Note Title 6/19/2006

#1 ABCD

Which of the following are completely true?

a) If you flip the graph of an odd function over the x-axis it will look the same as if you flipped it over the y-axis

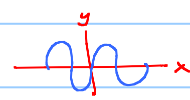
b) If you flip the graph of an even function over the y-axis it will look the same as the original function.

c) letter (a) above is exemplified by the fact that $\sin(-x) = -\sin(x)$ where $\sin(-x)$ represents flipping $\sin(x)$ over the y-axis and $-\sin(x)$ represents flipping $\sin(x)$ over the x-axis

d) Although $-\sin(-x)$ flips the graph of $\sin(x)$ over the x-axis and then the y-axis (or vice versa), it can also be thought of as flipping $\sin(x)$ over the x-axis twice or the y-axis twice. This is one way to think about the fact that $-\sin(-x) = \sin(x)$

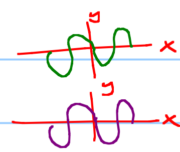
e) None of the above

a) true. Consider $\sin(x)$



flip it over x-axis $\Rightarrow -\sin(x)$

flip it over y-axis $\Rightarrow \sin(-x)$

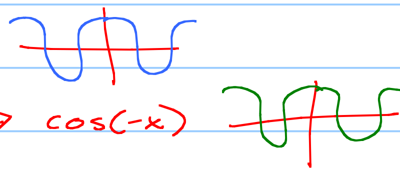


$\Rightarrow$ the same

$\Rightarrow -\sin(x) = \sin(-x)$

b) true. Consider $\cos(x)$

flip it over y-axis $\Rightarrow \cos(-x)$



$\Rightarrow$ the same

$\Rightarrow \cos(-x) = \cos(x)$

c) true. see (a)

d) true. If flipping over the x-axis is the same as flipping over the y-axis, then flipping over one and then the other is the same as flipping over one twice. Flip over an axis twice and you're right back where you started. Hence,

$-\sin(-x) = --\sin(x) = \sin(x)$, or

$-\sin(-x) = \sin(--x) = \sin(x)$

flip over x and y axis

flip over y axis twice

flip over x axis twice

gives you no change

#2 Which of the following are completely true?

AB

a) There are two even and four odd trig functions
b) The identity $\cos(\frac{\pi}{2}-x) = \sin(x)$ shows that although $\cos(x)$ is even, it's shifted version, $\cos(\frac{\pi}{2}-x)$ is an odd function.
c) Adding even functions produces an odd function.
d) Multiplying even functions produces an odd function.
e) None of the above

a) true. So far, we have seen
$\sin(x)$, $\tan(x)$, $\cot(x)$, $\csc(x)$ are odd and
$\cos(x)$ and $\sec(x)$ are even
Look at exam #1 solutions to see why $\sin(x)$ is odd + $\cos(x)$ is even if you don't remember. Now,
$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\tan(x) \Rightarrow \tan(-x) = -\tan(x) \Rightarrow \tan(x)$ is odd
Apply this to the others.

b) True. $\sin(x)$ is odd, so if $\cos(\frac{\pi}{2}-x) = \sin(x)$ then $\cos(\frac{\pi}{2}-x)$ is odd.

c) False. Consider $\cos(x) + \cos(x) = 2\cos(x)$ is still even

d) False. Multiplying two odd functions produces an even function.
(even)(even) = even (odd)(odd) = even (odd)(even) = odd

#3 Which of the following are completely true?

BC

a) Although $\sin(x)$ is 2π periodic, $\sin(4x)$ has 4 times as many periods as $\sin(x)$ and is therefore $2\pi \cdot 4 = 8\pi$ periodic
b) The period of $\sin(\omega x)$ is $T = \frac{2\pi}{\omega}$ but the period of $\tan(\omega x)$ is $T = \pi/\omega$ because $\tan(x)$ is π periodic
c) If $\omega > 1$ it makes the graph look horizontally compressed but if $\omega < 1$ it stretches it horizontally.
d) $3\sin(3x)$ has a graph 3 times as tall and a period 3 times as long as the graph of $\sin(x)$
e) None of the above

a) False. $\sin(4x)$ has 4 times as many periods as $\sin(x)$ and therefore they must be 4 times smaller $\Rightarrow T = \frac{2\pi}{4} = \frac{\pi}{2}$

b) True c) True

d) False: its period is 3 times shorter

#4 Which of the following are completely true?

