Cofunction Identities

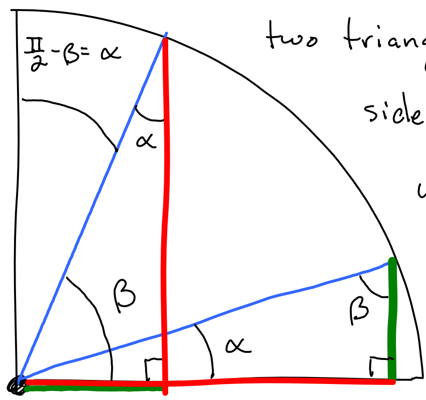
Cofunction Identities

$$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2}-\theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2}-\theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2}-\theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2}-\theta\right)$$

Consider quadrant 1 of our unit circle and the angles α and β as labeled. Since these two triangles are the same triangle but rotated, comparing the red sides we see that $\cos\alpha = \sin\beta$ and comparing green sides we see that $\sin\alpha = \cos\beta$. Since the angles of a triangle add up to $180°$ and one of our angles is $90°$ since it is a right triangle, we have



$$\alpha+\beta+\pi/2=\pi \iff \alpha+\beta=\pi/2 \iff \begin{cases}\alpha = \pi/2-\beta \\ \beta = \pi/2-\alpha\end{cases}$$

So,

$$\sin\alpha = \cos\beta \quad \text{and} \quad \alpha = \pi/2-\beta \Rightarrow \begin{cases}\sin\alpha = & \sin(\pi/2-\beta) = \cos\beta \\ \cos\alpha = & \cos(\pi/2-\beta) = \sin\beta\end{cases}$$

$$\sin\beta = \cos\alpha \quad \text{and} \quad \beta = \pi/2-\alpha \Rightarrow \begin{cases}\sin\beta = & \sin(\pi/2-\alpha) = \cos\alpha \\ \cos\beta = & \cos(\pi/2-\alpha) = \sin\alpha\end{cases}$$

In general $\sin\theta = \cos(\pi/2-\theta)$ and $\cos\theta = \sin(\pi/2-\theta)$.

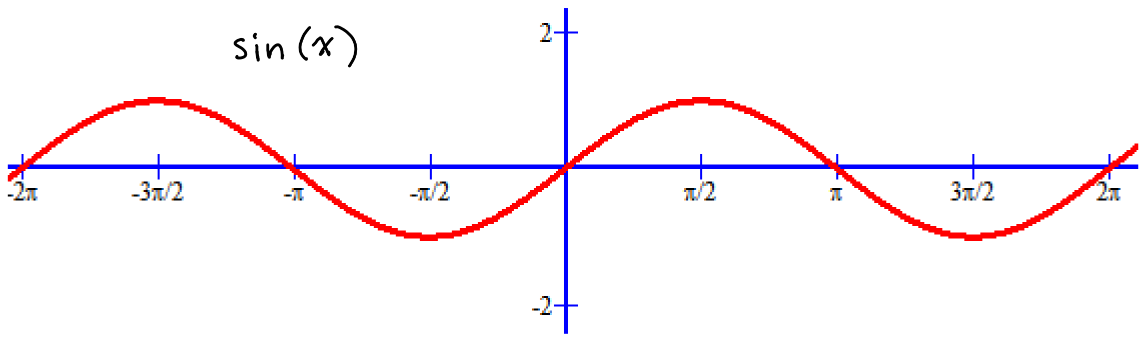
Put another way, given any right triangle, cosine of one acute angle is sine of the other, and vice versa.

That's the geometric representation of this identity. Let's look at it graphically.

