

PRECALCULUS QUIZ # 9

#1) WRITE DOWN THE SUM AND DIFFERENCE IDENTITIES FOR SINE AND COSINE AND USE THEM TO FIND SINE AND COSINE OF 15°

(i) SUM AND DIFFERENCE IDENTITIES

- $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$,
- $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$.

(ii) SINE AND COSINE OF 15°

let us use the fact that $15^\circ = 45^\circ - 30^\circ$.

- $\sin(15^\circ) = \sin(45^\circ - 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$
 $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
 $= \frac{\sqrt{2} \cdot \sqrt{3} - \sqrt{2}}{4}$
 $= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$,

- $\cos(15^\circ) = \cos(45^\circ - 30^\circ)$
 $= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$
 $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
 $= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$.

#2) WRITE DOWN ALL OF THE COFUNCTION IDENTITIES AND USE THEM TO FIND SINE AND COSINE OF 75°

(a) COFUNCTION

$$\bullet \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta, \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta.$$

$$\bullet \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta, \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta.$$

$$\bullet \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta, \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta.$$

(b) SINE AND COSINE OF 75° by cofunctions.

let us use the fact that $90^\circ - 75^\circ = 15^\circ$.

Then (ii) yields,

$$\bullet \underline{\sin(75^\circ)} = \cos(90^\circ - 75^\circ) = \cos(15^\circ) = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}},$$

$$\bullet \underline{\cos(75^\circ)} = \sin(90^\circ - 75^\circ) = \sin(15^\circ) = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}.$$