ABCD

a) If $\cos x > 0$ then $\cos x = \sqrt{1 - \sin^2 x}$

b) $\cos(x - \frac{\pi}{2}) = \cos[-(\frac{\pi}{2} - x)] = \cos(\frac{\pi}{2} - x) = \sin(x)$

c) $\cos(2x + \pi)$ is a shift of $\cos x$ to the left by $\frac{\pi}{2}$, not π, since $\cos(2x+\pi) = \cos[2(x + \frac{\pi}{2})]$ reveals that the amount being added to your x variable is $\frac{\pi}{2}$, not π, which is what is added to $2x$.

d) Choice (c) is an example of phase shift defined by ϕ/ω

e) None of the above

a) $x^2 + y^2 = r^2 \Rightarrow \frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2} \Rightarrow \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1 \Rightarrow \cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \cos\theta = \pm\sqrt{1 - \sin^2\theta}$ but $\cos\theta \geq 0 \Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} \Rightarrow$ true

b) true c) true d) true: here $\phi/\omega = -\frac{\pi}{2}$

#5 Which of the following are completely true?

ABCD

a) $\cos(x - \frac{17\pi}{2}) = \cos(x - 8\pi - \frac{\pi}{2}) = \cos(x - \frac{\pi}{2}) = \sin(x)$

b) $\sin(3x + \frac{16\pi}{3}) = \sin(3x + 8 \cdot \frac{2\pi}{3}) = \sin(3x)$

c) When going around the unit circle clockwise instead of counter-clockwise, the x-values are the same as if going counter-clockwise. This is one way to reason why $\cos(-\theta) = \cos\theta$

d) $\tan(\frac{\pi}{2} - x) = \cot x$ and $\sec(\frac{\pi}{2} - x) = \csc(x)$

e) None of the above

a) true: $\cos(x - \frac{17\pi}{2}) = \cos(x - 8\pi - \frac{\pi}{2}) =$

$= \cos(x - \frac{\pi}{2})$ since $\cos(x)$ is 2π periodic, a shift by 8π makes no difference

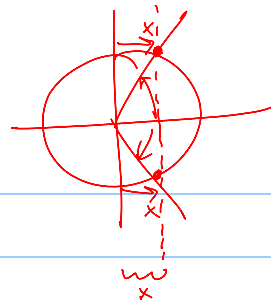
$= \cos[-(\frac{\pi}{2} - x)] = \cos(\frac{\pi}{2} - x) = \sin(x)$

drop minus sign since $\cos(x)$ is even

b) true: $\sin(3x + \frac{16\pi}{3}) = \sin(3x + 8 \cdot \frac{2\pi}{3}) = \sin(3x)$

this function has a period of $\frac{2\pi}{3}$ so a horizontal shift of any multiple of that period won't make any difference.

c) true

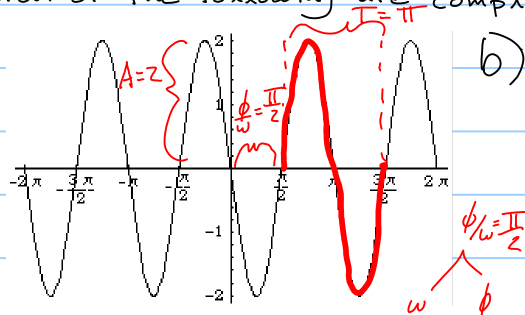


d) true: cofunctions of complementary angles are equal

#6

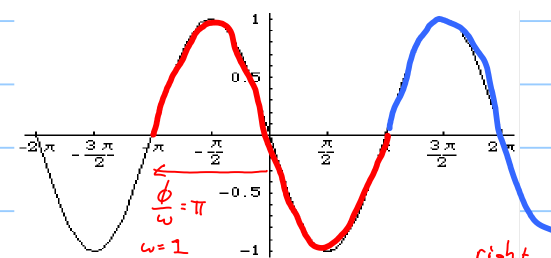
Which of the following are completely true?

a)



