Name____________________Banner____________

Spring 2008 Quiz #10 Solutions Precalculus

Clear your desk of everything but this quiz, a writing utensil (pen or pencil), and your student I.D. (I may verify your enrollment in this class if I don't recognize you). Do **NOT** use a calculator or formula/"cheat" sheet of any kind. Do not talk or look around the room.

Show your work. An answer without a solution receives no credit. Use proper notation. Think before you write or give up. Write on this paper only. Separate the pages and use the backsides for your scratchwork and to cover your work from the eyes of cheaters. Identify what scratchwork goes to which problem. Scratchwork is not a solution. Use your scratchwork to write a complete solution to each problem in the appropriate space provided. Box your answer at the end of your solution. Write legibly and make your arguments clear.

Please ask for clarification if a problem is illegible or ambiguous or if you suspect a typo. Do easy problems first. Check your work. Use the entire class time.

Do not leave this room with this quiz or a copy of any part of this quiz. Do not tell other students about any of the contents of this quiz as it can give them an unfair advantage (which is the same as an unfair disadvantage for you because it drives your grade down compared to the class average).

Your grade will be available in WebCT as soon as the grader enters it, which should be within a week or so.

If necessary, use the following identities to solve the problems on this quiz.

<u>Sum and Difference Identities for Sine and Cosine</u>

$\cos(\alpha+\beta) \equiv \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha-\beta) \equiv \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$\sin(\alpha+\beta) \equiv \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\sin(\alpha-\beta) \equiv \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

where α and β are real numbers (or angles).

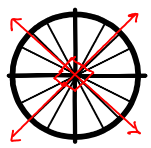
#1) Find all solutions for the following equations.

A) $\sin^2(2\theta) - \cos^2(2\theta) = 0$

$$\sin^2(2\theta) - \cos^2(2\theta) = 0 \iff \sin^2(2\theta) = \cos^2(2\theta)$$

$$\iff \sin(2\theta) = \pm\cos(2\theta)$$

2θ



$$\iff \left\{\begin{aligned} 2\theta &= \pi/4 + 2\pi K \\ 2\theta &= 3\pi/4 + 2\pi K \\ 2\theta &= 5\pi/4 + 2\pi K \\ 2\theta &= 7\pi/4 + 2\pi K \end{aligned}\right\} \iff 2\theta = \frac{\pi}{4} + \frac{\pi}{2}K$$

$$\iff \boxed{\theta = \frac{\pi}{8} + \frac{\pi K}{4}}$$

B) $\cos(2x) - \sin^2(x) = -2$

Hint: Consider the Sum and Difference Identities above when $\alpha = \beta = x$

$$\cos(2x) - \sin^2(x) = -2$$

$$\iff \cos(x+x) - \sin^2(x) = -2$$

$$\iff \cos(x)\cos(x) - \sin(x)\sin(x) - \sin^2(x) = -2$$

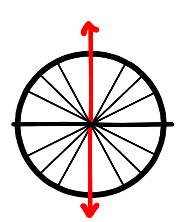
$$\iff \cos^2(x) - 2\sin^2(x) = -2$$

$$\iff 1 - \sin^2(x) - 2\sin^2(x) = -2$$

$$\iff 1 - 3\sin^2(x) = -2$$

$$\iff -3\sin^2(x) = -3$$

$$\iff \sin^2(x) = 1$$



$$\iff \sin(x) = \pm 1$$

$$\iff \boxed{x = \frac{\pi}{2} + \pi K}$$

#2) Prove the following identities:

A) $$\frac{\cot\theta + \tan\theta}{\sec\theta\csc\theta} \equiv 1$$

$$\frac{\cot\theta + \tan\theta}{\sec\theta\csc\theta} \equiv \frac{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}} \equiv \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta\sin\theta}{1}\right)$$

$$\equiv \frac{\cos^2\theta\cancel{\sin\theta}}{\cancel{\sin\theta}} + \frac{\cancel{\cos\theta}\sin^2\theta}{\cancel{\cos\theta}} \equiv \cos^2\theta + \sin^2\theta \equiv 1$$

B) $$1 - \cos(4\theta) \equiv 8\sin^2(\theta)\cos^2(\theta)$$

$$1 - \cos(4\theta) \equiv 1 - \cos(2\theta + 2\theta) \equiv 1 - [\cos(2\theta)\cos(2\theta) - \sin(2\theta)\sin(2\theta)]$$

$$\equiv 1 - \cos^2(2\theta) + \sin^2(2\theta)$$

$$\equiv \sin^2(2\theta) + \sin^2(2\theta)$$

$$\equiv 2\sin^2(2\theta)$$

$$\equiv 2[\sin(\theta+\theta)]^2$$

$$\equiv 2[\sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta)]^2$$

$$\equiv 2[2\sin(\theta)\cos(\theta)]^2$$

$$\equiv 2[4\sin^2(\theta)\cos^2(\theta)]$$

$$\equiv 8\sin^2(\theta)\cos^2(\theta)$$