Note Title 7/3/2006

#1 Which of the following are completely true?

a) $\cos(4\theta) = \cos^2(2\theta) - \sin^2(2\theta)$
b) $\cos(4\theta) = \cos(3\theta)\cos\theta - \sin(3\theta)\sin\theta$
c) $\cos(4\theta) = \cos\theta\cos(3\theta) - \sin\theta\sin(3\theta)$
d) $\cos(4\theta) = \cos(7\theta)\cos(3\theta) + \sin(7\theta)\sin(3\theta)$
e) None of the above

a) true.
let $\theta' = 2\theta$ then
$\cos(4\theta) = \cos(2\theta')$ and
$\cos^2(2\theta) - \sin^2(2\theta) = \cos^2(\theta') - \sin^2(\theta')$ and
$\cos(2\theta') = \cos^2(\theta') - \sin^2(\theta')$

b) true $\cos(4\theta) = \cos(3\theta+\theta) = \cos(3\theta)\cos\theta - \sin(3\theta)\sin\theta$
c) true $\cos(4\theta) = \cos(\theta+3\theta) = \cos\theta\cos(3\theta) - \sin\theta\sin(3\theta)$
d) true $\cos(4\theta) = \cos(7\theta-3\theta) = \cos(7\theta)\cos(3\theta) + \sin(7\theta)\sin(3\theta)$

#2 Which of the following are completely true?
a) $\sin(4\theta) = 2\sin(2\theta)\cos(2\theta)$
b) $\sin(4\theta) = \sin(10\theta)\cos(6\theta) - \cos(10\theta)\sin(6\theta)$
c) $\sin(4\theta) = \sin\theta\cos(3\theta) + \cos\theta\sin(3\theta)$
d) $\sin(4\theta) = -\cos^2(2\theta) + \sin^2(2\theta)$
e) None of the above

a) true let $\theta' = 2\theta$ see #1 (a)
b) tre $\sin(4\theta) = \sin(10\theta - 6\theta) = \sin(10\theta)\cos(6\theta) - \cos(10\theta)\sin(6\theta)$
c) true $\sin(4\theta) = \sin(\theta+3\theta) = \sin\theta\cos(3\theta) + \cos\theta\sin(3\theta)$
d) false : try $\theta = 0$
$\sin(4\cdot 0) = \sin(0) = 0$
$-\cos^2(2\cdot 0) + \sin^2(2\cdot 0) = -\cos^2(0) + \sin^2(0) = -1 \neq 0$

#3 Which of the following are completely true?

a) $\sin^2(17\alpha) + \cos^2(17\alpha) = 17$

b) $\sin\theta = 2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})$

c) $\cos\theta = \cos^2(\frac{\theta}{2}) - \sin^2(\frac{\theta}{2})$

d) $\sin^2\theta - 1 = \cos^2\theta$

e) None of the above

a) false let $\theta = 17\alpha$ then

$\sin^2(17\alpha) + \cos^2(17\alpha) = \sin^2\theta + \cos^2\theta = 1 \neq 17$

b) true $\sin(2\theta) = 2\sin\theta\cos\theta$ $\quad \theta \to \frac{\theta}{2}$

$\sin(\theta) = 2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})$

c) true $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\quad \theta \to \frac{\theta}{2}$

$\cos\theta = \cos^2(\frac{\theta}{2}) - \sin^2(\frac{\theta}{2})$

d) false $\cos^2\theta = 1 - \sin^2\theta \neq \sin^2\theta - 1$

#4 Which of the following are completely true?

a) If you use the sum/difference formula to find the exact value of $\sin(\frac{\pi}{12})$, you can get $\frac{\sqrt{6}-\sqrt{2}}{4}$.

b) If you use the sum/difference formula to find the exact value of $\cos(\frac{\pi}{12})$, you can get $\frac{\sqrt{6}+\sqrt{2}}{4}$.

