Note Title 4/23/2006

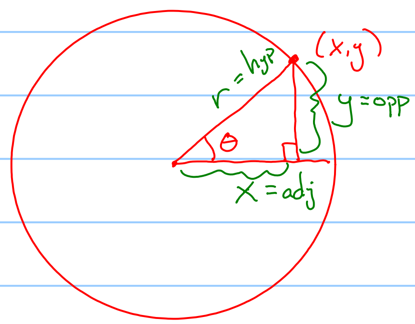
Which of the following is true for a right triangle?

a) $\sin\theta \cancel{=} \frac{hyp}{opp}$, $\cos\theta \cancel{=} \frac{opp}{adj}$, $\tan\theta \cancel{=} \frac{hyp}{opp}$

C b) $\sec\theta \cancel{=} \frac{opp}{adj}$, $\csc\theta \cancel{=} \frac{adj}{hyp}$, $\cot\theta \cancel{=} \frac{hyp}{adj}$

c) $\tan\theta = \frac{opp}{adj}$, $\cot\theta = \frac{adj}{opp}$, $\cos\theta = \frac{adj}{hyp}$

d) All of the above e) None of the above



$\sin\theta = \frac{y}{r} = \frac{opp}{hyp}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{hyp}{opp}$

$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{hyp}{adj}$

$\tan\theta = y/x = \frac{opp}{adj}$ $\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{adj}{opp}$

Which of the following is an example of the statement, "Cofunctions of complimentary angles are equal"?

a) $\sin(\pi) = \cos(\frac{5\pi}{2})$ b) $\tan(\theta) = \frac{1}{\cot\theta}$

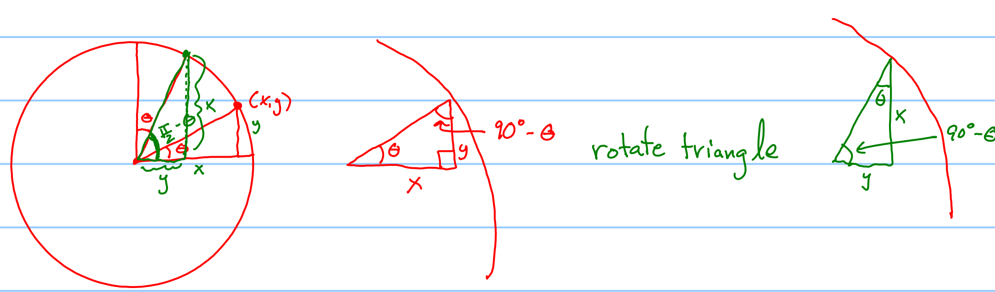
C c) $\sec(72°) = \csc(18°)$

d) All of the above e) None of the above

"Cofunctions of complimentary angles are equal" means

$\sin(\theta) = \cos(\frac{\pi}{2} - \theta)$ $\tan\theta = \cot(\frac{\pi}{2} - \theta)$

$\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$ $\cot\theta = \tan(\frac{\pi}{2} - \theta)$

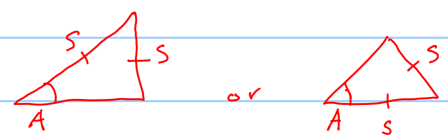


Which of the following type(s) of triangles is the "ambiguous" case?

C a) SSS b) SAS c) SSA

d) All of the above e) None of the above

The ambiguous case is ASS or SSA. These are the same

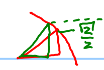


Given the following data for a triangle: $a=2,\ \gamma=50°,\ c=1$

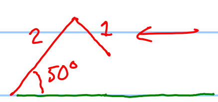
How many triangles fit this data?

E a) 1 b) 2 c) 3 d) 4 e) None of the above

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} \Rightarrow \sin\alpha = \frac{a\sin\gamma}{c} = 2\underbrace{\sin(50°)}_{>\frac{\sqrt{2}}{2}} > \sqrt{2}$$



$\Rightarrow \sin\alpha > \sqrt{2} \Rightarrow \alpha = \sin^{-1}(\sqrt{2})$ but $\sqrt{2} > 1 \Rightarrow$ No solution $\Rightarrow$ E



$\leftarrow$ too short to make a triangle.

