)$

A) $\log(x^3 z^{15}/y^8)$

B) $\log(y^8/x^3 z^{15})$

C) $\log(x^4 z^8/y^2)$

D) $\log(x^4 y^2 z^8)$

E) None of the above

#68) Solve for x: $3\log_2(x-1) - 5 = -\log_2(4)$

A) $x = 1$ B) $x = -1$ C) $x = 2$ D) $x = 0$ E) None of the above

---

$3\log_2(x-1) - 5 = -\log_2(4) = -2 \Leftrightarrow 3\log_2(x-1) = 3$

$\Leftrightarrow \log_2(x-1) = 1 \Leftrightarrow x - 1 = 2^1 = 2 \Leftrightarrow \boxed{x = 3}$

#69) Solve for x: $25^x - 2^3\cdot 5^x = -16$

A) $\log_2(5)$ B) 2 C) $\log_5(4)$ D) $\log_4(5)$ E) None of the above

---

$25^x - 2^3\cdot 5^x = -16 \Leftrightarrow (5^2)^x - 2^3\cdot 5^x + 2^4 = 0$

$\Leftrightarrow (5^x)^2 - 2\cdot 2^2\cdot 5^x + 2^4 = 0 \Leftrightarrow (5^x - 2^2)^2 = 0 \Leftrightarrow 5^x = 2^2 = 4$

$\Rightarrow \boxed{x = \log_5(4)}$

Compound Interest: The amount after t years with an initial amount, P, called the Principal, with an annual interest rate r, where interest is compounded n times per year, is given by:

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt} \xrightarrow[n\to\infty]{} Pe^{rt}$$

#70) How much money would you have if you invested \$100 at 6% compounded annually after two years? $n=1,\ t=2,\ r=0.06,\ P=\$100$

A) \$115 B) \$113.12 C) \$112.36 D) \$112.00 E) None of the above

---

$$A(2) = \$100\,[1+.06/1]^{1\cdot 2} = \$100\,(1.06)^2 = \$100\,(1.1236) = \boxed{\$112.36}$$

#71) How much money would you have if you invested \$100 at 6% compounded quarterly after two and a half years? $n=4,\ t=5/2,\ r=0.06,\ P=\$100$

A) $\$100\,(1.015)^{10}$

B) $\$100\,(1.15)^{10}$

C) $\$100\,(1.015)^{5}$

D) $\$100\,(1.06)^{10}$

E) None of the above

$$A(5/2) = \$100\,[1+.06/4]^{4\cdot 5/2} = \$100\,[1+.03/2]^{10} = \$100\,[1+.015]^{10} = \boxed{\$100\,(1.015)^{10}}$$

#72) How much money would you have if you invested \$100 at 6% compounded continuously after two and a half years? $n=\infty,\ t=5/2,\ r=0.06,\ P=\$100$

$$A(5/2) = \$100\,e^{.06(5/2)} = \boxed{\$100\,e^{0.15}}$$

A) \$112.17

B) $\$100\,e^{0.15}$

C) $\$100\,[1+e^{.06}]^{5/2}$

D) $\$100\,e^{5/2}$

E) None of the above

#73) How much money should you have invested two years ago at 6% compounded quarterly to have \$100 today? $r=0.06$, $n=4$, $A(2)=\$100$, $t=2$

A) $\$\,87.88$

B) $\$100(1.015)^{-8}$

C) $\$100(1.015)^{8}$

D) $\$100(.015)^{-8}$

E) None of the above

$$A(t)=P\left(1+\frac{r}{n}\right)^{nt} \Leftrightarrow P=A(t)\left(1+\frac{r}{n}\right)^{-nt}=\$100(1+.06/4)^{-4\cdot 2}=\boxed{\$100(1.015)^{-8}}$$

#74) What rate of interest compounded annually is required to double an investment in 10 years? $n=1$, $t=10$, $A(10)=2P$

A) $r=2$

B) $r=2^{1/10}$

C) $r=0.06$

D) $r=2^{1/10}-1$

E) None of the above

$$2P=A(10)=P\left(1+\frac{r}{1}\right)^{1\cdot 10}=P(1+r)^{10} \Leftrightarrow 2=(1+r)^{10} \Rightarrow 1+r=2^{1/10} \Rightarrow \boxed{r=2^{1/10}-1}$$

#75) What rate of interest compounded monthly is required to double an investment in 10 years? $n=12$, $t=10$, $A(10)=2P$

A) $r=12$

B) $r=0.073$

C) $r=12\cdot 2^{1/120}$

D) $r=(2^{1/120}-1)/12$

E) None of the above

$$2P=A(10)=P\left(1+\frac{r}{12}\right)^{12\cdot 10}=P\left(1+\frac{r}{12}\right)^{120}$$

$$\Leftrightarrow 2=\left(1+\frac{r}{12}\right)^{120} \Rightarrow 1+r/12=2^{1/120}$$

$$\Rightarrow r/12=2^{1/120}-1$$

$$\Rightarrow \boxed{r=12(2^{1/120}-1)}$$