Note Title 4/11/2006

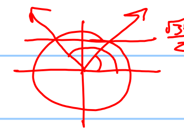
EXAM # 2 solutions

Find all solutions of $2\sin(3\theta) = \sqrt{3}$

a) $\frac{\pi}{3} + 2n\pi,\ \frac{2\pi}{3} + 2n\pi;\ n \in \mathbb{Z}$ b) $\frac{\pi}{9} + 2n\pi,\ \frac{2\pi}{9} + 2n\pi;\ n \in \mathbb{Z}$

c) $\frac{\pi}{9} + \frac{2}{3}n\pi,\ \frac{2\pi}{9} + \frac{2}{3}n\pi;\ n \in \mathbb{Z}$ d) $\frac{\pi}{9} + \frac{2}{3}n\pi;\ n \in \mathbb{Z}$

e) None of the above



$2\sin(3\theta) = \sqrt{3} \Rightarrow \sin(3\theta) = \frac{\sqrt{3}}{2}$

$\Rightarrow 3\theta = \frac{\pi}{3} + 2n\pi$ or $3\theta = \frac{2\pi}{3} + 2n\pi$

$\Rightarrow \theta = \frac{\pi}{9} + \frac{2}{3}n\pi$ or $\theta = \frac{2\pi}{9} + \frac{2}{3}n\pi$ where $n \in \mathbb{Z}$

$\sin[\sin^{-1}(0.34)] =$

a) $\cos[\cos^{-1}(0.34)]$ b) $\arccos[\cos(0.34)]$

c) $\tan[\tan^{-1}(0.34)]$ d) All of the above

e) None of the above

$\sin[\sin^{-1}(0.34)] = 0.34$ since 0.34 is in the domain of $\sin^{-1}(x)$.

$\cos[\cos^{-1}(0.34)] = 0.34$ since 0.34 is in the domain of $\cos^{-1}(x)$

$\arccos[\cos(0.34)] = 0.34$ since 0.34 is in the range of $\cos^{-1}(x)$

$\tan[\tan^{-1}(0.34)] = 0.34$ since 0.34 is in the domain of $\tan^{-1}(x)$

$\Rightarrow$ D

Which of the following is NOT true?

a) $\sec\theta + \tan\theta = \frac{\cos\theta}{1+\sin\theta}$ b) $\frac{\sin\theta}{\sin\theta - \cos\theta} = \frac{1}{1-\cot\theta}$

c) $\frac{1+\tan\theta}{1-\tan\theta} = \frac{\cot\theta + 1}{\cot\theta - 1}$ d) All of the above

e) None of the above

a) $\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \neq \frac{\cos\theta}{1+\sin\theta}$ ✗

b) $\frac{\sin\theta}{\sin\theta - \cos\theta} = \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}} \cdot \frac{\sin\theta}{\sin\theta-\cos\theta} = \frac{1}{1-\frac{\cos\theta}{\sin\theta}} = \frac{1}{1-\cot\theta}$ ✓

c) $\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\tan\theta}{1-\tan\theta} \cdot \frac{\cot\theta}{\cot\theta} = \frac{\cot\theta + \tan\theta\cot\theta}{\cot\theta - \tan\theta\cot\theta} = \frac{\cot\theta+1}{\cot\theta-1}$ ✓

$\Rightarrow$ A is Not true

If $\alpha = 150°$ and $\beta = 210°$, which of the following is true?

a) $\sin(2\alpha) = 2\sin\alpha\cos\alpha$ b) $\cos(\alpha-\beta) = \frac{1}{2}$

c) $\sin(\frac{\pi}{2}-\beta) = \cos\beta$ d) All of the above

e) None of the above

a) is the double angle identity for $\sin\theta$ and thus is always true ✓

b) $\cos(\alpha-\beta) = \cos(150°-210°) = \cos(-60°) = \cos(60°) = \frac{1}{2}$ ✓

c) is a basic identity. $\sin(\frac{\pi}{2}-\beta) = \sin(\frac{\pi}{2})\cos\beta - \cancel{\cos(\frac{\pi}{2})}\sin\beta = \cos\beta$ ✓

$\Rightarrow$ D

Find the exact value of $\tan(15°)$

a) $\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ b) $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$ c) $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$ d) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

e) None of the above

$\tan(15°) = \frac{\sin(15°)}{\cos(15°)}$ (You should remember these from class)