Which of the following is true?

A a) $b^2 = a^2 + c^2 - 2ac\cos\beta$ ✓ Law of cosines

b) $\frac{\sin\alpha}{a} = \frac{b}{\sin\beta}$ ✗

Law of sines is $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ or $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

c) $\frac{\sin^2\alpha}{a} = \frac{\sin^2\beta}{b} = \frac{\sin^2\gamma}{c}$ ✗

d) All of the above e) None of the above

Which of the following is true?

a) $A = \frac{1}{2}ab\sin\gamma$ ✓ b) $A = \frac{1}{2}bc\sin\alpha$ ✓ Both Area formulas

D c) $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$ Heron's Area formula

d) All of the above e) None of the above

What is the frequency of the oscillator defined by $d = 10\sin(5t)$?

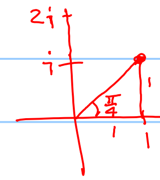
A a) $5/2\pi$ b) $2\pi/5$ c) 10 d) 2 e) None of the above

$d = 10\sin(5t) \Rightarrow \omega = 5 \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{5} \quad f = \frac{1}{T} = \frac{5}{2\pi}$

If $z = 1+i$, what is z^{12}?

C a) 2^6 b) 2^{12} c) -2^6 d) $12(1+i)$

e) None of the above



$z = 1+i = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

When multiplying complex numbers,

angles add + radius multiplies $\Rightarrow z^{12} = (\sqrt{2})^{12}\left[\cos\left(\frac{12\pi}{4}\right) + i\sin\left(\frac{12\pi}{4}\right)\right]$

$= 2^{12/2}\left[\cos(3\pi) + i\sin(3\pi)\right] = 2^6(-1 + i(0)) = -2^6 \Rightarrow C$

If $z = 1+i$ which of the following are roots of $z^{1/4}$?

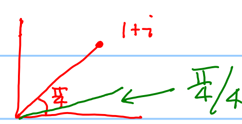
a) $2^{1/8}\left[\cos(25\pi/16) + i\sin(25\pi/16)\right]$ $\quad (2^{1/8})^4 = \sqrt{2}$

D b) $2^{1/8}\left[\cos(\pi/16) + i\sin(\pi/16)\right]$ $\quad 4 \cdot \frac{25\pi}{16} = \frac{100\pi}{16} = \frac{25\pi}{4} \sim \frac{\pi}{4}$

$4 \cdot \frac{\pi}{16} = \frac{\pi}{4}$

c) $2^{1/8}\left[\cos(9\pi/16) + i\sin(9\pi/16)\right]$ $\quad 4 \cdot \frac{9\pi}{16} = \frac{36\pi}{16} = \frac{9\pi}{4} \sim \frac{\pi}{4}$

d) All of the above e) None of the above



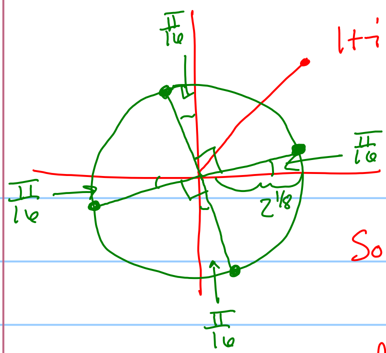
$z = 1+i = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right]$

Since angles add and you're looking for a 4th root, find a # with an angle 1/4 of the angle you want to end up with when raised to the 4th power. $\frac{\pi}{4}/4 = \frac{\pi}{16}$

Radius's multiply so take 4th root of radius (radius is a real, positive #)

$\Rightarrow$ Solution 1 $= (\sqrt{2})^{1/4}\left[\cos\left(\frac{\pi}{16}\right) + i\sin\left(\frac{\pi}{16}\right)\right]$

Solutions have the same radius when taking roots, so they all fall on a circle with radius $\sqrt{2}^{1/4} = 2^{1/8}$



$\Rightarrow$ Solutions are equally spaced around the circle.

