

# RECIPROCAL, QUOTIENT, AND PYTHAGOREAN IDENTITIES

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## RECIPROCAL IDENTITIES

$$\sin(\theta) := \frac{y}{r} = \frac{1}{r/y} = \frac{1}{\csc(\theta)} \Rightarrow \sin(\theta) \equiv \frac{1}{\csc(\theta)}$$

$$\csc(\theta) := \frac{r}{y} = \frac{1}{y/r} = \frac{1}{\sin(\theta)} \Rightarrow \csc(\theta) \equiv \frac{1}{\sin(\theta)}$$

$$\cos(\theta) := \frac{x}{r} = \frac{1}{r/x} = \frac{1}{\sec(\theta)} \Rightarrow \cos(\theta) \equiv \frac{1}{\sec(\theta)}$$

$$\sec(\theta) := \frac{r}{x} = \frac{1}{x/r} = \frac{1}{\cos(\theta)} \Rightarrow \sec(\theta) \equiv \frac{1}{\cos(\theta)}$$

$$\tan(\theta) := \frac{y}{x} = \frac{1}{x/y} = \frac{1}{\cot(\theta)} \Rightarrow \tan(\theta) \equiv \frac{1}{\cot(\theta)}$$

$$\cot(\theta) := \frac{x}{y} = \frac{1}{y/x} = \frac{1}{\tan(\theta)} \Rightarrow \cot(\theta) \equiv \frac{1}{\tan(\theta)}$$

# QUOTIENT IDENTITIES

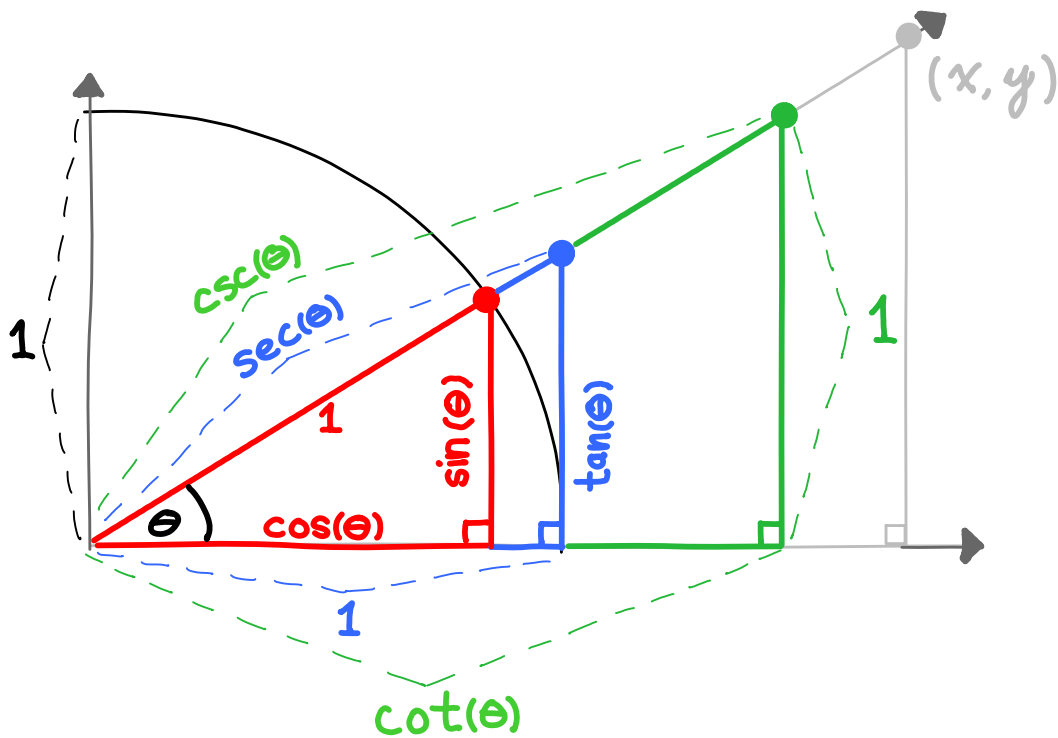
$$\tan(\theta) := \frac{y}{x} = \frac{y/r}{x/r} = \frac{\sin(\theta)}{\cos(\theta)} \Rightarrow \tan(\theta) \equiv \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) \equiv \frac{1}{\tan(\theta)} \equiv \frac{1}{\frac{\sin(\theta)}{\cos(\theta)}} \equiv \frac{\cos(\theta)}{\sin(\theta)} \Rightarrow \cot(\theta) \equiv \frac{\cos(\theta)}{\sin(\theta)}$$

$$\tan(\theta) := \frac{y}{x} = \frac{y/r}{x/r} \cdot \frac{\frac{1}{x/r}}{\frac{1}{y/r}} = \frac{r/x}{r/y} = \frac{\sec(\theta)}{\csc(\theta)} \Rightarrow \tan(\theta) \equiv \frac{\sec(\theta)}{\csc(\theta)}$$

$$\cot(\theta) \equiv \frac{\cos(\theta)}{\sin(\theta)} \equiv \frac{\frac{1}{\sec(\theta)}}{\frac{1}{\csc(\theta)}} \Rightarrow \cot(\theta) \equiv \frac{\csc(\theta)}{\sec(\theta)}$$

# PYTHAGOREAN IDENTITIES



$$x^2 + y^2 = r^2 \Leftrightarrow \frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \Leftrightarrow \cos^2(\theta) + \sin^2(\theta) = 1$$

$$x^2 + y^2 = r^2 \Leftrightarrow \frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2} \Leftrightarrow \cot^2(\theta) + 1 = \csc^2(\theta)$$

$$x^2 + y^2 = r^2 \Leftrightarrow \frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2} \Leftrightarrow 1 + \tan^2(\theta) = \sec^2(\theta)$$