Name______________________Banner_____________

Spring 2008 Quiz #4 Solutions Precalculus

Clear your desk of everything but this quiz, a writing utensil (pen or pencil), and your student I.D. (I may verify your enrollment in this class if I don't recognize you). Do **NOT** use a calculator or formula/"cheat" sheet of any kind. Do not talk or look around the room.

Show your work. An answer without a solution receives no credit. Use proper notation. Think before you write or give up. Write on this paper only. Separate the pages and use the backsides for your scratchwork and to cover your work from the eyes of cheaters. Identify what scratchwork goes to which problem. Scratchwork is not a solution. Use your scratchwork to write a complete solution to each problem in the appropriate space provided. Box your answer at the end of your solution. Write legibly and make your arguments clear.

Please ask for clarification if a problem is illegible or ambiguous or if you suspect a typo. Do easy problems first. Check your work. Use the entire class time.

Do not leave this room with this quiz or a copy of any part of this quiz. Do not tell other students about any of the contents of this quiz as it can give them an unfair advantage (which is the same as an unfair disadvantage for you because it drives your grade down compared to the class average).

Your grade will be available in WebCT as soon as the grader enters it, which should be within a week or so.

#1)

- Sine and Cosine are the first two trigonometric functions we learn about when studying trigonometry. These two functions can be represented using what is called a "Taylor Series" after a mathematician by the name of Taylor. Here are sine and cosine as represented by their Taylor series:

$$\sin(\#) = \# - \frac{\#^3}{3\cdot 2\cdot 1} + \frac{\#^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} - \cdots$$

$$\cos(\#) = 1 - \frac{\#^2}{2\cdot 1} + \frac{\#^4}{4\cdot 3\cdot 2\cdot 1} - \cdots$$

- These expressions can be simplified by using the factorial function, which is represented by an exclamation point "!" as follows:

$$M! = M(M-1)(M-2)\cdots 3\cdot 2\cdot 1 \quad = \quad M\cdot(M-1)!$$

For example: $3! = 3\cdot 2\cdot 1 = 3\cdot(2\cdot 1) = 3(2!) = 3\cdot 2!$

$2! = 2\cdot 1 = 2\cdot 1!$

$1! = 1$

From the recursive definition above: $M! = M\cdot(M-1)!$

We can see that $0! = 1$ since $1 = 1! = 1\cdot(1-1)! = 1\cdot 0! = 0!$

- Sigma notation can be applied to sums whose terms are all of the same form. Here is an example:

$$\sum_{p=-3}^{2} (-2)^p (p+3)! = (-2)^{-3}(-3+3)! + (-2)^{-2}(-2+3)! + (-2)^{-1}(-1+3)!$$
$$+ (-2)^0(0+3)! + (-2)^1(1+3)! + (-2)^2(2+3)!$$
$$= -\frac{0!}{2^3} + \frac{1!}{2^2} - \frac{2!}{2} + 3! - 2\cdot 4! + 2^2\cdot 5!$$

1A) Use sigma notation and the factorial function where appropriate to simplify the representations of the Taylor Series for:

i) $\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots = \frac{\theta^1}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{\theta^{2n+1}}{(2n+1)!} = \sin(\theta)}$

ii) $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \cos(x)}$

iii) $e^t = 1 + t + \frac{t^2}{2\cdot 1} + \frac{t^3}{3\cdot 2\cdot 1} + \frac{t^4}{4\cdot 3\cdot 2\cdot 1} + \cdots = \frac{t^0}{0!} + \frac{t^1}{1!} + \frac{t^2}{2!} + \cdots = \boxed{\sum_{n=0}^{\infty} \frac{t^n}{n!} = e^t}$

1B) The following problems require converting expressions from sigma notation forms to their standard forms. Show as many terms of the sum as is needed to be unambiguous. Properly use the "dot, dot, dot" notation, "...", where appropriate.

i) For $|r|<1$ and $b \in \mathbb{R}$ is a constant, "The Geometric Series" is represented by the following form. Expand the sigma sum.

$$\frac{b}{1-r} = \boxed{\sum_{n=0}^{\infty} br^n = br^0 + br^1 + br^2 + \cdots}$$

ii) Using the given information for part (i) above, give the proper sigma sum expression and its corresponding term by term expansion for $\frac{1}{1+x^2}$ when $|x^2|<1$

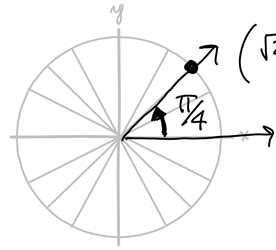
Hint: $\frac{1}{1+x^2} = \frac{b}{1-r}$ with $b=1$ and $r=x^2$

Since $\frac{b}{1-r} = \sum_{n=0}^{\infty} br^n = br^0 + br^1 + br^2 + \cdots$

We have $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} 1\cdot(x^2)^n = \sum_{n=0}^{\infty} x^{2n} = \boxed{x^0 + x^2 + x^4 + x^6 + \cdots = \frac{1}{1+x^2}}$

#2) Label the following angles and the corresponding points on the unit circle and evaluate sine and cosine for each angle.

Example: $\theta = \dfrac{\pi}{4}$

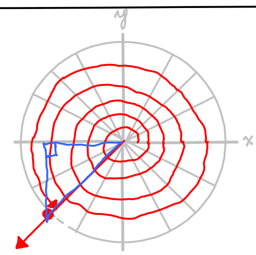


$\left(\sqrt{2}/2, \sqrt{2}/2\right)$

$\cos(\pi/4) = \sqrt{2}/2$

$\sin(\pi/4) = \sqrt{2}/2$

2A) $\theta = \dfrac{37\pi}{4}$



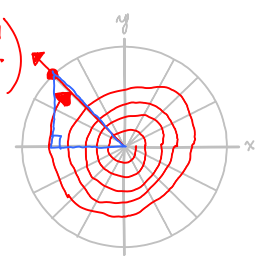
$\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

$\cos\left(\dfrac{37\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$\sin\left(\dfrac{37\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

2B) $\theta = -\dfrac{37\pi}{4}$

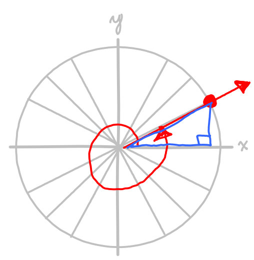
$\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$



$\cos\left(-\dfrac{37\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$\sin\left(-\dfrac{37\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

2C) $\theta = \dfrac{13\pi}{6}$

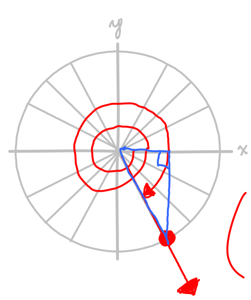


$\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$\cos\left(\dfrac{13\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

$\sin\left(\dfrac{13\pi}{6}\right) = \dfrac{1}{2}$

2D) $\theta = -\dfrac{13\pi}{3}$



$\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

$\cos\left(\dfrac{-13\pi}{3}\right) = \dfrac{1}{2}$

$\sin\left(\dfrac{-13\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$