Standard Math Tricks

"Smart" One

Multiplying by the number one doesn't change the value:

$1 \cdot 1 = 1 \qquad 2 \cdot 1 = 2 \qquad x \cdot 1 = x \qquad (3y+7) \cdot 1 = 3y+7$

The number one can be represented an infinite number of ways:

$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{x}{x} = \frac{x^2+6}{x^2+6} = \frac{180^\circ}{\pi \text{ radians}} = \frac{12 \text{ inches}}{1 \text{ foot}}$ etc.

The smart one trick is often used to change units:

$\$1 = \$1\left(\frac{100¢}{1\$}\right)$, $30^\circ = 30^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right) = \frac{\pi}{6} \text{ radians}$

The number changes when the unit changes but the value stays the same. One dollar is still worth 100 cents. We've only changed the <u>representation</u> of the value.

The smart one trick is also used to give fractions a common denominator so they may be added or subtracted:

$\frac{2}{3} + \frac{1}{2} = \frac{2}{3}\left(\frac{2}{2}\right) + \frac{1}{2}\left(\frac{3}{3}\right) = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$

They are also used to "rationalize" fractions: $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

Note: "Rationalizing" fractions is not necessary. Grade school teachers often make us feel like only "nonrational" people don't rationalize their fractions.

$\frac{1}{\sqrt{2}}$ IS A NUMBER. IT IS EQUAL TO $\frac{\sqrt{2}}{2}$, JUST AS $\frac{2}{4}$ EQUALS $\frac{1}{2}$.

You should become comfortable with leaving fractions unrationalized if it provides a prettier or more functional answer. I like $\frac{1}{\sqrt{2}}$ better than $\frac{\sqrt{2}}{2}$ because it is quicker to write and is easier to multiply because of the 1 on top. Make sure you are as comfortable with $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ as you are with $\frac{2}{4} = \frac{1}{2}$ and can recognize these kinds of equivalent numbers easily and can interchange them in your head.

"Smart" Zero

Just as multiplying something by the number one doesn't change it's value, adding zero to some thing doesn't change its value either:

$1+0=1$ $\quad x+0=x$ $\quad 3x+7+0=3x+7$ $\quad$ etc.

The number zero can also be represented an infinite number of ways:

$0=0$ $\quad 1-1=0$ $\quad 2-2=0$ $\quad 1+2-3=0$ $\quad x^2+7-7-x^2=0$

Sometimes it is useful to add and subtract the same thing from "something" in order to make that "something" have a desired form:

If you want to prove that $2\sin^2\theta+\cos^2\theta$ is equivalent to $2-\cos^2\theta$

and you know that $2\sin^2\theta+2\cos^2\theta=2$, you might try the following:

$$2\sin^2\theta+\cos^2\theta = 2\sin^2\theta+\cos^2\theta+\cos^2\theta-\cos^2\theta = 2\sin^2\theta+2\cos^2\theta-\cos^2\theta = 2-\cos^2\theta$$

We added and subtracted $\cos^2\theta$ in order to get the fact "$2\sin^2\theta+2\cos^2\theta=2$" linked to the expression we started with.

Always Try To Link What You Don't Know To What You Do Know!

The "Smart" One and "Smart" Zero have an infinite number of uses. These are a couple of basic tools like a hammer and a screwdriver. They're good for more than just nails and screws.

Because of the "unchanging" ability of 1 with multiplication and 0 with addition, they are sometimes called our <u>multiplicative identity</u> and <u>additive identity</u>. You'll see why we have this terminology if you study some Abstract Algebra. Algebra is much more than x's and y's!

"Force" Factor

After passing Algebra, most students have no problem factoring as long as they're using x's and y's and they're factoring a standard problem out of a standard Algebra book. Most students are not aware of how to actually apply the skills they've learned. This is another simple trick that is often used and often confusing to the Algebra novice.

Factoring $2x^2+4x$ is easy: $2x^2+4x = 2x(x+2)$

But when I ask students to factor out an ω from $\omega x - \phi$ I often get the response, "you can't because there isn't an ω with the ϕ.

There doesn't have to be: $\omega x - \phi = \omega\left(x - \frac{\phi}{\omega}\right)$

This is what I call "Force Factoring" because you force an ω out of the expression that isn't there. This is actually just an application of the "Smart One" trick:

$$\omega x - \phi = \omega x - \phi\left(\frac{\omega}{\omega}\right) = \omega\left(x - \frac{\phi}{\omega}\right)$$

We put the ω in and take it out.

<u>Some examples:</u>

$$3\xi + 2x\xi^2 - 4p = z\left(\frac{3\xi}{z} + \frac{2x\xi^2}{z} - \frac{4p}{z}\right)$$

$$1 = x\left(\frac{1}{x}\right)$$

$$2\eta - 3\eta^2 = (x+1)\left(\frac{2\eta}{x+1} - \frac{3\eta^2}{x+1}\right)$$