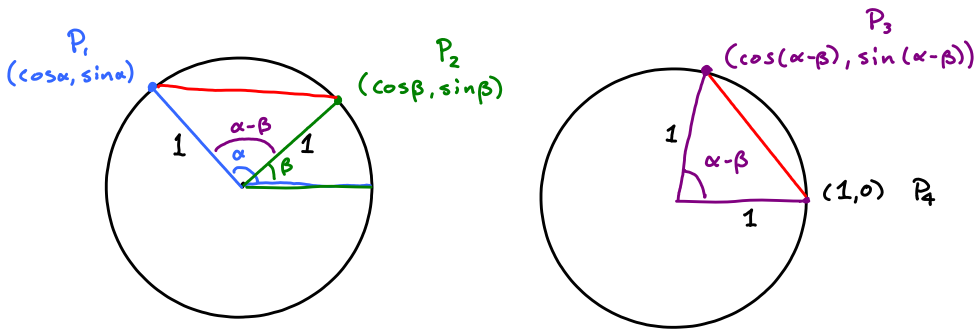
TRIGONOMETRY IDENTITIES DERIVED

Here is how to derive your basic trig identities. The first one is all the work.

Proof that $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



After wondering how to relate $\cos(\alpha-\beta)$ with $\cos\alpha$, $\sin\alpha$, $\cos\beta$, or $\sin\beta$ for a while, a picture is usually helpfull. It lead to finding a connection between them.

The red side of the two triangles above should be equal because the triangles are similar. We know this because we know two sides and the angle between those sides.

We set the length of the red sides equal using the distance formula.

The distance formula gives the distance between two points $P=(x_1,y_1)$ and $Q=(x_2,y_2)$

$$D(P,Q) = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

Notice how this reduces to the Pythagorean Theorem if P or Q is the origin.

So,

$$D(P_1,P_2) = D(P_3,P_4)$$

$$\Leftrightarrow \sqrt{(\cos\alpha-\cos\beta)^2 + (\sin\alpha-\sin\beta)^2} = \sqrt{(\cos(\alpha-\beta)-1)^2 + (\sin(\alpha-\beta)-0)^2}$$

$$\Rightarrow (\cos\alpha-\cos\beta)^2 + (\sin\alpha-\sin\beta)^2 = (\cos(\alpha-\beta)-1)^2 + (\sin(\alpha-\beta)-0)^2$$

$$\Leftrightarrow \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta$$

$$= \cos^2(\alpha-\beta) - 2\cos(\alpha-\beta) + 1 + \sin^2(\alpha-\beta)$$

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$$\Leftrightarrow 2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta = -2\cos(\alpha-\beta) + 2$$

$$\Leftrightarrow -2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta = -2\cos(\alpha-\beta)$$

$$\Leftrightarrow \boxed{\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)}$$

Since $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$ we can solve for $\cos(\alpha+\beta)$

$$\cos(\alpha+\beta) = \cos(\alpha-(-\beta)) = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$$

$$= \boxed{\cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos(\alpha+\beta)}$$

A common misconception occurs here. We know both $D(P_1,P_2)$ and $D(P_3,P_4)$ because we know P_1, P_2, P_3, P_4 and we know the distance formula. Plug the points into the distance formula and set the distances equal to one another. Don't try to derive $D(P_3,P_4)$ from $D(P_1,P_2)$. If you are trying to do this, it is because you have been programmed to work from the left hand side to the right hand side when proving an identity. There is a difference between proving an identity and deriving one. You can prove a given identity by showing that the left hand side indeed equals the right hand side. When deriving an identity, you don't assume to already know what the identity is. You have to find it. We are using the fact that $D(P_1,P_2) = D(P_3,P_4)$ to find out what $\cos(\alpha-\beta)$ is actually equal to in terms of $\cos\alpha$, $\sin\alpha$, $\cos\beta$, and $\sin\beta$.

Now for $\sin(\alpha \pm \beta)$. Remember your identities:

$$\boxed{\sin\left(\tfrac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\tfrac{\pi}{2}-\theta\right) = \sin\theta}$$

Think about the graphs of $\sin(x)$ and $\cos(x)$ and shifting one to be the other.