Solution 2 $= 2^{1/8}\left[\cos\left(\frac{\pi}{16}+\frac{2\pi}{4}\right)+i\sin\left(\frac{\pi}{16}+\frac{2\pi}{4}\right)\right]$

$= 2^{1/8}\left[\cos\left(\frac{9\pi}{16}\right)+i\sin\left(\frac{9\pi}{16}\right)\right]$

Solution 3 $= 2^{1/8}\left[\cos\left(\frac{9\pi}{16}+\frac{\pi}{2}\right)+i\sin\left(\frac{9\pi}{16}+\frac{\pi}{2}\right)\right]$

$= 2^{1/8}\left[\cos\left(\frac{17\pi}{16}\right)+i\sin\left(\frac{17\pi}{16}\right)\right]$

Solution 4 $= 2^{1/8}\left[\cos\left(\frac{17\pi}{16}+\frac{\pi}{2}\right)+i\sin\left(\frac{17\pi}{16}+\frac{\pi}{2}\right)\right]$

$= 2^{1/8}\left[\cos\left(\frac{25\pi}{16}\right)+i\sin\left(\frac{25\pi}{16}\right)\right]$

Notice the angles of solutions. If you multiply any of them by 4 you get an angle equivalent to $\frac{\pi}{4}$. You can also apply the formula which does the same as we just did.

If $z = r[\cos(\theta_0)+i\sin(\theta_0)] = |\sqrt{2}|[\cos(\frac{\pi}{4})+i\sin(\frac{\pi}{4})]$

then $r = |\sqrt{2}|$ and $\theta_0 = \frac{\pi}{4}$. Solving for x, $x^4 = z \Rightarrow x = z^{1/4}$, there are 4 solutions given by:

$$x_k = r^{1/4}\left[\cos\left(\frac{\theta_0}{4}+\frac{2\pi k}{4}\right)+i\sin\left(\frac{\theta_0}{4}+\frac{2\pi k}{4}\right)\right] \quad k=0,1,2,3$$

$x_0 = 2^{1/8}\left[\cos\left(\frac{\pi}{16}\right)+i\sin\left(\frac{\pi}{16}\right)\right]$

$x_1 =$ ⋮ plug in values of k

$x_2 =$

$x_3 =$

Which of the following is true?

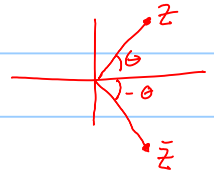
a) If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2+i\sin\theta_2)$ then

$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$ ✓ angles add + radius's multiply

D b) $z_1\bar{z}_1 = r_1^2$ ✓ $(x+iy)(x-iy) = x^2+y^2 = r^2$

c) $z_2^n = r_2^n[\cos(n\theta_2)+i\sin(n\theta_2)]$ ✓

d) All of the above e) None of the above



angles add $\Rightarrow$ $z\bar{z}$ has angle zero so is always a real # and positive since it is squared.

In polar form: $r(\cos\theta+i\sin\theta)\,r(\cos\theta-i\sin\theta) = r^2(\cos^2\theta+\sin^2\theta) = r^2$

B

Given $3^{2x} + 3^{x+1} - 4 = 0$, solve for x.

a) $x = 3$ b) $x = 0$ c) $x = -4$

d) All of the above e) None of the above

$3^{2x} + 3^{x+1} - 4 = 0 \Leftrightarrow (3^x)^2 + 3\cdot(3^x) - 4 = 0$

$\Leftrightarrow (3^x - 1)(3^x + 4) = 0$

$\Leftrightarrow 3^x = 1$ or $3^x = -4 \Leftrightarrow$

$\log_3(3^x) = \log_3(1)$ $\quad\quad$ $\log_3(3^x) = \log_3(-4) \Leftarrow$ not a real #