$= \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}}$ or $\frac{\frac{\sqrt{2-\sqrt{3}}}{2}}{\frac{\sqrt{2+\sqrt{3}}}{2}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \Rightarrow b$ or $\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$

remember

half angle identity

$$\sin(15°) = \sin\left(\frac{30°}{2}\right) = \sqrt{\frac{1-\cos(30°)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\sin(15°) = \sin(45°-30°) = \sin(45°)\cos(30°) - \cos(45°)\sin(30°)$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

similarly $\cos(15°) = \frac{\sqrt{2+\sqrt{3}}}{2}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$. These numbers are equal.

$\cos(115°)\cos(85°) + \sin(115°)\sin(85°) =$

a) $\sin(50°)\cos(10°) + \cos(50°)\sin(10°)$

b) $\sin(105°)\cos(15°) + \cos(105°)\sin(15°)$

c) $\cos(35°)\sin(155°) - \sin(35°)\cos(155°)$

d) All of the above   e) None of the above

$\cos(115°)\cos(85°) + \sin(115°)\sin(85°) = \cos(115°-85°) = \cos(30°)$

a) $\sin(50°)\cos(10°) + \cos(50°)\sin(10°) = \sin(50°+10°) = \sin(60°)$
$= \cos(90°-60°) = \cos(30°)$ ✓

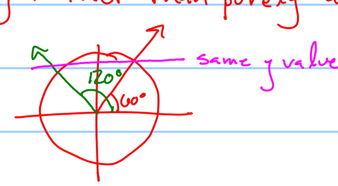
b) $\sin(105°)\cos(15°) + \cos(105°)\sin(15°) = \sin(105°+15°) = \sin(120°)$
$= \cos(90°-120°) = \cos(-30°) = \cos(30°)$ ✓

c) $\cos(35°)\sin(155°) - \sin(35°)\cos(155°) = \sin(155°-35°) = \sin(120°)$
$= \cos(90°-120°) = \cos(-30°) = \cos(30°)$ ✓

You can also view things geometrically rather than purely algebraically

a) gives $\sin(60°)$

b) gives $\sin(120°)$



c) gives $\sin(120°)$ obviously same as b

Remember to look at the triangles constructed in the unit circle by given angles & compare the sides of these triangles to compare trig quantities.

Find all solutions of $3\cos\theta + 3 = 2\sin^2\theta$

a) $\frac{n\pi}{3}$; $n \in \mathbb{Z}$ b) $\frac{n\pi}{4}, \frac{n\pi}{2}$; $n \in \mathbb{Z}$

c) $\frac{2\pi}{3} + 2n\pi,\ \frac{4\pi}{3} + 2n\pi,\ \pi + 2n\pi$; $n \in \mathbb{Z}$

d) $\frac{n\pi}{3}, \frac{n\pi}{4}$; $n \in \mathbb{Z}$ e) None of the above

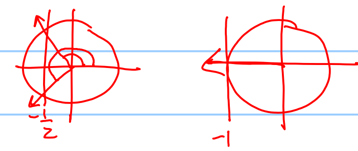
$$3\cos\theta + 3 = 2\sin^2\theta \Rightarrow -2\sin^2\theta + 3\cos\theta + 3 = 0$$

$$\Rightarrow -2(1-\cos^2\theta) + 3\cos\theta + 3 = 0 \Rightarrow -2 + 2\cos^2\theta + 3\cos\theta + 3 = 0$$

$$\Rightarrow 2\cos^2\theta + 3\cos\theta + 1 = 0 \Rightarrow (2\cos\theta + 1)(\cos\theta + 1) = 0$$

$$\Rightarrow 2\cos\theta + 1 = 0 \text{ or } \cos\theta + 1 = 0 \Rightarrow \cos\theta = -\tfrac{1}{2} \text{ or } \cos\theta = -1$$

$$\Rightarrow \begin{cases} \theta = \frac{2\pi}{3} + 2n\pi \\ \theta = \frac{4\pi}{3} + 2n\pi \end{cases} \text{ or } \theta = \pi + 2n\pi \quad \text{where } n \in \mathbb{Z}$$

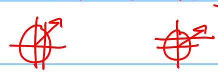


$\Rightarrow$ C

Find the exact value of $\sin\left[\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right]$