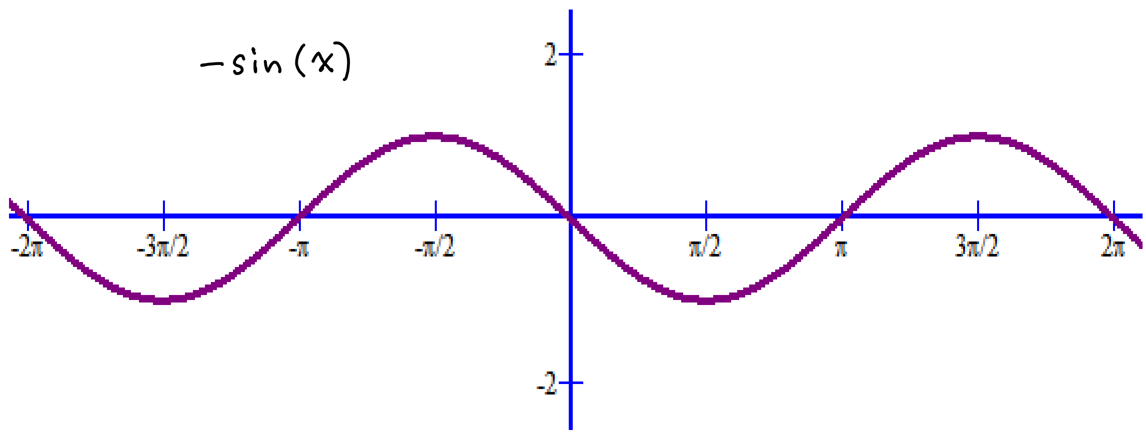
Recall that $\sin(x-\phi)$ is a shifted version of the graph of $\sin(x)$ to the right by ϕ.

Recall that $-\sin(x)$ produces a version of $\sin(x)$ flipped over the x axis.

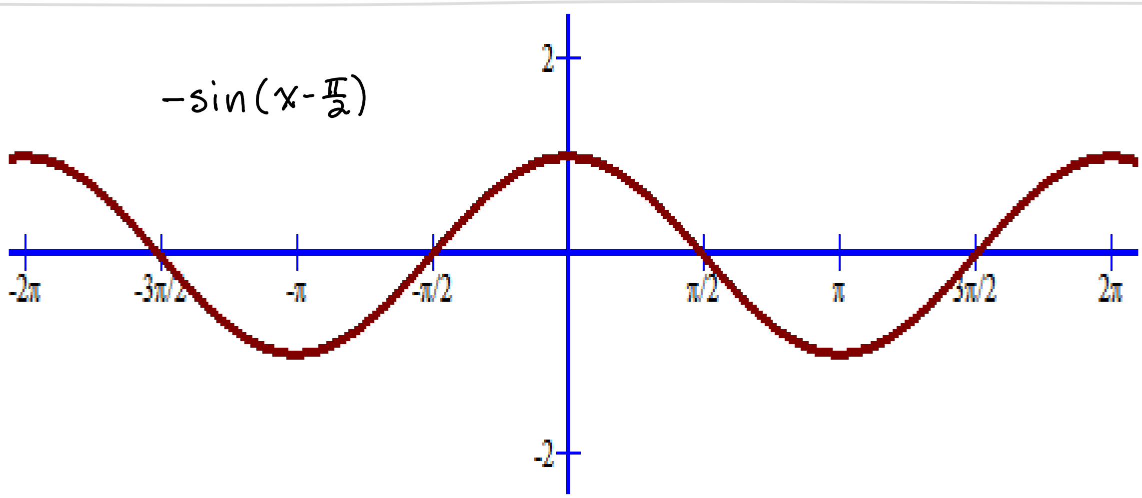
$\sin(\pi/2 - x) = \sin[-(x-\pi/2)] = -\sin(x-\pi/2) = \cos(x)$



Flip $\sin(x)$ over the x axis.



Shift $-\sin(x)$ to the right by $\frac{\pi}{2}$.



Compare to $\cos(x)$.

Likewise, $\cos(\pi/2 - x) = \cos[-(x-\pi/2)] = \cos(x-\pi/2) = \sin(x)$

The others are easy to show now that we have the cofunction identities for cosine and sine.

Derivations Of The Other Cofunction Identities

$$\tan\left(\frac{\pi}{2}-x\right) = \frac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)} = \frac{\cos(x)}{\sin(x)} = \cot(x)$$

$$\cot\left(\frac{\pi}{2}-x\right) = \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)} = \frac{\sin(x)}{\cos(x)} = \tan(x)$$

$$\sec\left(\frac{\pi}{2}-x\right) = \frac{1}{\cos\left(\frac{\pi}{2}-x\right)} = \frac{1}{\sin(x)} = \csc(x)$$

$$\csc\left(\frac{\pi}{2}-x\right) = \frac{1}{\sin\left(\frac{\pi}{2}-x\right)} = \frac{1}{\cos(x)} = \sec(x)$$