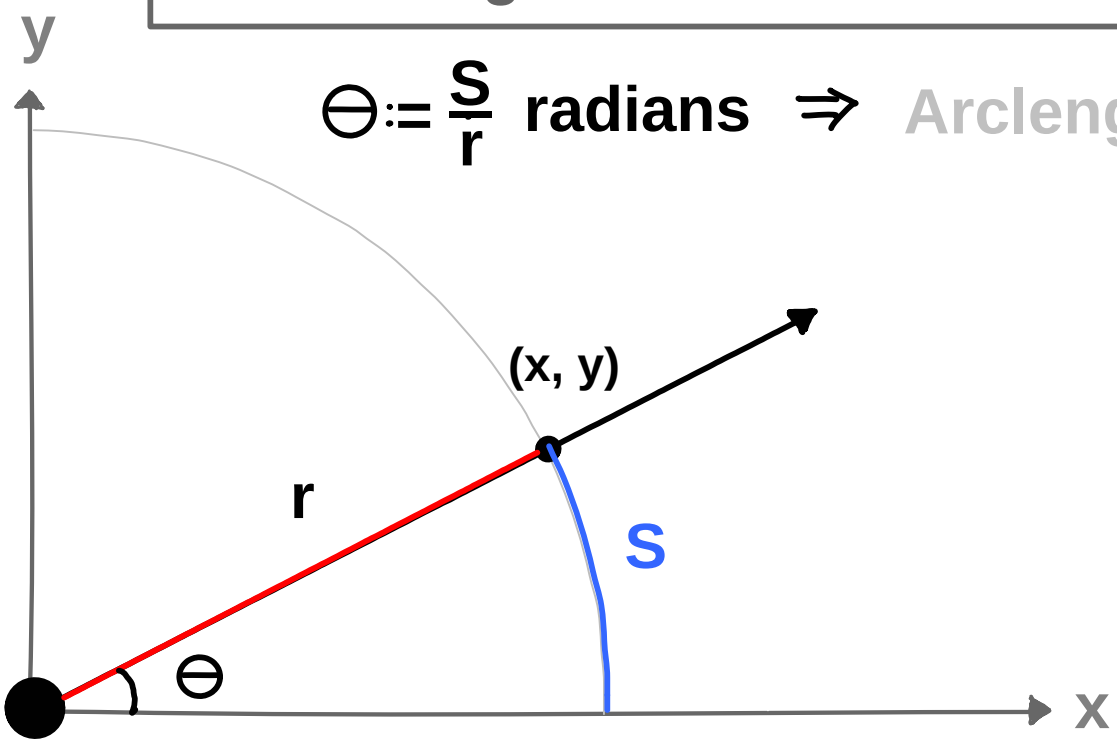
TRIG IN A NUTSHELL

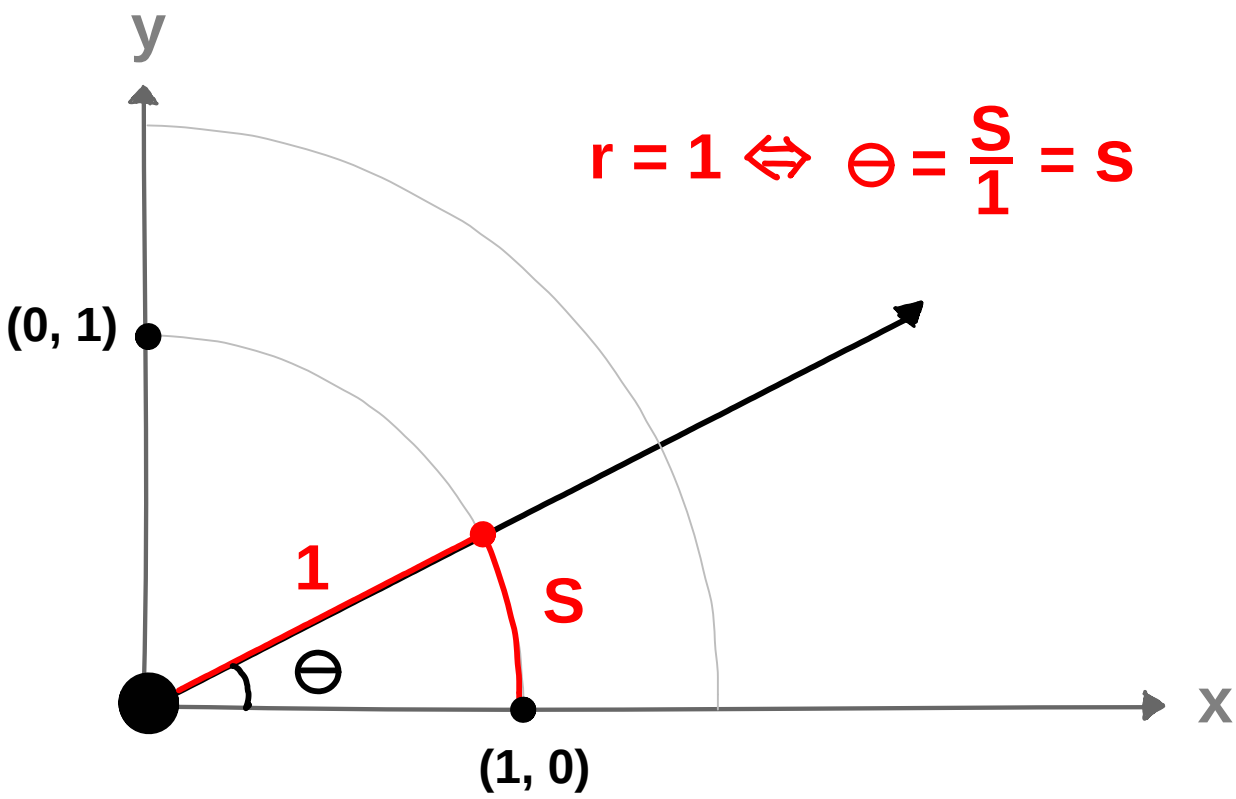
Angles and Circles

$\theta := \frac{s}{r}$ radians $\Rightarrow$ Arclength: $s = r\theta$