c) If you use the half angle formula to find the exact value of $\sin(\frac{\pi}{12})$, you can get $\frac{\sqrt{2-\sqrt{3}}}{2}$

d) If you use the half angle formula to find the exact value of $\cos(\frac{\pi}{12})$, you can get $\frac{\sqrt{2+\sqrt{3}}}{2}$

e) None of the above

a) true $\sin(\frac{\pi}{12}) = \sin(\frac{\pi}{3} - \frac{\pi}{4}) = \sin(\frac{\pi}{3})\cos(\frac{\pi}{4}) - \cos(\frac{\pi}{3})\sin(\frac{\pi}{4})$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

b) true $\cos(\frac{\pi}{12}) = \cos(\frac{\pi}{3} - \frac{\pi}{4})$

$= \cos(\frac{\pi}{3})\cos(\frac{\pi}{4}) + \sin(\frac{\pi}{3})\sin(\frac{\pi}{4}) = \frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

c) true $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{6}/2\right) = \sqrt{\frac{1-\cos\left(\frac{\pi}{6}\right)}{2}}$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2}{4} - \frac{\sqrt{3}}{4}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

d) true $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}/2\right) = \sqrt{\frac{1+\cos\left(\frac{\pi}{6}\right)}{2}}$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2}{4} + \frac{\sqrt{3}}{4}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

#5 $\cos\left(\tan^{-1}(1/2) + \sin^{-1}(1/4)\right) =$

a) $\frac{2\sqrt{15}-1}{4\sqrt{5}}$ b) $\frac{2\sqrt{15}+1}{4\sqrt{5}}$

c) $\cos(\tan^{-1}(\tfrac{1}{2}))\cos(\sin^{-1}(1/4)) - \sin(\tan^{-1}(\tfrac{1}{2}))\sin(\sin^{-1}(1/4))$

d) $\cos(\tan^{-1}(\tfrac{1}{2}))\sin(\sin^{-1}(1/4)) + \sin(\tan^{-1}(\tfrac{1}{2}))\cos(\sin^{-1}(1/4))$

e) None of the above

$\cos\left(\tan^{-1}(1/2) + \sin^{-1}(1/4)\right) = \cos(\alpha+\beta)$

where $\tan\alpha = 1/2$, $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and

$\sin\beta = 1/4$, $\beta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Since $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ we need to solve for $\cos\alpha$, $\cos\beta$, $\sin\alpha$, and $\sin\beta$

$\tan\alpha = 1/2 = y/x \Rightarrow$ let $y=1$, $x=2$ then

$r = \sqrt{4+1} = \sqrt{5}$

$\Rightarrow y/r = 1/\sqrt{5}$ but $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\alpha = 1/2$

$\Rightarrow \alpha \in$ Quadrant 1 $\Rightarrow \sin\alpha = \frac{1}{\sqrt{5}}$

$\Rightarrow \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \cos\alpha$

and

$\cos\beta = \pm\sqrt{1-\sin^2\beta} = \pm\sqrt{1-\frac{1}{16}} = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}$ but

$\beta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \cos\beta = \frac{\sqrt{15}}{4}$

$\Rightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{15}}{4}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{4}\right) = \frac{2\sqrt{15}-1}{4\sqrt{5}}$$

$\Rightarrow$ (a) true (b) false (c) true: apply sum formula (d) false

\#6 Which of the following are completely true?

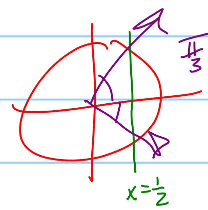
a) All solutions of $\cos\theta = \frac{1}{2}$ are $\theta = \frac{\pi}{3} + 2n\pi$, and $\theta = \frac{2\pi}{3} + 2n\pi$

b) All solutions of $\sin\theta = \frac{1}{2}$ are $\theta = \frac{\pi}{6} + 2n\pi$, and $\theta = \frac{5\pi}{6} + 2n\pi$

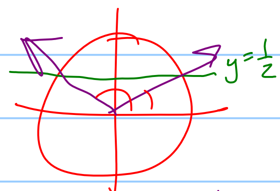
c) All solutions of $\cos\theta = 1$ are $\theta = 2n\pi$

d) All solutions of $\sin\theta = 1$ are $\theta = \frac{\pi}{2} + 2n\pi$

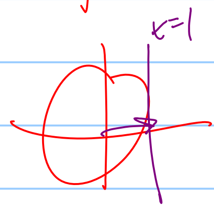
e) None of the above

a) 

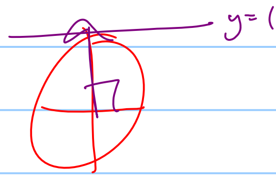
false $\cos\left(\frac{2\pi}{3}\right) \neq \frac{1}{2}$

b) 

true

c) 