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$ e) None of the above

$$\sin\left[\cos^{-1}\left(\tfrac{1}{2}\right) + \sin^{-1}\left(\tfrac{1}{2}\right)\right] = \sin\left[\tfrac{\pi}{3} + \tfrac{\pi}{6}\right] = \sin\left(\tfrac{2\pi}{6} + \tfrac{\pi}{6}\right) = \sin\left(\tfrac{3\pi}{6}\right)$$



$$= \sin\left(\tfrac{\pi}{2}\right) = 1 \Rightarrow E$$

Find the exact value of $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

a) undefined b) $\frac{3}{2}$ c) $\frac{\pi}{2}$ d) $\frac{\sqrt{3}}{2}$ e) None of the above

$$\cos\left(\sin^{-1}\left(\tfrac{\sqrt{3}}{2}\right)\right) = \cos\left(\tfrac{\pi}{3}\right) = \tfrac{1}{2} \Rightarrow E$$

Find the exact value of $\sin(105°)$

a) $\frac{\sqrt{2+\sqrt{3}}}{2}$ b) 0 c) $\frac{\sqrt{6}+\sqrt{2}}{4}$ d) a and c e) None of the above

$\sin(105°) = \sin(90°+15°) = \sin(90° - -15°) = \cos(-15°)$

$= \cos(15°) = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $\frac{\sqrt{2+\sqrt{3}}}{2} \Rightarrow A$ and $C \Rightarrow D$

The expression $K - \frac{1}{\sec^2 x} = \sin^2 x$ is an identity when K is equal to

a) 0 b) 1 c) $\cos^2 x$ d) $\sin^2 x$ e) None of the above

$K - \frac{1}{\sec^2 x} = \sin^2 x \Leftrightarrow K = \sin^2 x + \frac{1}{\sec^2 x} = \sin^2 x + \cos^2 x = 1$

check $1 - \frac{1}{\sec^2 x} = 1 - \cos^2 x = \sin^2 x$ ✓ $\Rightarrow b$

Find the exact value of $\sin(\tan^{-1}(\frac{1}{4}))$

a) $\sqrt{17}$ b) $-\sqrt{17}$ c) $\frac{1}{\sqrt{17}}$ d) $\frac{-1}{\sqrt{17}}$ e) None of the above

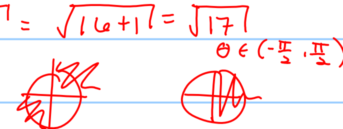
let $\tan^{-1}(1/4) = \theta$ then $\sin(\tan^{-1}(\frac{1}{4})) = \sin\theta$

and $\frac{1}{4} = \tan\theta$ where $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, (the range of $\tan^{-1}\theta$)

$\frac{1}{4} = \tan\theta \Rightarrow \frac{y}{x} = \frac{1}{4} = \tan\theta$. let $y=1$ and $x=4$. then

$r = \sqrt{x^2+y^2} = \sqrt{16+1} = \sqrt{17}$

$\Rightarrow \sin\theta = \pm y/r = \pm 1/\sqrt{17}$ plus or minus?

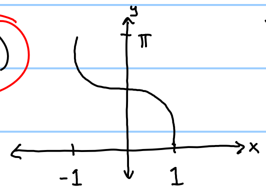


The angle must be in quadrant 1 where $\sin\theta$ is positive.

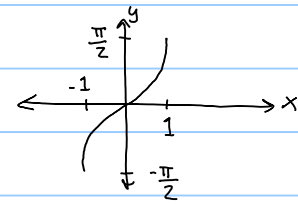
thus $\sin\theta = 1/\sqrt{17} \Rightarrow C$ is the answer.

Which of the following is a graph of $\arccos(x)$?