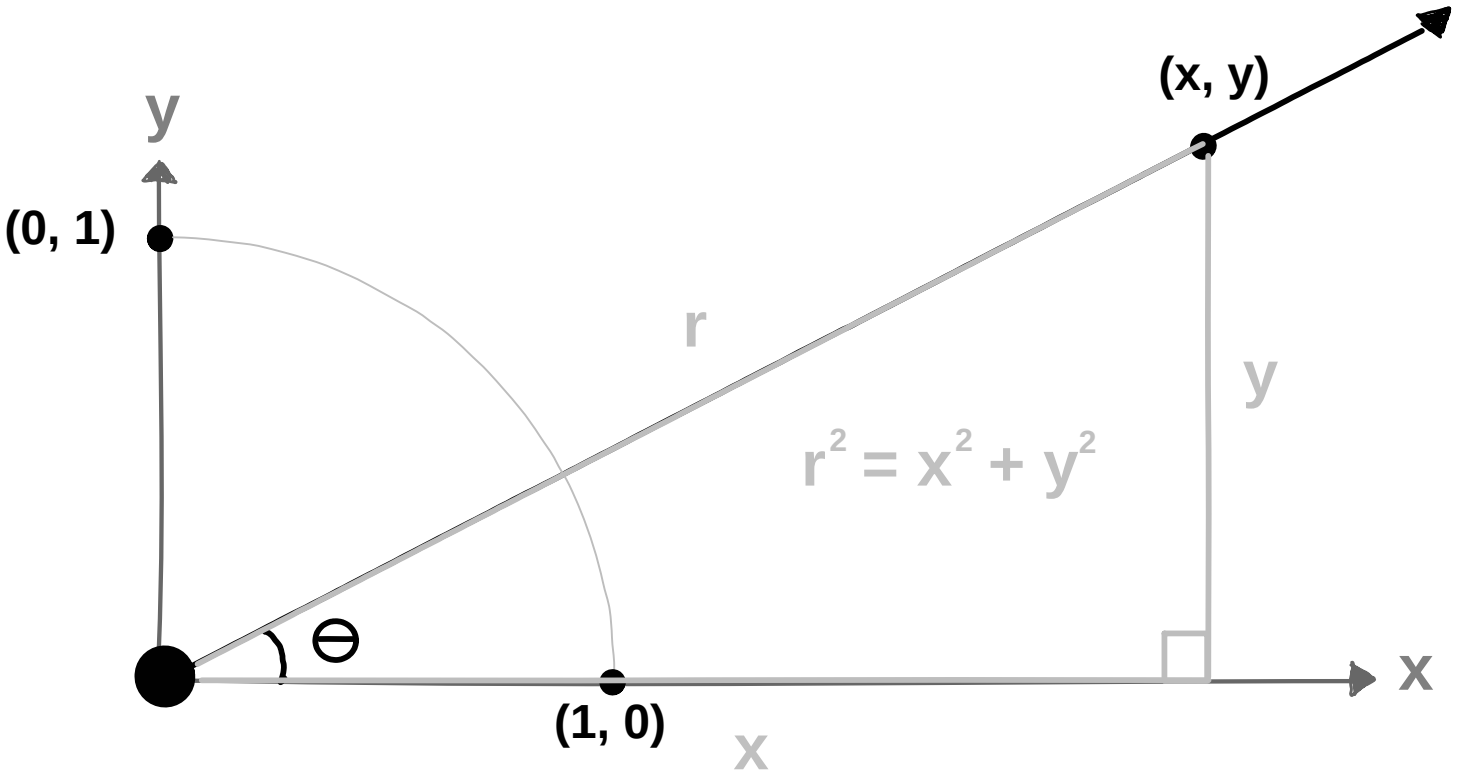


The Unit Circle: r = 1

$r = 1 \Leftrightarrow \theta = \frac{s}{1} = s$

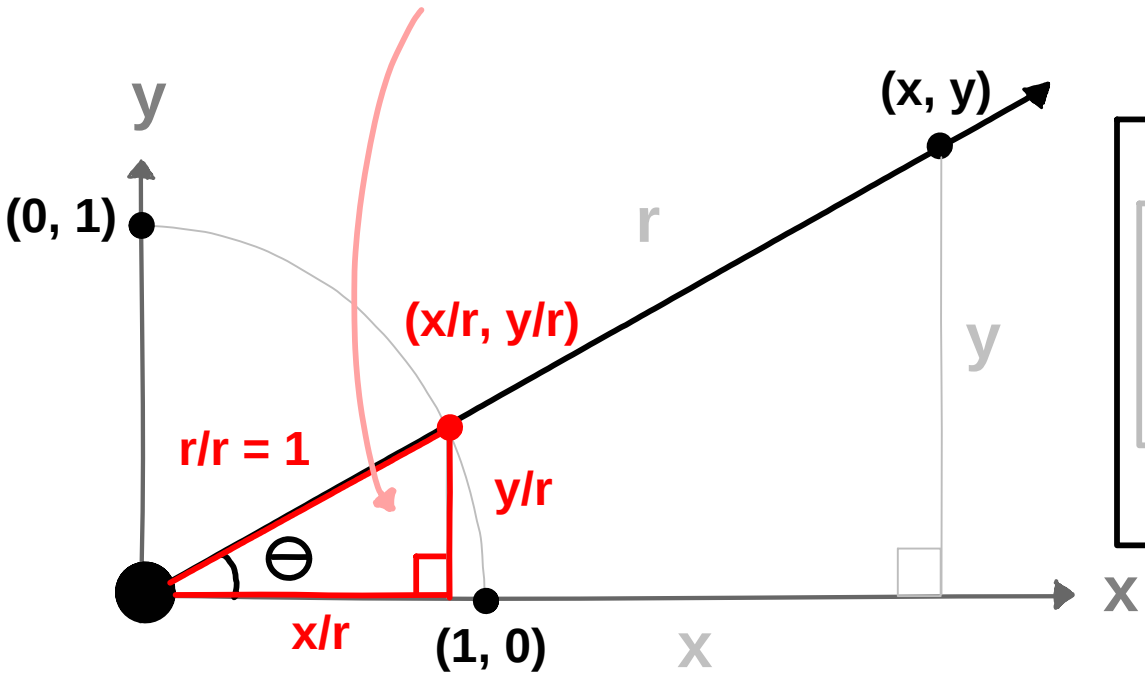


Angles and Triangles



Triangle with Hypotenuse = 1