Using $\sin\theta = \cos\left(\tfrac{\pi}{2}-\theta\right)$ where $\theta = \alpha+\beta$ here, we have

$$\sin(\alpha+\beta) = \cos\left(\tfrac{\pi}{2}-(\alpha+\beta)\right) = \cancel{\cos\left(\tfrac{\pi}{2}\right)}^{0}\cos(\alpha+\beta) + \cancel{\sin\left(\tfrac{\pi}{2}\right)}^{1}\sin(\alpha+\beta)$$

$$= \sin(\alpha+\beta) \quad ?$$

Try again but split the angles differently

$$\sin(\alpha+\beta) = \cos\left(\tfrac{\pi}{2}-(\alpha+\beta)\right) = \cos\left(\left(\tfrac{\pi}{2}-\alpha\right)-\beta\right)$$

$$= \underline{\cos\left(\tfrac{\pi}{2}-\alpha\right)}\cos\beta + \underline{\sin\left(\tfrac{\pi}{2}-\alpha\right)}\sin\beta$$

$\sin\alpha$

$\cos\alpha$

$$= \boxed{\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)} \qquad \text{and}$$

$$\sin(\alpha-\beta) = \sin(\alpha+(-\beta)) = \sin\alpha\cos(\underbrace{-\beta}_{\beta}) + \cos\alpha\sin(-\beta)$$

$$= \boxed{\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha-\beta)}$$

Now for $\tan(\alpha \pm \beta)$:

$$\tan(\alpha\pm\beta) = \frac{\sin(\alpha\pm\beta)}{\cos(\alpha\pm\beta)} = \frac{\sin\alpha\cos\beta \pm \cos\alpha\sin\beta}{\cos\alpha\cos\beta \mp \sin\alpha\sin\beta}$$

Notice the top is + when the bottom is − and vice versa.

$$= \frac{\left(\dfrac{\sin\alpha\cancel{\cos\beta}^{1}}{\cos\alpha\cancel{\cos\beta}} \pm \dfrac{\cancel{\cos\alpha}^{1}\sin\beta}{\cancel{\cos\alpha}\cos\beta}\right)}{\dfrac{\cancel{\cos\alpha\cos\beta}^{1}}{\cancel{\cos\alpha\cos\beta}} \mp \left(\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}\right)}$$

This is its raw form, but since it is a tangent identity, we'll convert things to tangents. Notice the $\cos\alpha\cos\beta$ on the bottom left. It is a convenient factor to use to convert things to tangent.

$$= \boxed{\frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} = \tan(\alpha\pm\beta)}$$

Now try the same technique for cotangent on your own

To summarize your sum and difference formulas:

$$\boxed{\begin{gathered}\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad \sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\ \tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}\end{gathered}}$$

REMEMBER THESE. ESPECIALLY $\sin(\alpha\pm\beta)$ and $\cos(\alpha\pm\beta)$

and don't forget your cofunction identities.

$\sin\left(\tfrac{\pi}{2}-\theta\right) = \cos\theta$ and $\cos\left(\tfrac{\pi}{2}-\theta\right) = \sin\theta$

Now, Double Angle Identies.

We want to write a double angle as trig of single angles.

Link what you don't know to what you do know (The sum formulas).

$$\sin(2\theta) = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta = \boxed{2\sin\theta\cos\theta = \sin(2\theta)}$$

$$\cos(2\theta) = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta = \boxed{\cos^2\theta - \sin^2\theta = \cos(2\theta)}$$

This can be rewritten two different ways by substituting.

$$\cos^2\theta - \sin^2\theta = (1-\sin^2\theta) - \sin^2\theta = \boxed{1 - 2\sin^2\theta = \cos(2\theta)}$$

$$\cos^2\theta - \sin^2\theta = \cos^2\theta - (1-\cos^2\theta) = \boxed{2\cos^2\theta - 1 = \cos(2\theta)}$$

$2\theta = \theta + \theta$
Just use your sum formula with $\alpha = \beta = \theta$

What About Half Angles

Well, relating 2θ to θ isn't much different from relating θ to $\frac{\theta}{2}$. Afterall, 2θ is twice θ and θ is twice $\theta/2$. It's just a matter of perspective. For instance,

$$\sin(2\theta) = 2\sin\theta\cos\theta \Leftrightarrow \begin{cases} \sin\theta = 2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2}) \\ \sin(4\theta) = 2\sin(2\theta)\cos(2\theta) \\ \sin(10\theta) = 2\sin(5\theta)\cos(5\theta) \end{cases}$$

Just relabel the angles.