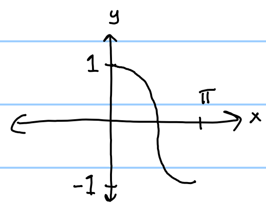
a)



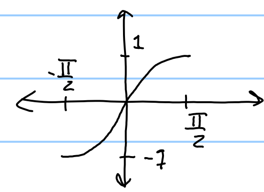
b)



c)

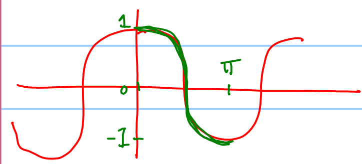


d)

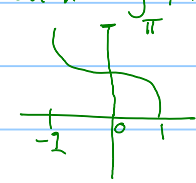


e) None of the above

remember which part of cosx we used to model $\cos^{-1}$ after



This peice passes the horizontal line test. Thus, when we switch $x \rightleftarrows y$, it looks like this



$\Rightarrow A$

If $\sin\alpha = 3/4$ and $\cos\beta = \frac{4}{5}$ where $0<\alpha<\frac{\pi}{2}$ and $-\frac{\pi}{2}<\beta<0$ find the exact value of $\sin(\alpha+\beta)$

a) $14\sqrt{3}-2$ b) $\sqrt{7}$ c) $\frac{3}{5} - \frac{3\sqrt{7}}{20}$ d) $\frac{\sqrt{7}-12}{10}$

e) None of the above

$\sin\alpha = 3/4 \Rightarrow \cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \pm\sqrt{1-\frac{9}{16}} = \pm\sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$ take + since $0<\alpha<\frac{\pi}{2}$

$\cos\beta = \frac{4}{5} \Rightarrow \sin\beta = \pm\sqrt{1-\cos^2\beta} = \pm\sqrt{1-16/25} = \pm\sqrt{\frac{9}{25}} = \frac{-3}{5}$ take − since $-\frac{\pi}{2}<\beta$

$\Rightarrow \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{3}{4}\cdot\frac{4}{5} - \frac{\sqrt{7}}{4}\cdot\frac{3}{5}$

$= \frac{12}{20} - \frac{3\sqrt{7}}{20} = \frac{3}{10} - \frac{3\sqrt{7}}{20} \Rightarrow C$

Express $\cos(6\theta)\cos(3\theta)$ as a sum without products

a) $\frac{1}{2}[\cos(3\theta)+\cos(9\theta)]$ b) $\frac{1}{2}[\cos(9\theta)-\cos(3\theta)]$

c) $\frac{1}{2}[\cos(3\theta)-\cos(9\theta)]$ d) $\frac{1}{2}[\sin(9\theta)-\sin(3\theta)]$

e) None of the above

product to sum identity:

$\cos\alpha\cos\beta = ?$ (it is even) consider

$$\left.\begin{array}{l}\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta\end{array}\right\}\text{ these are even}$$

$\Rightarrow \cos(\alpha+\beta)+\cos(\alpha-\beta) = 2\cos\alpha\cos\beta$

$\Rightarrow \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$ if you couldn't remember this, you should be able to derive it.

Applying, we have

$\cos(6\theta)\cos(3\theta) = \frac{1}{2}[\cos(6\theta+3\theta)+\cos(6\theta-3\theta)]$

$= \frac{1}{2}[\cos(9\theta)+\cos(3\theta)] \Rightarrow A$

Find all solutions of $\sin\theta+\cos\theta = 1$

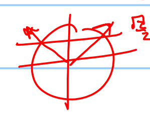
a) $2n\pi,\ \frac{\pi}{2}+2n\pi;\ n\in\mathbb{Z}$ b) $n\pi,\ \frac{\pi}{2}+n\pi;\ n\in\mathbb{Z}$

c) $\frac{\pi}{2}+2n\pi;\ n\in\mathbb{Z}$ d) $2n\pi;\ n\in\mathbb{Z}$ e) None of the above

$\sin\theta+\cos\theta = 1 \Rightarrow \frac{\sqrt{2}}{2}\sin\theta+\frac{\sqrt{2}}{2}\cos\theta = \frac{\sqrt{2}}{2}$

$\Rightarrow \cos\frac{\pi}{4}\sin\theta+\sin\frac{\pi}{4}\cos\theta = \frac{\sqrt{2}}{2}$

$\Rightarrow \sin(\frac{\pi}{4}+\theta) = \frac{\sqrt{2}}{2}$



$\Rightarrow \frac{\pi}{4}+\theta = \frac{\pi}{4}+2n\pi$ or $\frac{\pi}{4}+\theta = \frac{3\pi}{4}+2n\pi$

