Name____________________Banner____________

Spring 2008 Quiz #3 Solutions Precalculus

Clear your desk of everything but this quiz, a writing utensil (pen or pencil), and your student I.D. (I may verify your enrollment in this class if I don't recognize you). Do **NOT** use a calculator or formula/"cheat" sheet of any kind. Do not talk or look around the room.

Show your work. An answer without a solution receives no credit. Use proper notation. Think before you write or give up. Write on this paper only. Separate the pages and use the backsides for your scratchwork and to cover your work from the eyes of cheaters. Identify what scratchwork goes to which problem. Scratchwork is not a solution. Use your scratchwork to write a complete solution to each problem in the appropriate space provided. Box your answer at the end of your solution. Write legibly and make your arguments clear.

Please ask for clarification if a problem is illegible or ambiguous or if you suspect a typo. Do easy problems first. Check your work. Use the entire class time.

Do not leave this room with this quiz or a copy of any part of this quiz. Do not tell other students about any of the contents of this quiz as it can give them an unfair advantage (which is the same as an unfair disadvantage for you because it drives your grade down compared to the class average).

Your grade will be available in webCT as soon as the grader enters it, which should be within a week or so.

#1) Suppose Jerry was a race car driver who liked to drive too fast
But even when the curves were sharp
He never seemed to crash
Although his engine had more power
Any more than 200 miles per hour
Would throw his car from the track
But Jerry wasn't concerned about that
Because he only counted laps
And in the last half-hour he has counted ten.
Find the maximum radius that this track can have
If Jerry is going to live to claim another win

Max linear speed $= V_{max} = 200 \frac{mi}{hour}$

Current angular speed $= \omega = \frac{10 \text{ laps}}{\frac{1}{2} \text{ hour}} = 20 \frac{\text{laps}}{\text{hour}}$

$V = \omega r \Leftrightarrow V_{max} = \omega r_{max}$

$\Leftrightarrow r_{max} = \frac{V_{max}}{\omega} = 200 \frac{mi}{hour} \Big/ 20 \frac{laps}{hour}$

$= 200 \frac{mi}{\cancel{hour}} \left(\frac{\cancel{hour}}{20 \text{ laps}} \right)$

$= \frac{200}{20} \frac{mi}{lap} = 10 \frac{mi}{lap}$

$\Rightarrow$ There can be at most 10 miles in a lap

$\Rightarrow$ Maximum allowed length of 1 lap $= 2\pi r_{max} = 10$ miles

$\Leftrightarrow r_{max} = \frac{10}{2\pi}$ miles $=$ $\boxed{\frac{5}{\pi} \text{ miles} = V_{max}}$

Double Check our work:

- Maximum allowed length of 1 lap $= 2\pi r_{max}$
- Jerry goes 20 laps in 1 hour
- $V_{max} = 200 \frac{mi}{hour}$

So, $V_{max} = 200 \frac{mi}{hour} = \frac{s}{t} = \frac{\text{length of 20 laps}}{hour} = \frac{20 \cdot 2\pi r_{max}}{hour}$

$$\Rightarrow \quad r_{max} = 200 \frac{mi}{\cancel{hour}} \left(\frac{\cancel{hour}}{20 \cdot 2\pi} \right) = \frac{200}{40\pi}\, mi = \boxed{\frac{5}{\pi}\, mi = V_{max}}$$

Triple Check:

$$\omega = \frac{10 \text{ laps}}{\frac{1}{2} \text{ hour}} = \frac{20 \text{ laps}}{hour} = \frac{20}{hour} \cancel{laps} \left(\frac{2\pi \, rad}{1 \, \cancel{lap}} \right) = 40\pi \frac{rad}{hour}$$

$$V_{max} = r_{max}\,\omega \quad \Rightarrow \quad r_{max} = \frac{V_{max}}{\omega} = 200 \frac{mi}{hour} \Big/ \left(40\pi \frac{rad}{hour} \right)$$

$$= \frac{200 \, mi}{\cancel{hour}} \left(\frac{\cancel{hour}}{40\pi \, rad} \right) = \frac{200 \, mi}{40\pi \, rad} = \frac{5}{\pi} \frac{mi}{rad}$$

$$= \boxed{\frac{5}{\pi}\, mi = V_{max}}$$

We leave out the word radians since the end result is no longer referring to an angle. We can do this because radians are dimensionless units.

#2) The force of gravity causes masses to move toward one another. A peculiar consequence of the strength of this force for a given distance between the massive objects is that when a small object ends up orbiting a much larger object, the orbit is always an ellipse. The Earth's orbit about the Sun is one such ellipse but the Earth's orbit is very close to being circular. A circle is a special kind of ellipse, just as a square is a special kind of rectangle. We approximate Earth's orbit to be a perfect circle and we define a new unit of length equal to the radius of this circle called an "Astronomical Unit", or 1 AU.

A) In terms of years and AU's, what is the Earth's linear speed as it travels around the sun?

$$v := \frac{s}{t} = \frac{\text{Distance around the sun}}{1\ \text{year}} = \frac{2\pi r}{1\ \text{year}} = \frac{2\pi\,(1\ \text{AU})}{1\ \text{year}} = \boxed{\frac{2\pi\ \text{AU}}{\text{year}} = v}$$

B) In terms of orbits and years, what is the Earth's angular speed?

$$\boxed{\omega := \frac{\theta}{t} = \frac{1\ \text{orbit}}{\text{year}}}$$