$$\left(\frac{r}{r}\right)^2 = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 \qquad 1 = \cos^2(\theta) + \sin^2(\theta)$$



Sine

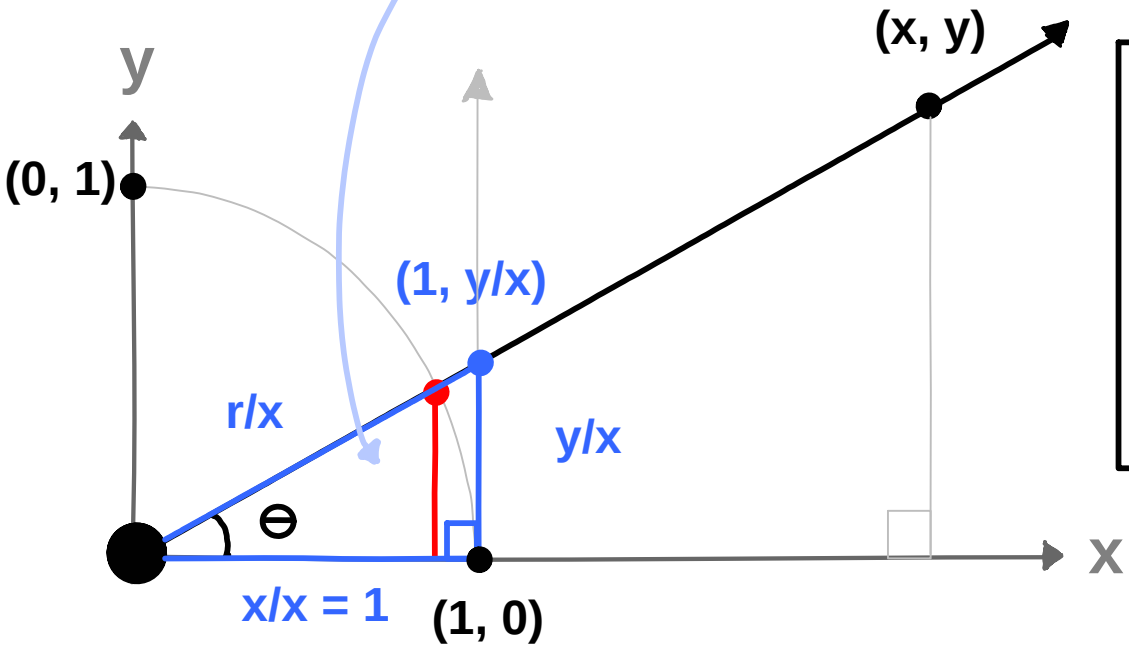
$$\sin(\theta) := \frac{y}{r}$$

$$\cos(\theta) := \frac{x}{r}$$

Cosine

Triangle with Adjacent Side = 1

$$\left(\frac{r}{x}\right)^2=\left(\frac{x}{x}\right)^2+\left(\frac{y}{x}\right)^2 \qquad \sec^2(\ominus) = 1 + \tan^2(\ominus)$$