$x = \log_3(1) = 0$

since $3^0 = 1$

$\Rightarrow x = 0 \Rightarrow$ B

A

Given $\log_4(x^2-9) - \log_4(x+3) = 3$, Solve for x.

a) $x = 4^3 + 3$ b) $x = 3$ c) $x = 3^4 - 4$

d) All of the above e) None of the above

$\log_4(x^2-9) - \log_4(x+3) = 3$

$\Leftrightarrow \log_4\left[\frac{x^2-9}{x+3}\right] = 3 \Leftrightarrow \log_4\left[\frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}\right] = 3 \Leftrightarrow \log_4(x-3) = 3$

$\Leftrightarrow 4^{\log_4(x-3)} = 4^3 \Leftrightarrow x - 3 = 4^3 \Leftrightarrow x = 4^3 + 3 \Rightarrow$ A

Given $\log_9(x) + 3\log_3(x) = 14$, Solve for x.

Hint: Change $\log_9(x)$ to base 3.

C a) $x = 9^{14}$ b) $x = 1$ c) $x = 3^4$

d) All of the above e) None of the above

$\underbrace{\log_9(x)} + 3\log_3(x) = 14$

change base $\longrightarrow$ $\log_9(x) = y \Leftrightarrow 9^y = x$

$\Leftrightarrow (3^2)^y = x \Leftrightarrow 3^{2y} = x \Leftrightarrow \log_3(3^{2y}) = \log_3(x)$

$\Leftrightarrow 2y = \log_3(x) \Leftrightarrow y = \frac{1}{2}\log_3(x)$

or use change of base formula

$\log_9(x) = \dfrac{\log_3(x)}{\log_3(9)} = \dfrac{1}{2}\log_3(x)$ so

$\log_9(x) + 3\log_3(x) = 14 \Leftrightarrow \frac{1}{2}\log_3(x) + 3\log_3(x) = 14$

$\Leftrightarrow \log_3(\sqrt{x}) + \log_3(x^3) = 14$

$\Leftrightarrow \log_3(\sqrt{x}\,x^3) = 14 \Leftrightarrow \log_3(x^{7/2}) = 14$

$\Leftrightarrow \frac{7}{2}\log_3(x) = 14 \Leftrightarrow \log_3(x) = 4 \Leftrightarrow 3^{\log_3(x)} = 3^4$

$\Leftrightarrow x = 3^4$

OR just plugin the choices you have into the equation

Given $(\sqrt[3]{2})^{2-x} = 2^{x^2}$, Solve for x.

B a) $x = \pm 2^{1/3}$ b) $x = -1, 2/3$ c) 1

d) All of the above e) None of the above

$(\sqrt[3]{2})^{2-x} = 2^{x^2} \Leftrightarrow (2^{1/3})^{2-x} = 2^{x^2} \Leftrightarrow 2^{\frac{1}{3}(2-x)} = 2^{x^2}$

$\Leftrightarrow \log_2(2^{\frac{1}{3}(2-x)}) = \log_2(2^{x^2}) \Leftrightarrow \frac{1}{3}(2-x) = x^2$

$\Leftrightarrow 3x^2 + x - 2 = 0 \Leftrightarrow (3x-2)(x+1) = 0$

$\Leftrightarrow x = 2/3, -1 \Rightarrow$ B

If you had \$100 and put it in the bank and collected 5% interest, compounded continuously for 100 years, how much money would you have?

C

a) \$50,000 b) \$5,000 c) \$14,841.30

d) All of the above e) None of the above

$A = Pe^{rt} \Rightarrow A = 100e^{.05(100)} = 14,841.30 \Rightarrow$ C

Which of the following is NOT true?