This angle is twice those angles

So, using our double angles for cosine, we can find our half angle identities

$$\cos(2\theta) = \begin{cases} \cos^2\theta - \sin^2\theta \\ 1 - 2\sin^2\theta \longleftarrow \text{Solve for } \sin^2\theta \text{ to get your half angle for sine} \\ 2\cos^2\theta - 1 \longleftarrow \text{Solve for } \cos^2\theta \text{ to get your half angle for cosine} \end{cases}$$

$$\cos(2\theta) = 1 - 2\sin^2\theta \Leftrightarrow \boxed{\sin^2\theta = \frac{1-\cos(2\theta)}{2}} \quad \text{and}$$

$$\cos(2\theta) = 2\cos^2\theta - 1 \Leftrightarrow \boxed{\cos^2\theta = \frac{1+\cos(2\theta)}{2}}$$

of course we can rename our angles to be more suggestive of the "half" in "half angle formula". So,

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2} \Leftrightarrow \boxed{\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}} \quad \text{and}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2} \Leftrightarrow \boxed{\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2}}$$

or taking the square root of both sides

$$\boxed{\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}} \qquad \boxed{\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}}$$

These are all different forms of the half angle formulas.

Here's A "Chart" Of The Identity Derivations

$\boxed{\cos(2\theta)}$ → sum formula $(\cos\theta\cos\theta - \sin\theta\sin\theta)$ → $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

↓ Pythagorean Theorem

$\cos(2\theta) = 1 - 2\sin^2\theta$

↓ solve for $\sin^2\theta$

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

↓ relabel angles $2\theta \to \theta,\ \theta \to \frac{\theta}{2}$

$\sin^2\left(\frac{\theta}{2}\right) = \dfrac{1-\cos\theta}{2}$

↓ square root both sides

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos(2\theta) = 2\cos^2\theta - 1$

↓ Solve for $\cos^2\theta$

$\cos^2\theta = \dfrac{\cos(2\theta)-1}{2}$

↓ relabel angles $2\theta \to \theta,\ \theta \to \frac{\theta}{2}$

$\cos^2\left(\frac{\theta}{2}\right) = \dfrac{1+\cos\theta}{2}$

↓ square root both sides

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\sin(2\theta)$ → sum formula → $\sin(2\theta) = 2\sin\theta\cos\theta$

We can use these to get a half angle formula for tangent.

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{\pm\sqrt{\frac{1-\cos\theta}{2}}}{\pm\sqrt{\frac{1+\cos\theta}{2}}} = \boxed{\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan\left(\frac{\theta}{2}\right)}$$

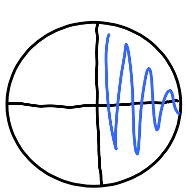
The $\pm$ are still needed because $\sin(\theta/2)$ could be negative when $\cos(\theta/2)$ is positive, or vice versa.

Notice from the chart above that this identity came from the cosine double angle identity alone, $\cos(2\theta)$. Well, ...

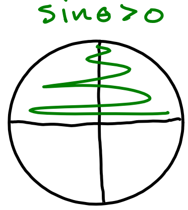
$\cos\theta$ is positive in quadrants 1 and 4 but $\tan\theta > 0$ in quadrant 1 and negative in quadrant 4. So knowing if $\cos\theta$ is positive or negative isn't enough to know if $\tan\theta$ is positive. Similarly, just knowing the sign of sine doesn't resolve the sign of $\tan\theta$.

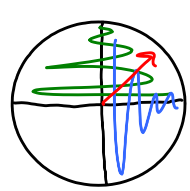
We need knowledge of both $\sin\theta$ and $\cos\theta$ to resolve the quadrant that θ is in and therefore resolve the sign of $\tan\theta$.

