Name________________________Banner____________

Spring 2008 **Quiz #1** **Solutions**

Show your work. Use proper notation. Think before you write or give up. Box your final answers. Write on this paper only. Do easy problems first.

#1) Solve for x: $3x^2 + 2x^1 - 1\cdot x^0 = -6x^2 - 3x^1 + 2x^0$

Quadratic Formula result

$$\Leftrightarrow 9x^2 + 5x - 3 = 0 \Leftrightarrow x = \frac{-5 \pm \sqrt{5^2 - 4(9)(-3)}}{2(9)} = \frac{-5 \pm \sqrt{25+108}}{18}$$

$$= \boxed{\frac{-5 \pm \sqrt{133}}{18} = x}$$

#2) Which values of x and y simultaneously satisfy the following two equations?

$$2x + y = 7 \quad \text{and} \quad y - 2x = 5y$$

Two Equations, Two Unknowns

$$2x + y = 7 \Rightarrow y = 7 - 2x$$

$$y - 2x = 5y \Rightarrow x = -2y = -2(7-2x) = -14 + 4x \Rightarrow 3x = 14 \Rightarrow \boxed{x = \frac{14}{3}}$$

$$\Rightarrow y = 7 - 2x = 7 - 2\left(\frac{14}{3}\right) = 7 - \frac{28}{3} = \frac{21}{3} - \frac{28}{3} = \boxed{\frac{-7}{3} = y}$$

#3) Use Pascal's Triangle to expand $(a+b-c)^5 = [(a+b)+(-c)]^5$

Use Pascal's Triangle to find the coefficients for $(x+y)^n$

n=0: 1

n=1: 1 1

n=2: 1 2 1

n=3: 1 3 3 1

n=4: 1 4 6 4 1

n=5: 1 5 10 10 5 1

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

If $x = a+b$ and $y = -c$, we have

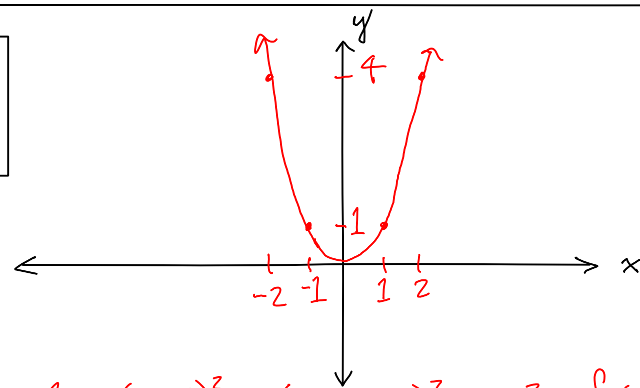
$$[(a+b)+(-c)]^5 = (a+b)^5 - 5(a+b)^4c + 10(a+b)^3c^2 - 10(a+b)^2c^3 + 5(a+b)c^4 - c^5$$

Now find $(a+b)^5, (a+b)^4, (a+b)^3, (a+b)^2$ using Pascal's Triangle and plug them into the expression above. We get

$$\boxed{\begin{aligned} &a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 - 5a^4c - 20a^3bc - 30a^2b^2c - 20ab^3c - 5b^4c \\ &+ 10a^3c^2 + 30a^2bc^2 + 30ab^2c^2 + 10b^3c^2 - 10a^2c^3 - 20abc^3 - 10b^2c^3 + 5ac^4 + 5bc^4 - c^5 \end{aligned}}$$

#4) A) Graph: $y = f(x) = z^2 + 2z + 1$ where $z = x - 1$

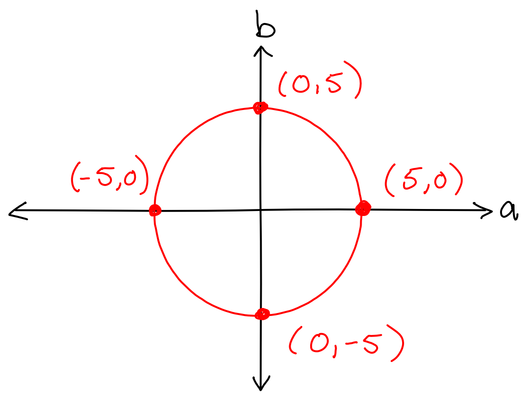
Notice I've labeled the axes for you.



$$f(x) = z^2 + 2z + 1 = (z+1)^2 = (x-1+1)^2 = x^2 = f(x)$$

B) Graph the solutions to the following equation: $a^2 + b^2 = 5^2$

Notice I've labeled the axes for you.