B

a) $A = P(1+\frac{r}{n})^{nt}$ when compounding interest n times a year ✓

b) $P = Ae^{rt}$ when compounding interest continuously ✗ $A = Pe^{rt}$

c) $I = Prt$ is the interest charged ✓

d) All of the above e) None of the above

A

Imagine that when you were 5 years old, you happened to inherent a signed, collector's edition pair of Power Ranger's Ultra underwear from your long lost uncle Jimbo, valued at \$100,000, and those underwear were your favorite underwear. In fact, you wore them every day until you were 30 years old and, because the picture of the Blue Ranger was on the back side of the underwear, you wore them backwards so you could see the picture when wearing them. If these underwear lost their value by 7% every ~~day~~ year you wore them, how much would they be worth after 25 years of wear? (Assume 365 days per year, and one wearing per day)

The solution given requires this change. I'm factoring this into my grading

$A = P(1+\frac{r}{n})^{nt}$ $n = 365$ $t = 25$

a) \$17,380.30 b) \$26,589.42 (daily compounding)

c) My imagination is not that good.

d) I only wear Spiderman underwear. $\Rightarrow$ A

e) Who needs underwear? $A = 100,000\left(1+\frac{.07}{365}\right)^{365(25)} = 17,380.30$

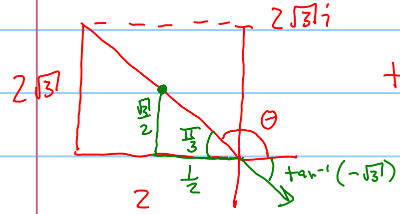
Convert $-2+2\sqrt{3}\,i$ to polar form.

A

a) $4\left[\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right]$ b) $\sqrt{13}\left[\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right]$

c) $2\sqrt{10}\left[\sin\left(\frac{2\pi}{3}\right)+i\cos\left(\frac{2\pi}{3}\right)\right]$

d) All of the above e) None of the above



$\tan\theta = \frac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \theta = \tan^{-1}(-\sqrt{3}) + \pi = \frac{2\pi}{3}$

$r = \sqrt{(-2)^2+(2\sqrt{3})^2} = \sqrt{4+12} = 4$

$\Rightarrow -2+2\sqrt{3}\,i = 4\left[\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right]$

Check: $4\left[\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right] = 4\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = -2+2\sqrt{3}\,i$ ✓

consider $-2+2\sqrt{3}\,i = 4\left(-\frac{1}{2}+\frac{\sqrt{3}\,i}{2}\right)$ these #'s should be familiar & you should be able to get the angle without using tan.

Given $x^4 = 64$, solve for x

C

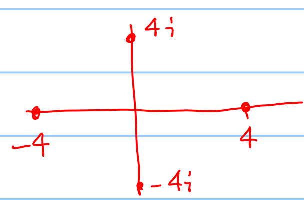
a) $x = \pm 4$ b) $x = \pm 4i$ c) $x = \pm 4, \pm 4i$

d) All of the above e) None of the above

$4^4 \neq 64$ $(4i)^4 \neq 64 \Rightarrow$ E

If the problem were $x^4 = 256$ then

$4^4 = 256 \Rightarrow x = 4$ is one solution. For taking roots, use your root formula or use your intuition. Solutions are equally spread around a circle. $\frac{2\pi}{4} = \frac{\pi}{2} \Rightarrow$ angles are separated by $\frac{\pi}{2}$



Check: $(-4)^4 = 256$ $(4i)^4 = 256$

$(-4i)^4 = 4^4 i^4 = 265$ ✓

$\Rightarrow$ would be C

Given a polynomial with real coefficients

a) Solutions will come in conjugate pairs ✓

D b) Solutions may not have the same radius ✓

c) If it is of order 4, it will have 4 solutions ✓

d) All of the above   e) None of the above

When taking roots (special polynomial of the form $x^4 - \# = 0$) solutions have the same radius.

polynomial form: $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$

examples $x^2+2x+1=0$, $x^9+3=0$. These have real coefficients and solutions will come in conjugate pairs.

$ix^3+x^2-2i=0$ has an imaginary coefficients so solutions won't come in conjugate pairs.

highest power of x is your order

$x^4+3x^2+1=0$ is of order $4 \Rightarrow 4$ solutions