$\cos\theta > 0$



$\sin\theta > 0$





We have this information and we can use it to find a better half angle identity for $\tan(\theta/2)$ that doesn't have $\pm$ in front of it.

Here is how we can get a different half angle formula for tangent by combining info from the sine and cosine double angle identities. Remember $\sin(2\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta = 2\cos\theta\sin\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta \iff \sin\theta = 2\sin(\theta/2)\cos(\theta/2)$ (relabeling angles)

same identity

Now, $\tan\left(\frac{\theta}{2}\right) = \dfrac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \left(\dfrac{2}{2}\right)\dfrac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\left(\dfrac{\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\right) = \dfrac{\sin\theta}{2\cos^2(\theta/2)} = \boxed{\dfrac{\sin\theta}{1+\cos\theta} = \tan(\theta/2)}$

we did this to get $\sin\theta$ involved

Here we get $\cos\theta$ involved

$\cos(2\theta) = 2\cos^2\theta - 1 \iff \cos\theta = 2\cos^2(\theta/2) - 1 \iff \cos\theta + 1 = 2\cos^2(\theta/2)$

THINK — DON'T MEMORIZE!

Let's review this last derivation and the actual thought process involved so it doesn't seem far-fetched that someone could think of this.
We wanted to write $\tan(\theta/2)$ in terms of trig of θ alone. We needed to use sine and cosine double angle identities so that we wouldn't end up with a $\pm$ in front like we did when only using the cosine double angle identity.

$\tan(\theta/2) = \dfrac{\sin(\theta/2)}{\cos(\theta/2)} = ?$ We want to relate this to $\cos\theta$ and to $\sin\theta$.
It has half angles.
Two identities relating angles to half angles that come from separate sources are:

Possible Thoughts

$\sin\theta = 2\cos(\theta/2)\sin(\theta/2)$ which came from $\sin(2\theta) = 2\sin\theta\cos\theta$
and
$\cos\theta = 2\cos^2(\theta/2) - 1$ which came from $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\dfrac{\sin(\theta/2)}{\cos(\theta/2)} = \underbrace{\left(\dfrac{2}{2}\right)\dfrac{\sin(\theta/2)}{\cos(\theta/2)}\left(\dfrac{\cos(\theta/2)}{\cos(\theta/2)}\right)}_{\text{numerator}=\sin\theta}$ First, I'll relate $\sin\theta$ by using 2 "smart" ones, $\dfrac{2}{2}$ and $\dfrac{\cos(\theta/2)}{\cos(\theta/2)}$

$= \dfrac{\sin\theta}{\underbrace{\boxed{2\cos^2(\theta/2)}}_{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$

Second, relate $\cos\theta$.
$\cos\theta = 2\cos^2(\theta/2) - 1 \Rightarrow 2\cos^2(\theta/2) = \cos\theta + 1$

We can rewrite this using what I call the "conjugate trick". The conjugate of $1+\cos\theta$ is $1-\cos\theta$. Use this in a "smart one".

$$\tan(\theta/2) = \frac{\sin\theta}{1+\cos\theta} = \left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right) =$$

$$\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}$$

So, $\boxed{\tan(\theta/2) = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}}$

TANGENT HALF ANGLE FORMULAS

Here's the Derivation Diagram for the 2nd version of the Tangent Half Angle Formula

$\cos(2\theta)$

$\downarrow$

sum formula

$\downarrow$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

Pythagorean Theorem substitution

$\cos(2\theta) = 1 - 2\sin^2\theta$ $\qquad$ $\cos(2\theta) = 2\cos^2\theta - 1$

Relabel angles

$\downarrow$

$\cos\theta = \boxed{2\cos^2(\theta/2)} - 1$

$\sin(2\theta)$

$\downarrow$

sum formula

$\downarrow$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\downarrow$

relabel angles

$\downarrow$

$\sin\theta = \boxed{2\sin(\theta/2)\cos(\theta/2)}$

$$\tan(\theta/2) = \frac{\sin(\theta/2)}{\underbrace{\cos(\theta/2)}} = \frac{\boxed{2\boxed{\sin(\theta/2)}\cos(\theta/2)}}{2\boxed{\cos(\theta/2)}\cos(\theta/2)} = \frac{\sin\theta}{\boxed{2\cos^2(\theta/2)}} = \frac{\sin\theta}{\cos\theta + 1}$$

Compare this diagram to the previous one.