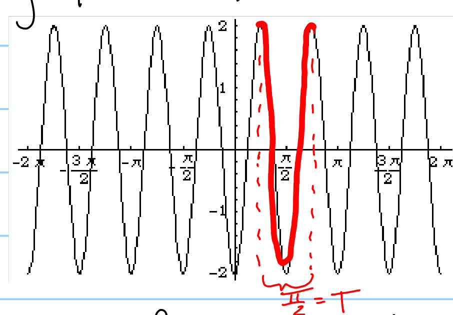
is a graph of $f(x) = 2\sin(2x-\pi)$

b)



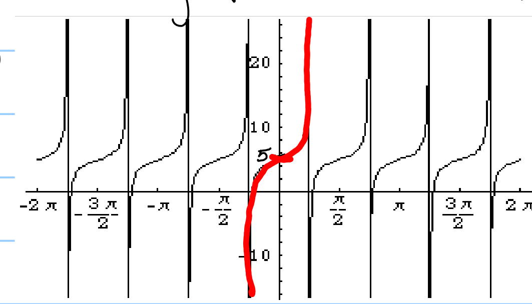
is a graph of $f(x) = \sin(x-\pi)$

c)



is a graph of $f(x) = 2\cos(2x-\pi)$

d)



is a graph of $f(x) = \tan(2x+5)$

e) None of the above

a) true $T=\pi=\frac{2\pi}{\omega} \Rightarrow \omega=2$

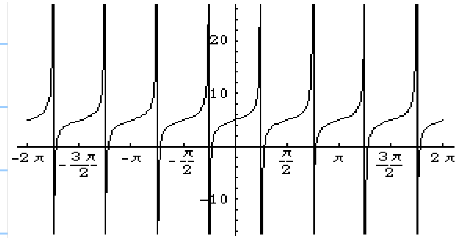
$\Rightarrow \frac{\phi}{\omega} = \frac{\phi}{2} = \frac{\pi}{2} \Rightarrow \phi=\pi$

b) true: $T=2\pi \Rightarrow \omega=1$, Shifted to the right by π

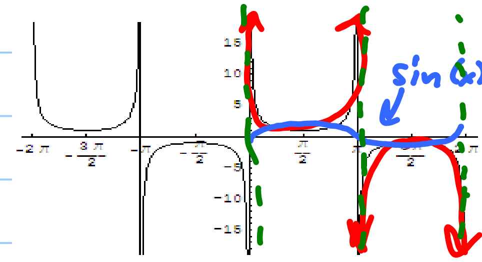
c) false: $T=\frac{\pi}{2}=\frac{2\pi}{\omega} \Rightarrow \omega=4\neq 2$

d) false: graph is shifted up 5, not left by 5

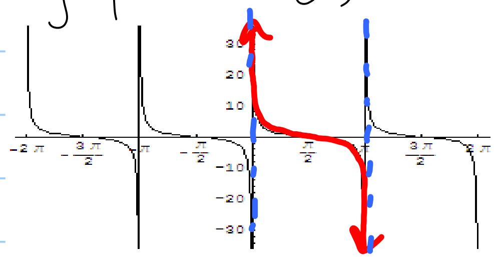
\#7 Which of the following are completely true?

a) 

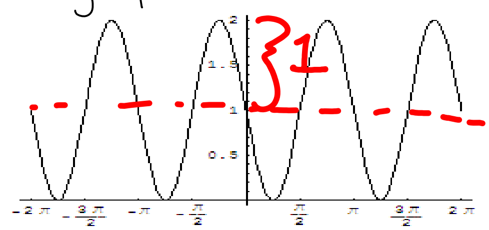
is a graph of $\sec(x)$

b) 

is a graph of $\csc(x)$

c) 

is a graph of $\cot(x)$

d) 

this function has an amplitude $A = 2$

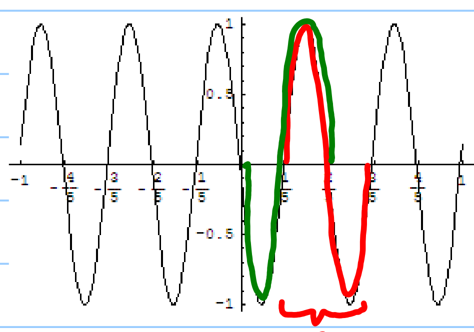
e) None of the above

a) false: this graph looks like $\tan(x)+5$

b) true: $\csc(x) = \frac{1}{\sin(x)}$ has asymptotes at $x = 0, \pi$

c) true: $\cot(x) = \frac{\cos(x)}{\sin(x)}$ has asymptotes at $x = 0, \pi$

d) False: $A = 1$

\#8 

$T = \frac{2}{5}$

is a graph of which of the following functions?