$\Rightarrow \theta = \boxed{2n\pi}$ or $\theta = \frac{3\pi}{4}-\frac{\pi}{4}+2n\pi = \frac{2\pi}{4}+2n\pi = \boxed{\frac{\pi}{2}+2n\pi}$ $n\in\mathbb{Z}$ $\Rightarrow A$

Find the exact value of $\cos(165°)$

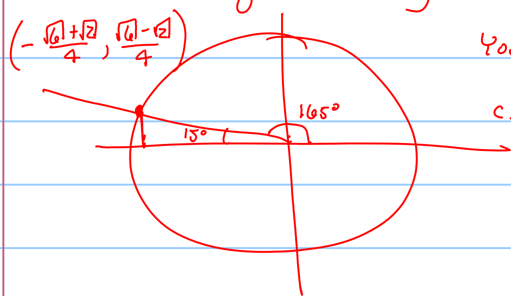
a) $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{\sqrt{6}+\sqrt{2}}{4}$ c) $-\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$ d) $-\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$

e) None of the above

$\cos(165°) = \sin(90° - 165°) = \sin(-75°) = -\sin(75°)$

$= -\cos(90° - 75°) = -\cos(15°) = -\frac{\sqrt{2+\sqrt{3}}}{2}$ or $-\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) \Rightarrow C$

or look at it geometrically



You should know the 15° triangle from class. If not apply half angle identity or sum identity.

Simplify $\frac{\sec^2\theta - 1}{\sec^2\theta}$

a) $\sin^2\theta$ b) $\csc\theta$ c) $2\sin^2\theta$ d) $\frac{\tan^2\theta}{\cos^2\theta}$

e) None of the above

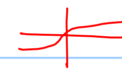
$\frac{\sec^2\theta - 1}{\sec^2\theta} = 1 - \frac{1}{\sec^2\theta} = 1 - \cos^2\theta = \sin^2\theta \Rightarrow A$

Which of the following is NOT true?

a) $\tan[\tan^{-1}(-3.9)] = -3.9$ b) $\sin^{-1}(-1) = -\frac{\pi}{2}$

c) $\sin[\tan^{-1}(-1)] = -\frac{\sqrt{2}}{2}$ d) $\cos^{-1}[\cos(-\frac{\pi}{6})] = -\frac{\pi}{6}$

e) None of the above



a) $\tan[\tan^{-1}(-3.9)] = -3.9$ since -3.9 is in the domain of $\tan^{-1}(x)$

b) $\sin^{-1}(-1) = -\frac{\pi}{2}$ ✓

c) $\sin[\tan^{-1}(-1)] = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ ✓

d) $\cos^{-1}[\cos(-\frac{\pi}{6})] = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \neq -\frac{\pi}{6} \Rightarrow$ D is not true

Find all solutions of $\sin(2\theta) = \sqrt{2}\cos\theta$

a) $\frac{\pi}{2} + 2n\pi \; ; n \in \mathbb{Z}$ b) $\frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi \; ; n \in \mathbb{Z}$

c) $\frac{\pi}{2} + 2n\pi, \frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi \; ; n \in \mathbb{Z}$

d) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}$ e) None of the above

$\sin(2\theta) = \sqrt{2}\cos\theta \Rightarrow \sin(2\theta) - \sqrt{2}\cos\theta = 0$

$\Rightarrow 2\cos\theta\sin\theta - \sqrt{2}\cos\theta = 0 \Rightarrow \cos\theta(2\sin\theta - \sqrt{2}) = 0$

$\Rightarrow \cos\theta = 0$ or $2\sin\theta - \sqrt{2} = 0 \Rightarrow \sin\theta = \frac{\sqrt{2}}{2}$

$$\Rightarrow \theta = \left\{\begin{array}{l}\frac{\pi}{2} + 2n\pi \\ \frac{3\pi}{2} + 2n\pi\end{array}\right\} = \frac{\pi}{2} + n\pi \quad \text{or} \quad \theta = \left\{\begin{array}{l}\frac{\pi}{4} + 2n\pi \\ \frac{3\pi}{4} + 2n\pi\end{array}\right. \Rightarrow E$$

be careful. It is too easy to divide out $\cos\theta$:

$\cancel{\cos\theta}(2\sin\theta - \sqrt{2}) = 0$. This loses solutions.

Never divide by your variable. Also, if you multiply both sides by your variable, you may be adding solutions.