Now for Product to Sum Formulas

There are 3 basic varieties:

$\sin\alpha\sin\beta$, $\cos\alpha\cos\beta$, and $\sin\alpha\cos\beta$

We want to write each of these as a sum. Let's start with $\sin\alpha\sin\beta$

Well, $\sin\theta$ has odd symmetry ($\sin(-\theta) = -\sin\theta$), so $\sin\alpha\sin\beta \rightarrow (\text{odd})(\text{odd}) = \text{even}$ is an even function.

Remember, you can derive all these identities using only the Pythagorean Theorem and your sum formulas in one or two steps.
Where does $\sin\alpha\sin\beta$ show up? Well, it's even, so look at an even identity.

Namely, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Aha, so we can just solve for $\sin\alpha\sin\beta$....

$\Rightarrow \sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos\alpha\cos\beta$ .... No, wait. This has a product in it, too.

We need another source of information. Another identity related to $\sin\alpha\sin\beta$ that will help us get rid of the $\cos\alpha\cos\beta$. How about

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ and

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ together

If we add them together, our $\sin\alpha\sin\beta$ will disappear because it's positive in one but negative in the other. We want $\cos\alpha\cos\beta$ to go away, so maybe we should subtract them.

$$\cos(\alpha-\beta) - \cos(\alpha+\beta) = (\cos\alpha\cos\beta + \sin\alpha\sin\beta) - (\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

or writing this in a manner easier on our eyes

$$\begin{array}{rl} & [\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta] \\ - & [\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta] \\ \hline \end{array}$$

$$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha\sin\beta$$

$$\begin{array}{rl} & A = C+D \\ - & B = C-D \\ \hline \end{array}$$

$A - B = C + D - (C - D) = C - C + D + D$

or

$A - B = 2D$

$\Rightarrow$ $\boxed{\frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] = \sin\alpha\sin\beta}$

Use the same trick to get the other 2 versions.

PRODUCT TO SUM FORMULAS

$$\boxed{\begin{aligned} \sin\alpha\sin\beta &= \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \\ \cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)] \\ \sin\alpha\cos\beta &= \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)] \end{aligned}}$$

SUM-TO-PRODUCT FORMULAS

From here it is easy to get the SUM-TO-PRODUCT formulas. Just reverse the Product-To-SUM formulas and relabel the angles with the following method:

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right] \Leftrightarrow \cos(\alpha-\beta)+\cos(\alpha+\beta) = 2\cos\alpha\cos\beta$$

Let $x = \alpha+\beta$ and $y = \alpha-\beta$ so that $\cos(\alpha-\beta)+\cos(\alpha+\beta)$ becomes $\cos y+\cos x = \cos x+\cos y$. Now solve for α and β in terms of x and y:

$$\left.\begin{aligned} x &= \alpha+\beta \\ y &= \alpha-\beta \end{aligned}\right\} \Rightarrow \begin{cases} x+y = 2\alpha \Rightarrow \alpha = \dfrac{x+y}{2} \\ x-y = 2\beta \Rightarrow \beta = \dfrac{x-y}{2} \end{cases}$$

Now, $2\cos\alpha\cos\beta$ becomes $2\cos\left[\frac{x+y}{2}\right]\sin\left[\frac{x-y}{2}\right]$. Put it together,

$$\cos x+\cos y = \cos(\alpha+\beta)+\cos(\alpha-\beta) = 2\cos\alpha\cos\beta = 2\cos\left[\frac{x+y}{2}\right]\cos\left[\frac{x-y}{2}\right]$$

$$\Rightarrow \boxed{\cos x+\cos y = 2\cos\left[\frac{x-y}{2}\right]\cos\left[\frac{x+y}{2}\right]}$$

The others come using the same method and application of the odd and even identities.

SUM TO PRODUCT

$$\sin\alpha+\sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

Now write your own Identity Derivation Tutorial so you never forget.

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