Fundamental Theorem Of Algebra

$\boxed{\text{A polynomial of n-th degree has n factors}}$

First off, I'll make sure you know what a polynomial is. You've seen plenty of examples.

$3x^2+2x+1$, $4z^9$, $16y^4+2$, $999t^{326}+99t^{26}+9t^6$, --- etc

Here is some notation that you should embrace. A polynomial has the form

$$\sum_{i=0}^{n} a_i x^i = a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$$

The i here is NOT $\sqrt{-1}$. It is an index. The $\sum$ symbol means "sum". Plug in values of i from 1 to n to get the sum $a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$.

All regular polynomials have this form. The examples above are written in reverse order. Let's look at some of them.

$3x^2+2x+7 = 7+2x+3x^2$ So $a_0 = 7$, $a_1 = 2$, and $a_2 = 3$

$x^0 = 1$

For $999t^{326}+99t^{26}+9t^6$, $a_{326} = 999$, $a_{26} = 99$, $a_6 = 6$

What about $a_1, a_2, \ldots$? They're all equal to zero.

$16y^4+2 = 16y^4+0y^3+0y^2+0y^1+2y^0$.

Now, n factors means n solutions to a polynomial equation such at

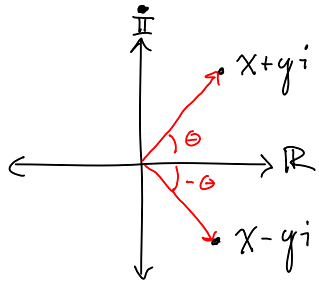
$z^4-16=0$ The 4 solutions are $\pm 2, \pm 2i$, so the 4 factors are

$(z-2)(z+2)(z-2i)(z+2i)=0$

Go ahead and multiply these together and you'll get $z^4-16=0$.

Or factor $z^4-16 = (z^2-4)(z^2+4) = (z-2)(z+2)(z-2i)(z+2i)$

Notice $2i$ came with its complex conjugate $-2i$. Also, $2 = 2+0i$ so $\bar{2} = 2-0i = 2$. So it is its own complex conjugate, $\bar{2}=2$. Likewise, $-2 = -2+0i \Rightarrow -\bar{2} = -2-0i = -2$. So -2 is its own complex conjugate.



To get a complex conjugate, just flip the number over the real axis. So if you're number is ON the axis and you flip it over the axis, you're still on the real axis. So, the complex conjugate of a real number is just that number.

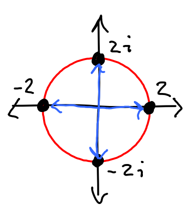
So, the solutions of $z^4 - 16 = 0$ are $2, -2, 2i$, and -2. They seem to come in complex conjugate pairs. $2i$ and $-2i$, 2 is its own conjugate and -2 is its own conjugate.

Any time all of the coefficients of your polynomial are real numbers, your solutions will come in complex conjugate pairs.

The coefficients are the a_i's in $\sum_{i=1}^{n} a_i x^i$. They are the numbers in front of the variable to be solved for.

$16x^4 + 3x^2 - 1 = 0$ will have 4 solutions that come in complex conjugate pairs.

$z^4 + 16 = 0$ will have 4 solutions that come in complex conjugate pairs and will be equally spaced around a circle of $\sqrt[4]{16} = 2$.



$z^4 + 16i = 0$ will have 4 solutions that will <u>NOT</u> come in complex conjugate pairs since $16i$ is not a real number.

But they will be equally spaced around a circle of $\sqrt[4]{16} = 2$ since we are taking a 4th root