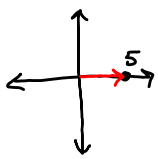
Part Three: Rectangular To Polar Examples

Examples: Conversion From Standard To Polar Form:

1) Convert $z = 5$ to polar form.

$z = 5 = 5 + 0i$ has a radius of 5 and an angle zero: $r = 5$, $\theta = 0$



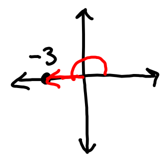
$\Rightarrow \boxed{z = 5 = 5[\cos(0) + i\sin(0)]}$

You can convert back to standard form simply by evaluating the trig.

2) Convert $z = -3$ to polar form.

$z = -3 = -3 + 0i$ has a radius of $|-3| = 3$ and an angle π : $r = 3$, $\theta = \pi$

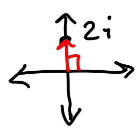
$\Rightarrow \boxed{z = -3 = 3[\cos(\pi) + i\sin(\pi)]}$



3) Convert $z = 2i$ to polar form.

$z = 2i = 0 + 2i$ has a radius of $|2i| = 2$ and an angle $\frac{\pi}{2}$: $r = 2$, $\theta = \frac{\pi}{2}$

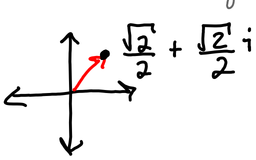
$\Rightarrow \boxed{z = 2i = 2[\cos(\pi/2) + i\sin(\pi/2)]}$



But those were trivial. Let's try some a little more meaty.

4) Convert $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ to polar form.

$z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ has a radius of 1. This should be obvious since you know $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is a point on your unit circle. Remember, its called the unit circle because it has a radius of 1. You should also recognize that it has an angle of $\pi/4$.



$\Rightarrow \boxed{z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = 1[\cos(\pi/4) + i\sin(\pi/4)]}$

5) Convert $z = \sqrt{2} + \sqrt{2}i$ to polar form.

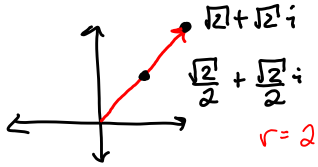
z has a radius $r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$ and an angle of $\pi/4$. We can tell the angle is $\pi/4$ because $x = y = \sqrt{2}$. $r = 2$, $\theta = \pi/4 \Rightarrow \boxed{z = \sqrt{2} + \sqrt{2}i = 2[\cos(\pi/4) + i\sin(\pi/4)]}$

5) Yes, #5 again. Convert $z = \sqrt{2} + \sqrt{2}\,i$ to polar form.

We need to be able to recognize those complex numbers related to the numbers we see on the unit circle.

$z = \sqrt{2} + \sqrt{2}\,i = 2\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right]$ You should know $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ has a radius of 1

so $2\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right]$ has a radius of 2 and the angle doesn't change. It's just twice as far away.



$r = 2,\ \theta = \pi/4 \Rightarrow \boxed{z = \sqrt{2} + \sqrt{2}\,i = 2[\cos(\pi/4) + i\sin(\pi/4)]}$

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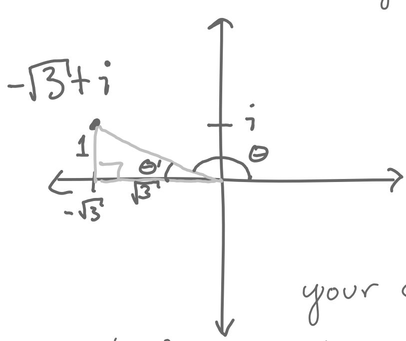
6) Convert $z = -\sqrt{3} + i$ to polar form.

$r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{3})^2 + (1)^2} = \sqrt{4} = 2$ A common error is to include the i with the y when using the Pythagorean Theorem. This is an easy mistake. Remember, $z = x + yi$ where the x and y are REAL numbers. Don't include the i with the y when plugging into the Pythagorean Theorem. This error is most commonly made when the imaginary part is just i without a number in front. Remember, $i = 1i$, your y is 1. Now let's find the angle.



We can use tangent to get θ' and then find $\theta = \pi - \theta'$ for this example.

$\tan\theta' = \frac{opp}{adj} = \frac{1}{\sqrt{3}} \Rightarrow \theta' = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1/2}{\sqrt{3}/2}\right) = \frac{\pi}{6}$

So $\theta = \pi - \theta' = \pi - \frac{\pi}{6} = 5\pi/6$. Sometimes you'll need to use your calculator to find the inverse tangent.

Instead of using the triangle definition of tangent ($\tan\theta = opp/adj$) we could have used its circle definition: $\tan\theta = y/x$.

$\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{1/2}{\sqrt{3}/2} \Rightarrow \theta' = \tan^{-1}\left(-\frac{1/2}{\sqrt{3}/2}\right) = -\frac{\pi}{6}$ Remember, the equation $\tan\theta = -\frac{1}{\sqrt{3}}$ has an infinite number of solutions, but the range of inverse tangent is $(-\frac{\pi}{2}, \frac{\pi}{2})$. $-\pi/6$ is <u>not</u> our angle. When in doubt, look at the quadrant your number is in and compare it to your angle. Here $\theta = -\frac{\pi}{6} + \pi = 5\pi/6$. So, $\boxed{z = -\sqrt{3} + i = 2\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right]}$.

6) Yes, #6 again. Convert $z = -\sqrt{3} + i$ to polar form.

Notice in the previous solution we did not need to use a calculator to find θ. All of the tangent analysis was really unnecessary.

$z = -\sqrt{3} + i = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ reveals we are merely multiplying a known point on the unit circle by 2. $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ has an angle $\frac{5\pi}{6}$ and radius 1 so

$2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ has a radius 2 and angle $\theta = \frac{5\pi}{6}$.

So, $\boxed{z = -\sqrt{3} + i = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2\left[\cos(5\pi/6) + i\sin(5\pi/6)\right]}$

7) Convert $z = \frac{7}{12} - \frac{7}{12\sqrt{3}}i$ to polar form.

Tip: If the number doesn't look familiar, it may still be related to something familiar on the unit circle. Always find your radius first. Then factor the radius out of your number to see if the remaining part (with radius 1) resembles something familiar from the unit circle, thus making it easier to find the angle.

$z = \frac{7}{12} - \frac{7}{12\sqrt{3}}i \Rightarrow r = \sqrt{\left(\frac{7}{12}\right)^2 + \left(\frac{7}{12\sqrt{3}}\right)^2} = \sqrt{\frac{49}{144} + \frac{49}{3\cdot 144}} = \sqrt{\frac{3\cdot 49}{3\cdot 144} + \frac{49}{3\cdot 144}} = \sqrt{\frac{4\cdot 49}{3\cdot 144}} = \frac{2\cdot 7}{12\sqrt{3}} = \frac{7}{6\sqrt{3}}$

$\Rightarrow z = \frac{7}{6\sqrt{3}}\left[\frac{\sqrt{3}}{2} - \frac{i}{2}\right]$ and $\frac{\sqrt{3}}{2} - \frac{i}{2}$ has an angle $-\frac{\pi}{6}$ so, so does z.

$r = \frac{7}{6\sqrt{3}}$, $\theta = -\frac{\pi}{6} \Rightarrow$ $\boxed{z = \frac{7}{12} - \frac{7}{12\sqrt{3}}i = \frac{7}{6\sqrt{3}}\left[\frac{\sqrt{3}}{2} - \frac{i}{2}\right] = \frac{7}{6\sqrt{3}}\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]}$