Tangent

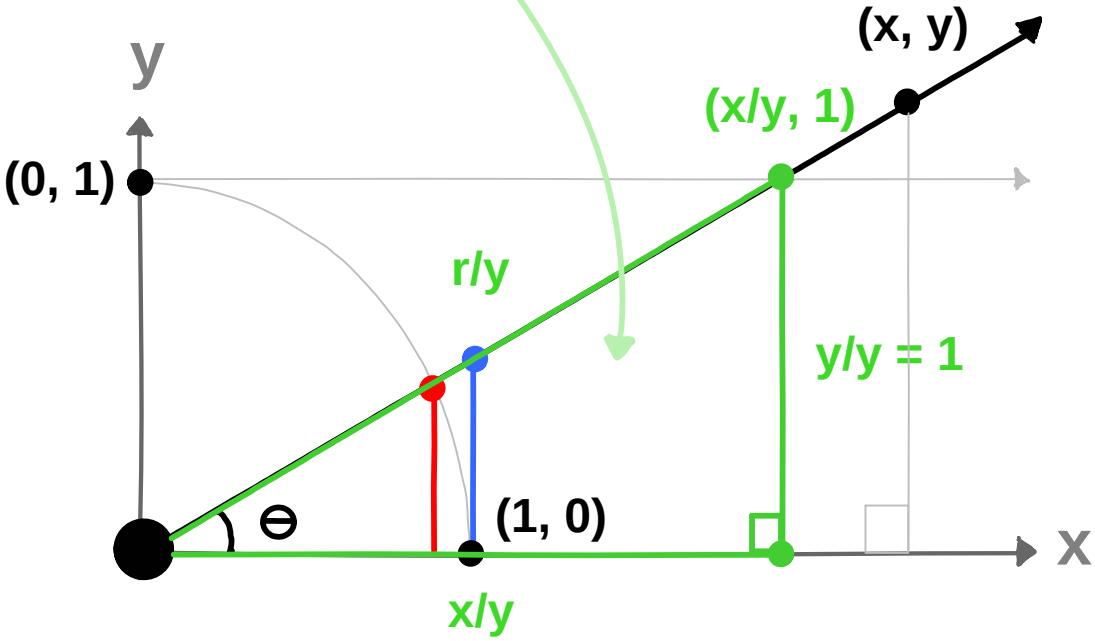
$$\tan(\ominus) := \frac{y}{x}$$

$$\sec(\ominus) := \frac{r}{x}$$

Secant

Triangle with Opposite Side = 1

$$\left(\frac{r}{y}\right)^2=\left(\frac{x}{y}\right)^2+\left(\frac{y}{y}\right)^2 \qquad \csc^2(\ominus) = \cot^2(\ominus) + 1$$