Equation of a circle with radius 5.

C) Evaluate the following:

(i) $2^3/3^{-2} = 2^3 \cdot 3^2 = 8 \cdot 9 = \boxed{72}$

(ii) $3^{-2} \cdot (2^3)^{-2} + 3^{-2} = \frac{1}{(2^3)^2 \cdot 3^2} + \frac{1}{3^2} = \frac{1}{2^6 \cdot 3^2} + \frac{2^6}{2^6 \cdot 3^2} = \boxed{\frac{65}{576}}$

(iii) $\frac{2^3/3^2}{(3^2)^{-1}} + \frac{3^2/2^{-3}}{(2^{-3})^{-1}} = \frac{2^3}{3^2} \cdot \frac{1}{(3^2)^{-1}} + \frac{3^2}{2^{-3}} \cdot \frac{1}{(2^{-3})^{-1}} = \frac{2^3}{\cancel{3^2}} \cdot \cancel{3^2} + 3^2 \cdot \cancel{2^3} \cdot \frac{1}{\cancel{2^3}}$

$= 2^3 + 3^2 = 8 + 9 = \boxed{17}$

#5) Solve for x: $\sqrt{x^2} = (\sqrt{x})^2$

Hint: Will any real number work or only integers? Positives or negatives? What is $\sqrt{-1}$? How many solutions are there?

$\sqrt{x^2} = (\sqrt{x})^2 \Leftrightarrow (x^2)^{1/2} = (x^{1/2})^2 \Leftrightarrow |x| = x \Leftrightarrow x \geq 0 \Leftrightarrow \boxed{x \in [0, \infty)}$

Test: Try $x = -1$: $(\sqrt{-1})^2$ = undefined if we are concerned only with real numbers

but $\sqrt{(-1)^2} = \sqrt{1} = 1$

Try $x = 1$: $\sqrt{1^2} = \sqrt{1} = 1$ and $(\sqrt{1})^2 = 1^2 = 1$

Note 1: If we are concerned only with real numbers then the $\sqrt{\ }$ symbol refers to a positive quantity, for instance, $\sqrt{4} = 2$.

You get a $\pm$ when you solve equations such as $x^2 = 4$. The solutions are $x = \pm\sqrt{4} = \pm 2$.

So, $\sqrt{4} = 2$ and $-\sqrt{4} = -2$. Don't automatically put $\pm$ when you see the $\sqrt{\ }$ symbol.

Note 2: $\sqrt{x^2}$ is the same as $|x|$.

$\sqrt{(-1)^2} = \sqrt{1} = 1 = |-1|$

$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2|$ etc.

#6) Solve for a in terms of x, y, and z: $xa^2 + ya + z = 0$ (show your work)

$xa^2 + ya + z = 0 \Leftrightarrow a^2 + \frac{y}{x}a + \frac{z}{x} = 0 \Leftrightarrow a^2 + \frac{y}{x}a = -\frac{z}{x}$

$\Leftrightarrow \underbrace{a^2 + \frac{y}{x}a + \frac{y^2}{4x^2}} = \frac{y^2}{4x^2} - \frac{z}{x} = \frac{y^2}{4x^2} - \frac{4xz}{4x^2} = \frac{y^2 - 4xz}{4x^2}$

$\Leftrightarrow \left(a + \frac{y}{2x}\right)^2 = \frac{y^2 - 4xz}{4x^2} \Leftrightarrow a + \frac{y}{2x} = \pm\sqrt{\frac{y^2 - 4xz}{4x^2}} = \frac{\sqrt{y^2 - 4xz}}{2x}$

$\Leftrightarrow a = -\frac{y}{2x} \pm \frac{\sqrt{y^2 - 4xz}}{2x} = \boxed{\frac{-y \pm \sqrt{y^2 - 4xz}}{2x} = a}$ (The Quadratic Formula)