$\theta = 0 + 2n\pi = 2n\pi$ true

d) 

true

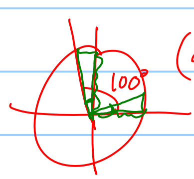
\#7 $\cos(80°)\sin(20°) + \sin(80°)\cos(20°) =$

a) $\cos(10°)$ b) $\sin(100°)$ c) $2\sin(50°)\cos(50°)$

d) π e) None of the above

$\cos(80°)\sin(20°) + \sin(80°)\cos(20°) = \sin(80°+20°) = \sin(100°)$

$\Rightarrow$ (b) true



(a) true $\sin(100°) = \sin(90° + 10°) = \sin(90°)\cos(10°) + \cancel{\cos(90°)}^{0}\sin(10°)$

$= \cos(10°)$

or use geometry of the circle

c) $2\sin(50^\circ)\cos(50^\circ) = \sin(2\cdot 50^\circ) = \sin(100^\circ) \Rightarrow$ true

d) false

#8 The expression $K - \frac{1}{\sec^2 x} = \sin^2 x$ is an identity when K is equal to

a) 1 b) $\frac{1}{\csc^2 x}$ c) $\tan^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right)$ d) $-\sin^2 x$ e) None of the above

$K - \frac{1}{\sec^2 x} = \sin^2 x \Rightarrow K = \sin^2 x + \frac{1}{\sec^2 x} = \sin^2 x + \cos^2 x = 1$

$\Rightarrow$ (a) true

(b) false $\frac{1}{\csc^2 x} \neq 1$

(c) true $\tan^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right) = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = 1$

(d) false $-\sin^2 x \neq 1$

#9 $\frac{\sec^2\theta - 1}{\sec^2\theta} =$

a) $\cos^2\theta$ b) $\sin^2\theta$ c) $\tan^2\theta$ d) $\csc^2\theta$ e) None of the above

$\frac{\sec^2\theta - 1}{\sec^2\theta} = 1 - \frac{1}{\sec^2\theta} = 1 - \cos^2\theta = \sin^2\theta \Rightarrow$ (b) true

a) false $\cos^2\theta \neq \sin^2\theta$

c) false d) false

#10 Which of the following is NOT true?

a) $\sec\theta + \tan\theta = \frac{\cos\theta}{1+\sin\theta}$ b) $\frac{\sin\theta}{\sin\theta - \cos\theta} = \frac{1}{1-\cot\theta}$

c) $\frac{1+\tan\theta}{1-\tan\theta} = \frac{\cot\theta + 1}{\cot\theta - 1}$ d) $\cos^2\theta = \sin^2\theta - 1$

e) None of the above

(a) LHS $= \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \neq \frac{\cos\theta}{1+\sin\theta}$ false

try plugging in $\theta = \frac{\pi}{2}$

(b) $\frac{\sin\theta}{\sin\theta - \cos\theta} = \left(\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}\right)\left(\frac{\sin\theta}{\sin\theta - \cos\theta}\right) = \frac{1}{1 - \frac{\cos\theta}{\sin\theta}} = \frac{1}{1-\cot\theta}$ true

(c) $\frac{1+\tan\theta}{1-\tan\theta} = \left(\frac{1+\tan\theta}{1-\tan\theta}\right)\left(\frac{\cot\theta}{\cot\theta}\right) = \frac{\cot\theta + \tan\theta\cot\theta}{\cot\theta - \tan\theta\cot\theta}$ true

$= \frac{\cot\theta + 1}{\cot\theta - 1}$ true

d) $\cos^2\theta \neq \sin^2\theta - 1$ false

A + D are not true

& Bonus: Derive the quadratic formula on your scratch sheet. This means: show that the solutions to an equation of the form $ax^2+bx+c=0$ are

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

This can also be said:

Prove the assertion

$$ax^2+bx+c=0 \Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Hint: Reverse engineer OR use easier "complete the square"

$(m+n)^2 = m^2+2mn+n^2$

$ax^2+bx+c=0 \Rightarrow ax^2+bx = -c$

$\Rightarrow x^2 + \frac{b}{a}x = -\frac{c}{a}$ Now complete the square

add the correct amount to both sides so that the left side is something squared

$\Rightarrow x^2 + \frac{b}{a}x + \underline{\left(\frac{b}{2a}\right)^2} = -\frac{c}{a} + \underline{\left(\frac{b}{2a}\right)^2}$

$\Rightarrow \underline{\left(x+\frac{b}{2a}\right)^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$$\Rightarrow \quad x + \frac{b}{2a} = \pm \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$$

$$\Rightarrow \quad x = -\frac{b}{2a} \pm \sqrt{\frac{-4ac}{4a^2} + \frac{b^2}{4a^2}} = -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}} = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$