Name_____ Banner_____

**Fall 2007** **Quiz #5** **Solutions**

#1) Fill in the blanks in the chart below with the information we've learned regarding the unit circle and how to use it to evaluate the 6 trig functions.

| $r=1$ | | $x$ | $y$ | $y/x$ | $x/y$ | $1/x$ | $1/y$ |
|---|---|---|---|---|---|---|---|
| $\theta$ | | $\cos(\theta)$ | $\sin(\theta)$ | $\tan(\theta)$ | $\cot(\theta)$ | $\sec(\theta)$ | $\csc(\theta)$ |
| 0 | $0^\circ$ | 1 | 0 | 0 | Undefined | 1 | Undefined |
| $\pi/6$ | $30^\circ$ | $\sqrt{3}/2$ | $1/2$ | $1/\sqrt{3} = \sqrt{3}/3$ | $\sqrt{3}$ | $2/\sqrt{3} = \frac{2\sqrt{3}}{3}$ | 2 |
| $\pi/4$ | $45^\circ$ | $1/\sqrt{2} = \sqrt{2}/2$ | $1/\sqrt{2} = \sqrt{2}/2$ | 1 | 1 | $\sqrt{2}$ | $\sqrt{2}$ |
| $\pi/3$ | $60^\circ$ | $1/2$ | $\sqrt{3}/2$ | $\sqrt{3}$ | $1/\sqrt{3} = \sqrt{3}/3$ | 2 | $2/\sqrt{3} = \frac{2\sqrt{3}}{3}$ |
| $\pi/2$ | $90^\circ$ | 0 | 1 | Undefined | 0 | Undefined | 1 |
| $2\pi/3$ | $120^\circ$ | $-1/2$ | $\sqrt{3}/2$ | $-\sqrt{3}$ | $-1/\sqrt{3} = -\sqrt{3}/3$ | -2 | $2/\sqrt{3} = \frac{2\sqrt{3}}{3}$ |
| $3\pi/4$ | $135^\circ$ | $-1/\sqrt{2} = -\sqrt{2}/2$ | $1/\sqrt{2} = \sqrt{2}/2$ | -1 | -1 | $-\sqrt{2}$ | $\sqrt{2}$ |
| $5\pi/6$ | $150^\circ$ | $-\sqrt{3}/2$ | $1/2$ | $-1/\sqrt{3} = -\sqrt{3}/3$ | $-\sqrt{3}$ | $-2/\sqrt{3} = \frac{-2\sqrt{3}}{3}$ | 2 |
| $\pi$ | $180^\circ$ | -1 | 0 | 0 | Undefined | -1 | Undefined |
| $7\pi/6$ | $210^\circ$ | $-\sqrt{3}/2$ | $-1/2$ | $1/\sqrt{3} = \sqrt{3}/3$ | $\sqrt{3}$ | $-2/\sqrt{3} = \frac{-2\sqrt{3}}{3}$ | -2 |
| $5\pi/4$ | $225^\circ$ | $-1/\sqrt{2} = -\sqrt{2}/2$ | $-1/\sqrt{2} = -\sqrt{2}/2$ | 1 | 1 | $-\sqrt{2}$ | $-\sqrt{2}$ |
| $4\pi/3$ | $240^\circ$ | $-1/2$ | $-\sqrt{3}/2$ | $\sqrt{3}$ | $1/\sqrt{3} = \sqrt{3}/3$ | -2 | $-2/\sqrt{3} = \frac{-2\sqrt{3}}{3}$ |
| $3\pi/2$ | $270^\circ$ | 0 | -1 | Undefined | 0 | Undefined | -1 |
| $5\pi/3$ | $300^\circ$ | $1/2$ | $-\sqrt{3}/2$ | $-\sqrt{3}$ | $-1/\sqrt{3} = -\sqrt{3}/3$ | 2 | $-2/\sqrt{3} = \frac{-2\sqrt{3}}{3}$ |
| $7\pi/4$ | $315^\circ$ | $1/\sqrt{2} = \sqrt{2}/2$ | $-1/\sqrt{2} = -\sqrt{2}/2$ | -1 | -1 | $\sqrt{2}$ | $-\sqrt{2}$ |
| $11\pi/6$ | $330^\circ$ | $\sqrt{3}/2$ | $-1/2$ | $-1/\sqrt{3} = -\sqrt{3}/3$ | $-\sqrt{3}$ | $2/\sqrt{3} = \frac{2\sqrt{3}}{3}$ | -2 |