a) $\sin\left(\frac{\pi x}{5}\right)$ b) $-\sin(5\pi x)$

c) $\sin(5\pi x - 3)$ d) $\sin(5\pi x - 5)$

e) None of the above

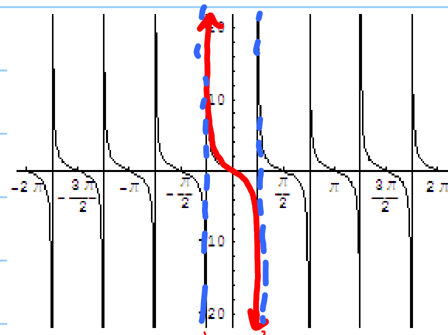
a) False $T = \frac{2}{5} = \frac{2\pi}{\omega} \Rightarrow \omega = 5\frac{2\pi}{2} = 5\pi$

b) True: $\omega = 5\pi$ & graph is folded over x-axis

c) False $\phi/\omega = \frac{3}{5\pi}$ but graph is shifted by $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \ldots$

d) False Same reason as (c)

#9



$T = \frac{\pi}{2}$

is a graph of which of the following functions?

a) $\cot(2x)$ b) $\tan(-2x)$

c) $-\tan(2x)$ d) $-\tan(2x-\pi)$

e) None of the above

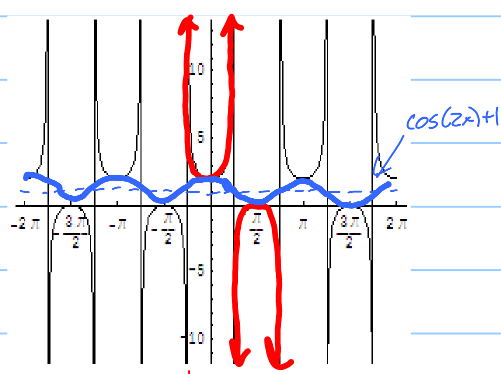
a) False: $\cot(2x) = \frac{\cos(2x)}{\sin(2x)}$ has asymptotes at 0 and $\frac{\pi}{2}$

b) True: $T = \frac{\pi}{2} = \frac{\pi}{\omega}$ and $\omega = 2$. The minus sign flips the graph.

c) True: $\tan(x)$ is odd so $\tan(-2x) = -\tan(2x)$. see (b)

d) True: The graph has a period of $\frac{\pi}{2}$, so shifting it horizontally by any multiple of $\frac{\pi}{2}$ will leave the graph unchanged. So, $-\tan(2x-\pi) = -\tan(2x)$

#10



$T = \pi$

is a graph of which of the following functions?

a) $\sec(2x)$ b) $\sec(x)+1$

c) $\sec(2x+1)$ d) $2\sec(x)$

e) None of the above

a) False: Period is correct but graph is shifted up by 1

b) False: Shift up is correct but period is not

c) False: The plus 1 should be outside paranthesis for a vertical shift

d) False: Amplitude, Period, + shift are wrong.

$\Rightarrow$ e) None of the above is true

BONUS The graph of $f(x) = x^2 + 2x + 1$

a) is a shift of the graph of $f(x) = x^2$ by 1 up
b) is a shift of the graph of $f(x) = x^2$ by 1 to the left
c) has even symetry
d) Would have even symetry if it was shifted to the right by 1.
e) None of the above

$f(x) = x^2 + 2x + 1 = (x+1)^2 \Rightarrow$ shift of x^2 to the left by 1.

$\Rightarrow$ b is true

Since x^2 is even, if we took $(x+1)^2$ and shifted it to the right by 1 it would be x^2 and would be even

$\Rightarrow$ d is true