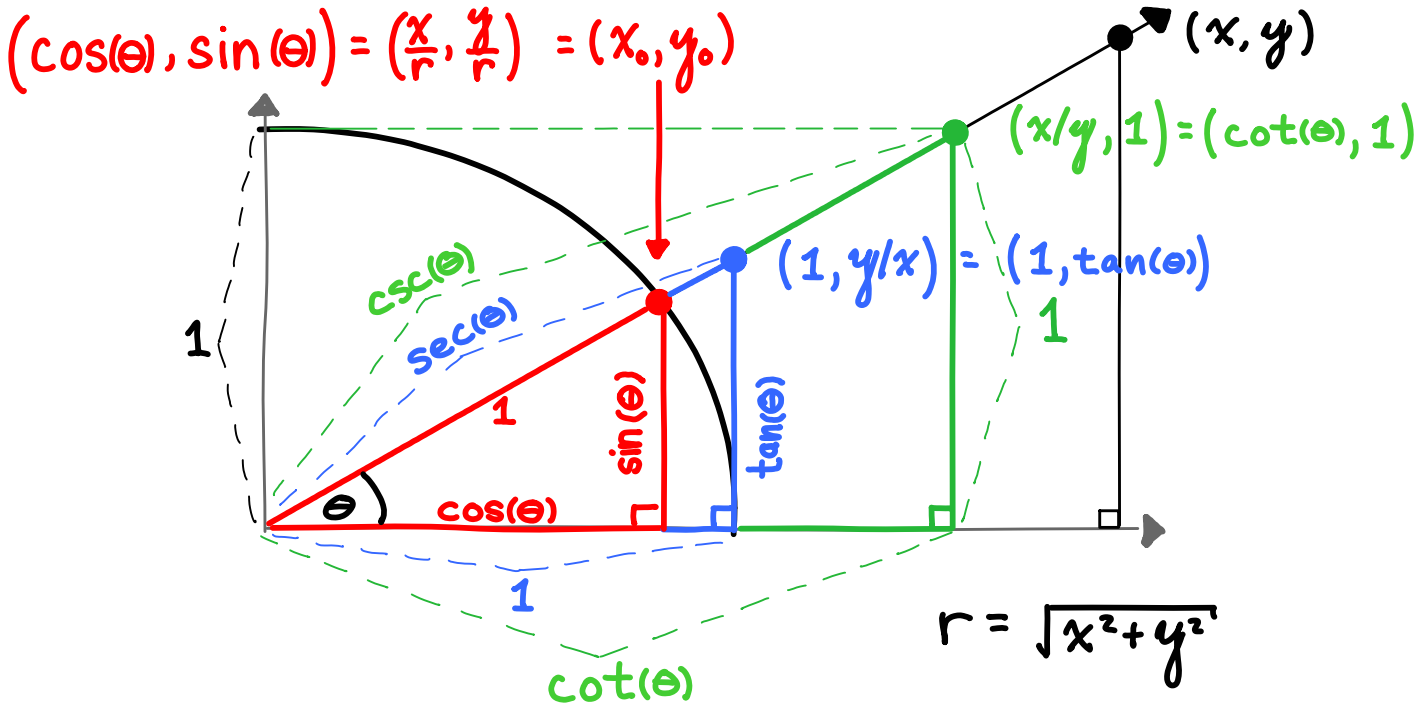
Cotangent

$$\cot(\ominus) := \frac{x}{y}$$

$$\csc(\ominus) := \frac{r}{y}$$

Cosecant

Trigonometry Summary Diagram



Geometric definitions of the six trig functions and the

"Reciprocal Identities"

Sine	Cosecant
$\sin(\theta) = y_0 = \frac{y}{r} = \frac{1}{\csc(\theta)}$	$\csc(\theta) = \frac{1}{y_0} = \frac{r}{y} = \frac{1}{\sin(\theta)}$
Cosine	**Secant**
$\cos(\theta) = x_0 = \frac{x}{r} = \frac{1}{\sec(\theta)}$	$\sec(\theta) = \frac{1}{x_0} = \frac{r}{x} = \frac{1}{\cos(\theta)}$
Tangent	**Cotangent**
$\tan(\theta) = \frac{y_0}{x_0} = \frac{y}{x} = \frac{1}{\cot(\theta)}$	$\cot(\theta) = \frac{x_0}{y_0} = \frac{x}{y} = \frac{1}{\tan(\theta)}$

The "Quotient Identities"

$$\tan(\theta) := \frac{y}{x} = \frac{y/r}{x/r} =: \frac{\sin(\theta)}{\cos(\theta)} \quad \Leftrightarrow \quad \tan(\theta) \equiv \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) \equiv \frac{1}{\tan(\theta)} \equiv \frac{1}{\frac{\sin(\theta)}{\cos(\theta)}} \equiv \frac{\cos(\theta)}{\sin(\theta)} \quad \Leftrightarrow \quad \cot(\theta) \equiv \frac{\cos(\theta)}{\sin(\theta)}$$

The "Pythagorean Identities"

$x^2+y^2=r^2$	$\Leftrightarrow \quad \frac{x^2}{r^2}+\frac{y^2}{r^2}=\frac{r^2}{r^2}$	$\Leftrightarrow \quad \cos^2(\theta)+\sin^2(\theta)\equiv 1$
$x^2+y^2=r^2$	$\Leftrightarrow \quad \frac{x^2}{y^2}+\frac{y^2}{y^2}=\frac{r^2}{y^2}$	$\Leftrightarrow \quad \cot^2(\theta)+1\equiv \csc^2(\theta)$
$x^2+y^2=r^2$	$\Leftrightarrow \quad \frac{x^2}{x^2}+\frac{y^2}{x^2}=\frac{r^2}{x^2}$	$\Leftrightarrow \quad 1+\tan^2(\theta)\equiv \sec^2(\theta)$