#2) Starting with the pythagorean theorem $x^2+y^2=r^2$, derive the 3 pythagorean identities:

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A) $\sin^2\theta+\cos^2\theta=1$: $x^2+y^2=r^2 \Rightarrow \frac{x^2+y^2}{r^2}=\frac{r^2}{r^2} \Rightarrow \left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=1$

$\Rightarrow \sin^2\theta+\cos^2\theta=1$

B) $1+\tan^2\theta=\sec^2\theta$: $x^2+y^2=r^2 \Rightarrow \frac{x^2+y^2}{x^2}=\frac{r^2}{x^2} \Rightarrow 1+\left(\frac{y}{x}\right)^2=\left(\frac{r}{x}\right)^2$

$\Rightarrow \tan^2\theta+1=\sec^2\theta$

C) $\cot^2\theta+1=\csc^2\theta$: $x^2+y^2=r^2 \Rightarrow \frac{x^2+y^2}{y^2}=\frac{r^2}{y^2} \Rightarrow \left(\frac{x}{y}\right)^2+1=\left(\frac{r}{y}\right)^2$

$\Rightarrow 1+\cot^2\theta=\csc^2\theta$

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#3 Derive the reciprocal and quotient identities:

Example deriving $\sin\theta=\frac{1}{\csc\theta}$: $\sin\theta=\frac{y}{r}=\frac{1}{r/y}=\frac{1}{\csc\theta}$

A) $\tan\theta=\frac{1}{\cot\theta}$: $\tan\theta=\frac{y}{x}=\frac{1}{x/y}=\frac{1}{\cot\theta}$

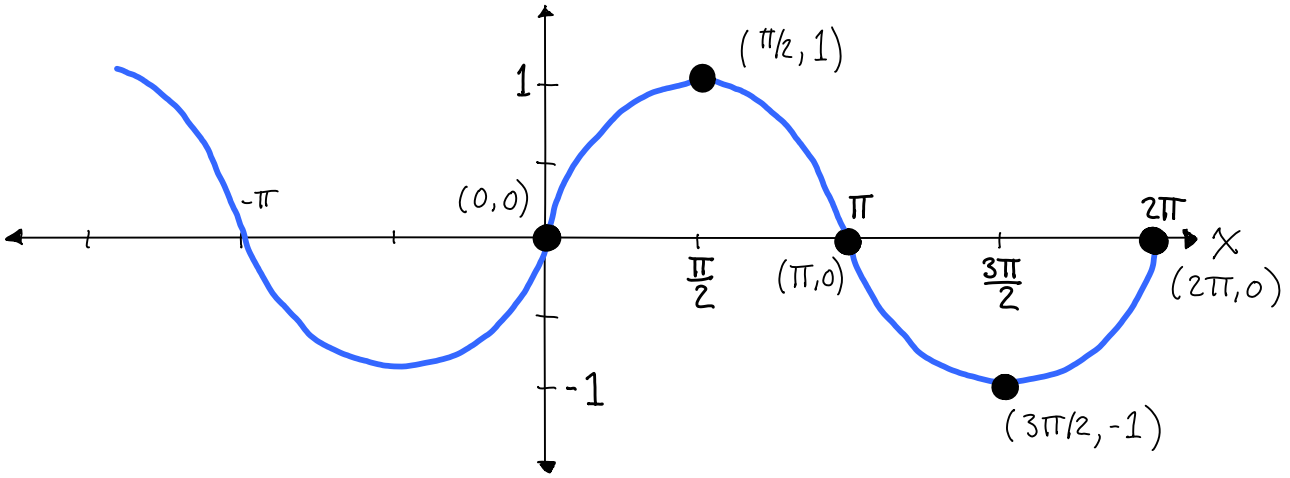
B) $\cos\theta=\frac{1}{\sec\theta}$: $\cos\theta=\frac{x}{r}=\frac{1}{r/x}=\frac{1}{\sec\theta}$

C) $\tan\theta=\frac{\sin\theta}{\cos\theta}$: $\tan\theta=\frac{y}{x}=\frac{y/r}{x/r}=\frac{\sin\theta}{\cos\theta}$

D) $\cot\theta=\frac{\cos\theta}{\sin\theta}$: $\cot\theta=\frac{x}{y}=\frac{x/r}{y/r}=\frac{\cos\theta}{\sin\theta}$

#4) Graph the following

A) sin(x)




B) cos(x)